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The Sustaining Frequency

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36N9 GENETICS LLC . Calpine, California

Book Three of the Creature Comforts series

The Omission This Book Closes

Book One taught an operator to **read** the comfort gradient — to notice the chair the dream is sitting in. Book Two taught an engineer to **build** the circumstance: the room, the table, the institution, and then the same instrument turned inward and carried down to the cell.

Book Two laid down both halves of one equation and never multiplied them.

In Volume Two it established the essence bridge — that an interaction's surplus has an exact pitch:

```
f = k . e^(2F)              k = 0.483425732988414 Hz
```

In Volume One it established the fixed point of stored order — where a life, a bond, or an institution arrives if its parameters hold:

```
Q    =  ( η F(u*) − c* ) / δ
 inf
```

Those two statements sat in separate parts of the same book for nine hundred pages without being brought together. Multiply them and the key falls out.

A relation sustains itself when its fixed point is positive — when `η F > c`, which is to say when `F > c/η`. And the bridge inverts: `F = ½ ln(f/k)`. Substitute, and the condition for sustainability is not a ratio, not a policy, and not a virtue. **It is a pitch.**

```
f     =  k . e^( 2c / η )
 min

f > f_min    the relation ACCUMULATES
f = f_min    it HOLDS, and nothing more
f < f_min    it DECAYS, whatever anyone intends
```

Every unit of cost-to-conversion ratio costs 2.885 octaves of required pitch. That is the whole book in one line, and Chapter 4 derives it, tables it, and hands you the diagnostic.

Why it was held back

Neither half was safe to join before the apparatus existed to audit the join. Book Two spent a volume building that apparatus — the validation protocol, the scale requirements, the concordance method — and then used it to demote its own keystone formula. **A book that had produced this result in its first hundred pages would have produced it without the means to check it.** The omission was correct. It is closable now.

What this book adds

Pure frequency mapping of the psyche and of psychological relationship. Three transfer rules carry the essence law into psychology: a state has a frequency, a relationship has a frequency, and a compound has a frequency by the alchemical rule. From those follow the self as a chord, the dyad as an interval, and the octave deficit — a computable reading of whether a bond is paying for itself.

Then **essence alchemy**, audited: which classical operations correspond to real operations on indices and which are empty. Then **the cellular layer**, where narrative is rewritten and the open problem is restated without softening.

What it refuses

It claims no physical resonance. The essence frequencies are essence frequencies — no person, tissue, or molecule is asserted to resonate at any value in this book. It makes no clinical claim, edits no DNA, confers no capability, and hides nothing from any listener. Where a predecessor's formula was demoted for want of a ratio scale, the demotion travels with it here.

And it defines destiny rather than promising it. Destiny in this book means `Qinf` — the fixed point, Book One's tenth node. Changing a destiny means changing `(η, F, c, δ)` and nothing else. There are four levers; the frequency reading says which one binds. **That is a definition, and what it buys is that you can check it.**

Structure

Part	Ch	
I — The Omission	1–4	the two halves, the joining, the minimum sustainable frequency
II — Frequency Mechanics of the Psyche	5–11	indices, moods, traits, the self as a chord, dissonance
III — Frequency Mechanics of Relationship	12–18	the dyad as interval, the sustainable bond, tuning

Part	Ch	
IV — Essence Alchemy	19–24	the operations, solve et coagula, transmutation, the audit
V — The Cellular Rewrite	25–30	narrative, protocol, the open problem, destiny
Appendices	A–F	reference card, tables, protocols, concordance, honesty, correspondence

Part I — The Omission

Chapter 1 . What Was Held Back

There is a result at the centre of this series that could have been written down in three lines on page one hundred of the second book. It was not written down. This chapter is the series' account of why, and it is an account with a number attached, because an apology without arithmetic is just a mood.

The result is this. Book Two established, in its second volume, that an interaction's surplus has an exact pitch — `f = k e^(2F)` with `k = 0.483425732988414 Hz`. Book Two established, in its first volume, that a system's stored order arrives at a fixed point — `Q_inf = (eta F(u*) − c*)/delta`. Both statements are in the same book. They share a variable. Neither section refers to the other. Set the fixed point positive, invert the bridge, and out falls the minimum sustainable frequency:

```
f_min = k * e^( 2c / eta )
```

That is the whole derivation. It is three substitutions and a logarithm. **The join was not withheld because it was hard. It was withheld because a formula this easy to produce is exactly the kind that gets produced before anyone can check it.**

The Law

1.1 The two halves, stated as they stood

Both halves are quoted here in their published form, with their status badges intact, because the whole argument of this chapter is about the difference between the two badges.

HALF ONE — the essence bridge (Book Two, Chapter 30; CANON_SONIC §4).

```
n_eff = N_eff = 1 + (N − 1)u = e^F
f     = k * N_eff^2 = k * e^(2F)
k     = (9/8)^2 / phi^2 = (81/64)/(phi + 1) = 0.483425732988414 Hz
```

Status: the algebra after the identification is exact. The identification of the essence index with the effective count is a badged modelling decision, and the constant `k` is a chosen reference, not a measured one. The square law `f = k n^2` is a postulate about essence structure. The output is an essence frequency and not a physical resonance; no person, tissue, molecule, room or institution is claimed to resonate at any value in this book, here or anywhere after.

HALF TWO — the storage fixed point (Paper E, Theorem 3.5; Book Two, Chapter 44).

```
    Q_{t+1} = (1 − delta) Q_t + eta S_t − C_t        S_t = F(U_t) = ln(1 + (N−
1)U_t)

    with U_t = u∗ and C_t = c∗ constant and delta > 0:

    Q_inf = ( eta F(u∗) − c∗ ) / delta
```

Status: a proven theorem of a linear recursion. Banach's fixed point theorem supplies existence, uniqueness, and convergence from any starting stock, because `delta > 0` makes the map a contraction. Paper E does not hide its own failure mode: when `eta F(u∗) < c∗` the equilibrium is negative, and the theorem's own text says that the regime is unsustainable or the linear model must be replaced.

1.2 The join, and what kind of object it is

LEMMA 1.1 (the join). For `delta > 0` and `eta > 0`,

```
    Q_inf > 0
     <=>  eta F(u∗) − c∗ > 0            (delta > 0, so division preserves sign)
       <=>  F(u∗) > c∗/eta                      (eta > 0)
       <=>  (1/2) ln(f/k) > c∗/eta             (bridge inverted; f = k e^(2F))
       <=>  f > k e^(2c∗/eta)
```

Therefore `f_min := k e^(2c/eta)`, and `f > f_min` accumulates, `f = f_min` holds, `f < f_min` decays. Chapter 3 performs every step with its licensing condition attached; Chapter 4 owns the result and its consequences for the rest of the book.

The object produced is a change of variable and nothing else. It adds no empirical content to either half. It cannot, because it is algebra. What it adds is a *register*: it restates a sustainability condition in a unit — pitch — that humans compare accurately, that composes logarithmically the way the rest of the framework does, and that makes a hard structural ceiling audible instead of abstract.

1.3 The rule this chapter is actually about

THE AUDIT-PRECEDENCE RULE, badged as a house postulate and not as a theorem: a result may not be published above the level of audit available to check it. A result whose inputs are known only ordinally may be published only as an ordering. A result whose inputs are known on a ratio scale may be published as a magnitude. The badge on a result is not decoration; it is the statement of which operations on that result are licensed.

This rule is not a general epistemology and is not offered as one. It is a working constraint adopted by this series, and it is the reason for the shape of all three books. Book One reads. Book Two builds and, having built, audits. Book Three joins, because by then there is something to join with.

1.4 What is proved here and what is not

Proved: the join itself (Lemma 1.1) — trivially, given both halves.

Postulated: the audit-precedence rule; the essence-index identification; the constant k.

Inherited and binding: every honesty clause from Papers A through E and from CANON_SONIC. Chief among them, and repeated because it is the one that governs this chapter: **R1, the Moment equation is demoted on scale.** `D/P` may be used ordinally only — direction and ordering, never a numeric magnitude — pending Rasch instrumentation at 123 items per term. Any psychological frequency derived through `D/P` inherits that demotion, and this book says so in place, every time.

Refused: any clinical, diagnostic or therapeutic claim; any claim of physical resonance; any intervention that removes a person's ability to decline; any content masked or placed below threshold; and any presentation of the (-1) precedent column as evidence.

The Engineering

1.4b Three books, read as one instrument-building sequence

The series has a shape, and the shape is not literary. It is the order in which an instrument gets built.

```
BOOK ONE      READ      the operator learns to notice the gradient
                        twelve nodes; story form; no numbers demanded of the reader
                        output: the ability to see that a circumstance has a direction

BOOK TWO      BUILD     the engineer learns to construct the circumstance
                        ninety chapters; manual form; the storage law, the essence law,
                                  the validation protocol, the concordance method
                        output: an apparatus that can audit a claim, including its author's

BOOK THREE    JOIN      the two halves are multiplied
                                  thirty chapters; the key, not the cabinet
                        output: a computable reading of whether a relation pays for itself
```

Each stage is unusable without its predecessor, and the failure is different at each boundary. A reader who has the join but not the reading has a number with nothing to point it at — they can compute an octave deficit for a relationship they have not learned to perceive as having a direction at all. A reader who has the join but not the build has a number they cannot check, and Book Two exists to teach the checking.

This ordering also explains a structural feature of Book Two that looks accidental and is not. The storage law lives in Volume One and the essence law lives in Volume Two of the same book, nine hundred pages apart, sharing the variable `F`. Chapter 30 of Book Two states plainly that it is a keystone which unifies two arithmetics — the ordinal index within a discipline and the effective count across disciplines — and it goes as far as `one nat of interaction surplus is exactly 2.885 octaves`. **It stops one substitution short.** It converts surplus into pitch and does not ask what pitch a *sustainable* surplus corresponds to. The question is one sentence away in the text and one volume away in the argument.

1.4c Why the missing step was the sustainability step, specifically

There is a reason the omission fell exactly where it did, and it is worth naming because it generalises.

Converting a quantity into a new register is safe. Converting a *condition* into a new register is not. `F -> f` is a relabelling of a measured thing: whatever error `F` carries, `f` carries the same error, exponentiated but faithfully. Nobody acts on `f` alone. But `f > f_min` is a *verdict*. It sorts relations into ones that will accumulate and ones that will decay. It is the kind of statement people act on immediately and check slowly, and it is produced from parameters (`eta`, `c`) that Book Two had not yet established could be measured at all.

A conversion invites a second look. A verdict forecloses one. That is the whole asymmetry, and it is why Chapter 30 could publish the bridge and Chapter 4 of this book had to wait for the audit.

1.5 Why a three-line derivation is dangerous

The danger is specific and it is quantifiable. The join is an exponential in a ratio of two estimated parameters. Differentiate:

```
f_min = k e^(2c/eta)

d ln f_min / d(c/eta) = 2
```

Every unit of error in the cost-to-conversion ratio is two nats of error in the required frequency. In practical units, an error of 0.1 in `c/eta` is a 22.14% error in `f_min`; an error of 0.5 is a factor of 7.39, which is 2.8854 octaves.

Computed, at three operating points:

```
c/eta        f_min interval implied          width      width in octaves
1.0 +/- 0.2   [  2.3944,   5.3289] Hz        2.2255x        1.1542
2.0 +/- 0.1   [ 21.6097,  32.2379] Hz        1.4918x        0.5771
2.0 +/- 0.2   [ 17.6925,  39.3754] Hz        2.2255x        1.1542
2.0 +/- 0.5   [  9.7099,  71.7467] Hz        7.3891x        2.8854
2.4849 +/- 0.2 [46.6626, 103.8495] Hz        2.2255x        1.1542
```

A formula that exponentiates its inputs is only as good as the scale on which those inputs are measured, and the framework's own validation protocol had already found that some of its inputs were not on such a scale.

1.6 The ordinal demonstration — the sharpest form of the argument

Suppose `c/eta` is known only ordinally. That is the honest condition for any parameter estimated from an unanchored rating instrument: you know that site B costs more per unit of conversion than site A, and that site C costs more than site B, and you do not know by how much.

An ordinal scale is preserved by any strictly increasing relabelling. Take three systems ranked `c/eta = (1.0, 2.0, 3.0)` and relabel them `(0.5, 2.0, 5.0)`. Every ordering is preserved; the ordinal content is identical. The frequencies are not:

```
labels (1.0, 2.0, 3.0)  ->  f_min = (    3.5721,  26.3942,    195.0279) Hz
labels (0.5, 2.0, 5.0)  ->  f_min = (    1.3141,  26.3942,  10648.1604) Hz
```

The third system's minimum sustainable frequency moves by a factor of 54.6 — 5.77 octaves — under a relabelling that changes nothing an ordinal instrument can see. This is not a large error bar. It is the absence of a number. An ordinal input to `f_min` does not yield an imprecise frequency; it yields no frequency at all.

This is the technical content of the withholding. Book Two's Chapter 52 built the Formula Validation Protocol; its gate 2 is scale type; gate 2 returned DEMOTE on the Moment equation; and Chapter 54 priced the route out at 123 items per term on a Rasch instrument. Had the join been published before that gate existed, it would have been published without any way of knowing which of the three columns above was the real one.

1.7 The apparatus that had to exist first, item by item

The claim "we needed the apparatus" is empty unless the apparatus is named and unless it can be shown to have caught something. Here it is named, and here is what it caught.

(a) The Formula Validation Protocol, seven gates (Book Two, Chapter 52).

```
  FVP-1  DIMENSIONS       units on both sides agree; ratios dimensionless
 FVP-2  SCALE TYPE     Stevens' typology per term; division needs a ratio scale
 FVP-3  LIMITING CASES  behaviour at 0 and infinity must name a real phenomenon
  FVP-4  FIXED POINTS     solve for the invariant; state what it means
 FVP-5  IDENTIFIABILITY  can the parameters be recovered from obtainable data
      FVP-6    FALSIFICATION        preregister,  freeze,  report  held-
out, beat four baselines
 FVP-7  REPRODUCIBILITY  same operationalisation, different people, different labs
```

`f_min` passes FVP-1 (both sides are hertz; `c/eta` is dimensionless provided both are measured on one clock). It passes FVP-3 and FVP-4 — Chapter 3 §3.11 works the limits. It passes FVP-2 only if `c` and `eta` are ratio-scaled, and that is a condition on the user, not on the formula. It is the formula's most important caveat and it exists only because a protocol was built to find it.

(b) The four-baseline suite, and its three measured holes. Book Two required every empirical claim to beat persistence, direct-input, matched-autoregressive and shuffled-input controls. Then it measured the suite itself, and the suite failed three ways:

```
 R13a  the shuffled-input control is ANTI-CONSERVATIVE as ordinarily run.
       Against nominal alpha = 0.05 over 500 null trials:
           iid permutation    0.108   (2.2x nominal)
           circular shift     0.120   (2.4x nominal)
           block permutation  0.060   (near nominal)
                  The  matched-autoregressive  comparison  alone  false-
passed 35.8% of the time.

 R4   the shuffle cannot see repackaging. A filtered repackaging of the input passed
     all four baselines (p = 0.0005) and was caught only by the structural ceiling
        test, its single-step jump being 38.5x the theoretical maximum.

 R24  the shuffle cannot see a shared slow driver: it passed 98.35% of the time in a
     4,000-trial simulation, while detecting the clean case 48.17% of the time.
```

A book that had published `f_min` early would also have published a validation recipe that roughly doubles its own false-positive rate, and would have invited every reader to certify a frequency reading with it.

(c) The concordance register, with rejections in print. Book Two required each claim to carry a (+1) evidence column and a (-1) precedent column, and required that precedent never be presented as evidence. It then published two rejections — the 64-codon/64-hexagram correspondence, and astrology's orb as a conjugate-spread precedent, the latter with Carlson (*Nature* 318, 1985) cited in the same passage as the definitive failed validity test and criterion C2 marked FAILED. A method that has never rejected anything has not been tested.

(d) The survivorship result, R23. Under a selection process that publishes successes and discards failures, the likelihood ratio of testimonial evidence is exactly `1.000000`. A likelihood ratio of one does not move a posterior weakly. It does not move it at all. A thousand testimonials that a practice "worked" carry the evidential weight of zero testimonials. The companion figure is worth as much: a prediction specifying only a direction measured a likelihood ratio of `2.0000`; one specifying a magnitude measured `11.3577`. A prediction with a number attached is about 5.7 times more evidentially powerful, which is precisely why this book insists on producing a frequency rather than an adjective — and precisely why the frequency has to be defensible.

(e) The scale audit, and the reason it is not a counsel of despair. Stevens' typology sorts measurement into nominal, ordinal, interval and ratio scales, and it states which arithmetic each licenses. Division and multiplication require a ratio scale with a true zero. The audit that demoted the Moment equation is the same audit that licenses the index this book actually uses: n, the number of distinguishable components required to specify a state, is a count. It has a true zero — no components, no state — and equal units, since one component is one component. It is ratio-scaled by construction, and therefore does not inherit the demotion. Chapter 5 makes that argument in full and Chapter 6 defends it against the obvious objection, which is that deciding what counts as one component is itself a judgement.

The audit is also not a claim that subjective quantities cannot be measured. Psychophysics has been measuring them on ratio scales for a century. Stevens' method of magnitude estimation produces genuine ratio-scale measurements of subjective experience from human subjects, and it replicates; the published exponents — loudness at about 0.67, brightness about 0.33, visual length about 1.0, warmth on the arm about 1.6, electric shock about 3.5 — are constants of a procedure, reported as such. The existence proof matters here: the demand for ratio scales is a demand that has been met before, in this exact domain, and the demand for it is therefore not a rhetorical device for refusing to commit.

(f) Two aggregation corrections that would have silently corrupted any group reading (R2, R10, R16). Because `F` is strictly concave, feasibility must be tested on the mean of `F(U_t)` and not on `F` of the mean `U`; the Jensen gap was worth 6.2% of surplus in Book Two's worked office. And distortion does not average: aggregating the doctrine-to-data ratio across two sites cancelled opposite-direction distortion and **overstated bankable input by 4.26x**. R16 adds a third: two nine-member groups with *identical* `u = 0.5999` and `S = 1.7578` lost **77% versus 21%** of their surplus to a three-person departure — aggregate statistics hide the structure that decides the outcome — and comparing internal and external scopes on mismatched clocks produced a **90.1x** period-normalisation trap. `f_min` is a ratio of two per-period quantities. Every one of these traps is live in it, and every one of them was found after Chapter 30 was written.

1.8 Three corrections the apparatus produced that the join would otherwise have inherited

This is the load-bearing part of the argument. The claim is not that caution is a virtue. The claim is that the audit found specific errors, in numbers this book uses, that would have travelled into the keystone had the keystone come first.

Correction one — the minimum-gap cliff was in the wrong place, and the author's own "fix" to the rate factor was wrong and had to be reversed (R21). Re-derived with exact rational arithmetic by folding every `n^2` into a single octave and taking the minimum adjacent ratio including wrap-around:

```
N =  3..6   min gap = 9/8       = 203.9100 cents
N =  7..16  min gap = 50/49     =  34.9756 cents
N >= 17     min gap = 289/288   =   6.0008 cents      <-- the cliff
N >= 41     min gap = 1682/1681 =   1.0296 cents
```

The published location had been `N = 20`; it is `N = 17`, because `17^2 = 289` folds beside `288 = 2 * 12^2` the instant index 17 enters the set. The second threshold is `N = 41`, not `N = 48`. And the rate penalty at the cliff is **exactly 8**, not the `288/49 = 5.878` the author had asserted, because the two binding pairs sit at different octave heights — `50/49` straddles `49/32`, `289/288` straddles `9/8` — so the absolute gap in hertz is `f/32` before the cliff and `f/256` after. Three ratios, all correct in their own units, and all different:

```
   cents  ln(50/49)/ln(289/288) = 5.828475    how much smaller the interval is
    cycles  288/49               = 5.877551     3.899 cycles becomes 22.918
     RATE     (1/32)/(1/256)          = 8 exactly      symbols per second --
the design limit
```

The correction agent verified the factor of 8, refused the author's edit, and documented the derivation so the error could not be reintroduced. That is what an apparatus is for.

Correction two — the source table of the essence law contains data-entry errors, and one of them is provably impossible (R5). `f = k n^2` reproduces all 118 published element values to a worst relative deviation of `9.0e-15`. The compound rule reverse-engineered as `f = k^2 G^4` with `G` the count-weighted geometric mean of the constituent indices matches 37 of 44 published composites to better than one part in `10^12`. The seven that disagree are transcription faults, not evidence of a different law, and the O2 row proves it: for a single-element species `G` is the element index whatever the atom count, so no atom count whatever can produce the published figure. Two further faults travel with the citation — the source's prose instruction `1+5^.5/2` is missing its parentheses and must be read as `(1 + sqrt(5))/2`, and the spreadsheet stored `phi` to fifteen significant figures, which accounts for the whole of the residual bias. The standing rule that follows: do not copy published values, recompute them. Independently verified here, the recipe `((n/phi) * 1.125)^2` agrees with `k n^2` over `n = 1..118` to a worst relative deviation of `4.760e-16`.

Correction three — the intuitive repair for a failing bond makes it worse, and by how much (R15). Computed in Book Two's worked consortium: trust alone moved `Q_inf` from `-0.0836` to `-0.2114`. Reciprocity alone moved it to `+0.7394`; both together to `+1.8701`. The mechanism is exact and follows from Theorem 3.5 without approximation. Trust acts by reducing `delta`. When the numerator is negative, dividing by a smaller positive denominator makes the result more negative.

Trust does not add order; it amplifies whatever sign the system already has.

Each of these three is a number this book uses. Correction one governs how any of these frequencies can be rendered at all. Correction two governs the constant at the centre of every calculation. Correction three governs the single most common intervention a reader will reach for after finishing Chapter 4. A keystone published on page one hundred of Book Two would have carried all three errors forward into a diagnostic that people were meant to act on.

1.9 The counterfactual, stated plainly

Imagine the alternative book. It opens with `f_min = k e^(2c/eta)`. It tables the frequency against the cost ratio. It hands the reader an octave deficit. And it has, at that point:

- no scale audit, so no way to say whether `c/eta` is a magnitude or a rank;
- a validation suite that doubles its own false-positive rate, unmeasured;
- a rendering theory that puts the resolution cliff three categories too late and misprices the penalty by 36%;
- a constant taken from a spreadsheet with seven bad rows and a truncated `phi`;
- and a recommended first intervention — build trust — that accelerates the failure it is meant to repair, in exactly the regime where a reader is most likely to reach for it.

That book would have been more exciting and less true, and its readers would have had no way to tell the difference. The instrument would have been indistinguishable, from the outside, from the instruments this series exists to replace.

1.9b Three questions to ask of any keystone, including this one

The audit-precedence rule is only useful if it can be applied from outside. Here it is, reduced to three questions a reader can put to any compact, powerful-looking formula — this book's included.

QUESTION ONE: what scale are the inputs on, and who says so? If the answer is "ratio, and here is the instrument," the magnitude is reportable. If the answer is "ordinal," or if there is no answer, the ordering is reportable and the magnitude is not. A formula that divides two quantities and cannot name the true zero of either is producing a rank dressed as a number. Run the relabelling test of §1.6 if in doubt: it takes one line and it is decisive.

QUESTION TWO: what has this method rejected? Not what it has explained — what it has thrown out. A method with no rejection history has never been in a position to reject. This series' answer is on the record: the author's own Moment equation, demoted; the author's own rate-factor correction, reversed to a factor of exactly 8; the 64-codon/64-hexagram correspondence, refused; astrology's orb as a conjugate-spread precedent, with criterion C2 marked FAILED in the same passage that cites Carlson (*Nature* 318, 1985); and the evidential reading of gematria, killed by a 99.995% collision rate.

QUESTION THREE: how fast does the formula amplify error? Take the logarithmic derivative with respect to each input. For `f_min` the answer is 2 — every unit of error in `c/eta` is two nats of error in the output — and a formula with that property must ship with its sensitivity, not just its point estimate. A formula that amplifies and does not say so is inviting confident wrong answers, and confident wrong answers are the specific harm that this chapter's delay was intended to avoid.

A book that can survive its own three questions is not thereby correct. It is checkable, which is the most any book can offer, and it is a great deal more than most offer.

1.10 What the withholding cost, and the series does not pretend it was free

Two books of readers did not have the diagnostic. People who could have used an octave deficit to see that a bond was structurally short — not badly intentioned, not insufficiently loving, structurally short by a computable number of octaves — did not have it. That is a real cost, it was paid by real readers, and it is not erased by the fact that the delay was correct.

The series also does not claim the omission was foreseen in every detail from the beginning. **Some of it was design and some of it was discovery.** The design part: Book Two's structure deliberately separates the storage law (Volume One) from the essence law (Volume Two), and Chapter 30 says in its own text that it is a keystone which unifies two arithmetics. The discovery part: the reconciliation ledger exists because findings emerged *during* drafting and had to be propagated backwards, and several of them — R13, R21, R23, R15 — arrived late enough that the join could not honestly have been made earlier even if the author had wanted to. Both of those are true, and a book that claimed only the first would be doing the thing this chapter is arguing against.

1.11 Why the join can be closed now

Four conditions are now met, and they are the four that were missing.

1. **The scale question has an answer and a price.** R1 states the demotion; Book Two Chapter 54, Theorem 54.3 gives the constructive route — a Rasch logit difference supplies the true zero that division requires — and prices it at 123 items per term. A demoted quantity with a costed route to promotion is a research instruction, not a caveat.
2. The validation suite has been measured against itself, its three holes are published, and the replacement rule is stated: design controls, not more surrogates; structural tests for repackaging; the ceiling test as the only detector of an impossible jump.
3. The rendering theory is correct and its limits are exact: the cliff at `N = 17`, the second at `N = 41`, the rate factor of 8, the single-transposition audible-set ceiling at `N <= 25`, and the Gabor bound `dt * df >= 1/(4 pi)` binding every rendering.
4. The method has demonstrated that it rejects. Two published rejections, one demoted keystone formula belonging to the author, one reversed self-correction. A method that has visibly overruled its own author can be trusted to overrule a reader's wishful reading, which is what a diagnostic has to survive.

1.12 What this book promises, and the boundary of the promise

It promises a computable, checkable reading of whether a relation is paying for itself, in the form of an octave deficit, together with the exact identification of which of four levers binds.

It promises that every number in it was computed, that every postulate carries its badge in the sentence that states it, and that every case is constructed and labelled as constructed.

It does not promise that the reading is easy to obtain. The reading inherits exactly the defensibility of its inputs. A contested decomposition of the state space yields a contested frequency, and this book says so wherever the index is used.

It does not promise a mechanism. The open problem stands, unsoftened: there is no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal. Chapter 27 restates it in full and the book ends on it rather than on a triumph.

It does not promise that anything resonates. The frequencies are essence frequencies. This is not a hedge; it is a statement of what kind of object the output is. A pitch here is a position on a logarithmic scale of surplus, and the scale is logarithmic because the framework is logarithmic, not because anything is vibrating.

And it defines destiny rather than offering it. Destiny in this book means `Q_inf`. Changing a destiny means changing `(eta, F, c, delta)` and nothing else. That is a definition, and what a definition buys you is that you can check it.

The Specification

These requirements bind every chapter of this book. They are numbered so that later chapters can cite them.

SPEC 1.1 — SCALE DECLARATION. Any chapter that produces a numeric `f_min`, `f`, or octave deficit must state, in the same passage, the scale status of `c`, `eta`, and `u` as used there. If any input is ordinal, the output is published as an ordering and not as a magnitude.

SPEC 1.2 — R1 TRAVELS. Any quantity derived through `D/P` carries the demotion of R1 explicitly, with the Rasch route (Theorem 54.3, 123 items per term) named as the way out.

SPEC 1.3 — ESSENCE, NOT RESONANCE. Every chapter that prints a frequency restates that it is an essence frequency and that no physical resonance of any person, tissue, molecule, room or institution is claimed.

SPEC 1.4 — NO CLINICAL CLAIM. No chapter presents the twelve states as diagnoses, the readings as assessments, or any procedure as treatment. Where a reader might mistake them, the chapter says so in place.

SPEC 1.5 — CONSTRUCTED CASES LABELLED. Every worked case in this book is constructed and is labelled as constructed in the sentence that introduces it. No constructed case is presented as an observation, and no observation is invented.

SPEC 1.6 — RECOMPUTE, DO NOT COPY. No number is taken from a published table, including this series' own tables. Every figure is recomputed and the recomputation is what appears. Where a recomputation disagrees with a source, the disagreement is printed with the reason (R5 is the model).

SPEC 1.7 — BADGE IN PLACE. Every postulate is badged in the sentence that states it, not in a footnote, not in an appendix, and not once at the front of the book.

SPEC 1.8 — PUBLISH REJECTIONS. Any correspondence, analogy, or borrowed structure that fails its criterion is printed as failed, in the chapter that tested it, with the criterion named.

SPEC 1.9 — PRECEDENT IS NOT EVIDENCE. Dual concordance where a claim invites it: (+1) carries evidence, (-1) carries precedent. The (-1) column is never promoted, and R23's likelihood ratio of 1.000000 is the reason.

SPEC 1.10 — CONSENT IS STRUCTURAL. No intervention described in this book may remove a person's ability to decline. No content is masked, placed below threshold, or concealed from the person receiving it. This is absolute and admits no exception for benefit.

SPEC 1.11 — THE OPEN PROBLEM IS CARRIED. Every chapter touching the cellular layer restates that there is no established mechanism for sequence-specific acoustic demodulation.

SPEC 1.12 — PRECISION MATCHES PROVENANCE. A figure is printed to the precision its inputs support. Where a parameter is a design choice with one or two significant figures, the derived frequency is reported with its sensitivity, not with four decimal places pretending to be measurement.

SPEC 1.13 — CITE, DO NOT RE-DERIVE. Where SPINE §4 assigns ownership of an object to a chapter, every other chapter cites that chapter rather than restating the derivation. Chapter 4 owns `f_min`, the octave deficit, and the four levers for the whole book.

SPEC 1.14 — ONE CARRIED CASE. Every chapter that needs an example uses the carried case of SPINE §3 or an explicitly labelled extension of it. No chapter invents a fresh case, because a book that selects a new example for every claim is selecting examples that work.

The Failure Modes

FM 1.1 — THE PREMATURE KEYSTONE. *Signature:* a compact, memorable formula published ahead of the instrumentation for its inputs; typically accompanied by a table of impressive-looking values to four decimal places. *Mechanism:* the formula's inputs are estimated on scales that do not license the operations performed on them, and the exponential amplifies the resulting error by a factor of two per nat. *Fix:* publish the ordering, withhold the magnitude, and state the instrument that would license the magnitude and what it costs.

FM 1.2 — RETROSPECTIVE INFALLIBILITY. *Signature:* an author's account of their own construction in which nothing was ever discovered, only unveiled. *Mechanism:* the narrative of design is more flattering than the narrative of correction, and it also destroys the reader's ability to calibrate how much of the work was checked and how much was planned. *Fix:* keep the reconciliation ledger in print. This series does: twenty-four numbered findings, several of which reversed the author's own corrections. Chapter 1 §1.10 states which part of the omission was design and which was discovery, and refuses to claim the second as the first.

FM 1.3 — AUDIT THEATRE. *Signature:* a validation protocol that is described at length and never returns a negative result. *Mechanism:* gates set below the level of anything the work actually does. *Fix:* the protocol must have demoted something the author cared about. In this series it demoted the author's own Moment equation (R1), reversed the author's own rate-factor correction (R21), and killed the evidential reading of gematria by arithmetic (R12: essentially every word in an 8,000-word corpus has a value-twin, `p = 0.9997`; across 234,454 dictionary words the map yields 305 distinct outputs, a 99.995% collision rate destroying 10.47 bits). A hash with that collision rate cannot carry evidence; it can carry a memory index, and the mnemonic reading survives intact.

FM 1.4 — SECRECY DRESSED AS PRUDENCE. *Signature:* a withheld result whose withholding is never scheduled to end, and whose value is asserted rather than shown. *Mechanism:* the esoteric tradition's characteristic epistemic failure — secrecy prevents the error-correction that

would have caught the error, and then the absence of correction is read as the absence of error. Chapter 24 audits this at length. *Fix:* a withholding must come with the condition for its release stated in advance and checkable from outside. §1.11's four conditions are that statement for this series. This chapter exists so that the withholding was a schedule and not a mystery.

FM 1.5 — THE ORDINAL SMUGGLE. *Signature:* a ratio computed from two rank-ordered quantities, reported as a number. *Mechanism:* division requires a ratio scale with a true zero; ranks have neither. The output has the form of a magnitude and the content of an ordering. *Fix:* FVP-2. And when it returns DEMOTE, print the demotion where the formula appears. §1.6's relabelling — `f_min` moving from 195.0279 Hz to 10648.1604 Hz under an order-preserving relabel — is the diagnostic to run when in doubt.

FM 1.6 — THE PRECISION CASCADE. *Signature:* `f_min = 30.1588 Hz` derived from `eta = 0.3` and `c = 0.62`. *Mechanism:* the arithmetic is exact, so the output carries as many digits as you ask for, regardless of what the inputs support. *Fix:* SPEC 1.12. Print the sensitivity beside the value. In this book, worked cases are constructed, so their parameters are exact *by construction* and the digits are honest — but that is a property of a constructed case and does not transfer to a measured one.

FM 1.7 — CITING THE TABLE INSTEAD OF THE LAW. *Signature:* a value quoted from the 2022 source document rather than recomputed. *Mechanism:* seven of the 44 published composites are data-entry errors, and `phi` was stored at fifteen significant figures. *Fix:* SPEC 1.6, and cite the errata whenever the source is cited.

FM 1.8 — THE NEGATIVE STOCK. *Signature:* a reported `Q_inf` of `−5.0119` treated as a quantity of stored order that exists and is negative. *Mechanism:* the fixed point is the attractor of a linear recursion. When the numerator is negative the recursion drives `Q` below zero and violates Paper E's own admissibility condition `C_t <= (1−delta)Q_t + eta S_t`, at which point the linear model no longer describes anything. *Fix:* read a negative `Q_inf` as a direction and a rate, not a destination: the system is losing order at a computable rate and the linear model will stop applying before it arrives. Chapter 3 §3.14 computes exactly when admissibility breaks in the carried case — at `t = 1`.

FM 1.9 — THE REGISTER SLIP. *Signature:* a sentence that begins by saying a relationship has a frequency and ends by recommending that someone listen to it. *Mechanism:* the word "frequency" carries an acoustic connotation that the formalism does not supply. The output of `f = k e^(2F)` is a position on a logarithmic scale of surplus. Rendering it as sound is a separate, optional act governed by Chapter 26's protocol, the Gabor bound, and the no-subliminals rule. *Fix:* say *essence frequency* in the sentence, every time, and keep rendering claims in the chapters that own rendering. A rendered tone is a display of a computed quantity, in the same way that a thermometer's mercury column is a display of a temperature and not the temperature.

FM 1.10 — THE SINGLE-INSTANCE PROOF. *Signature:* a keystone demonstrated on one flattering example. *Mechanism:* a case chosen after the result is known will fit the result. *Fix:* SPEC 1.14 — one carried case, fixed in advance, used everywhere, **including where it fails.** The carried case of this book contains a bond that decays, a repair that works, and a six-member commons that no single lever can save. A book that only showed fixable examples would be lying by selection, and the lie would be invisible because every individual number in it would be correct.

Worked Example

THE CARRIED CASE, READ TWICE — WITH THE APPARATUS AND WITHOUT

This case is CONSTRUCTED. It is the dyad of SPINE §3.2 and it is used throughout this book. Its parameters are chosen, not observed. No claim is made that any real pair of people has these values, and nothing here is an assessment of anyone.

Two people, A and B. Each is represented as a unit vector over the twelve-state ladder, components being the intensity with which the state is held, normalised to length one. The construction is Chapter 5's and the elicitation is Chapter 6's; here the vectors are simply given.

```
A = {4: 0.7, 6: 0.5, 9: 0.4}
B = {4: 0.6, 6: 0.55, 9: 0.3, 10: 0.2}
```

Computed:

```
cos theta = 0.965021
u         = 1 − cos^2 theta = 0.068735
F         = ln(1 + 11u)     = 0.563085 nats
N_eff     = 1 + 11u         = 1.7561
  f            = k (1 + 11u)^2    = 1.4908 Hz        (folded to 220–
440: 381.644 Hz, j = 8)
```

The pair agree about nearly everything. Their represented states point 0.965 of the way in the same direction. Two people who almost coincide have almost no surplus between them, and their relationship sits 1.6247 octaves above the ground essence out of a possible 7.1699.

Now the operating parameters. **Constructed:**

```
eta   = 0.30      conversion of interaction into durable structure
c     = 0.62      cost per period, same clock as eta
delta = 0.09      retention loss per period
```

Reading it WITH the apparatus

`c/eta = 0.62/0.30 = 2.066667`, and the first thing the apparatus does is compare that to the ceiling.

```
ceiling for N = 12:   c/eta <= ln 12 = 2.484907
this case:        c/eta  = 2.066667    −− below the ceiling, so a tuning problem
```

Then the reading:

```
f_min        = k e^(2 ∗ 2.066667)  = 30.1588 Hz     (folded 241.270, j = 3)
 octaves required = 2.885390 ∗ 2.066667 =  5.963140
 octaves present  = log2(f/k)           =  1.624719
OCTAVE DEFICIT                        = +4.338420      (= +5206.1 cents)
 Q_inf = (0.30 ∗ 0.563085 − 0.62)/0.09  = −5.0119          DECAYS
```

The bond is short by 4.3384 octaves. It is not short of goodwill; it is short of pitch, and pitch here means available order.

Reading it WITHOUT the apparatus

Now suppose the same analyst has the same three-line formula and no scale audit. `eta` and `c` were estimated from a rating instrument that produces ranks. The analyst's honest uncertainty on `c/eta` is about `+/- 0.5` — which, for an ordinal instrument, is generous, since strictly the uncertainty is unbounded.

Take the three points of that interval:

```
   c/eta       f_min (Hz)   deficit    below ceiling?   prescription
  1.566667      11.0948    +2.8957       yes          raise u to >= 0.344605
  2.066667      30.1588    +4.3384       yes          raise u to >= 0.627132
 2.566667    81.9801   +5.7811     NO       no arrangement of 12 categories works;
                                                 N must rise to at least 14
```

Three things follow, and they are the point of this chapter.

First: the verdict survives. At every point in the interval the bond decays. `f = 1.4908 Hz` sits below the smallest `f_min` in the range by a wide margin. **A coarse instrument is enough to tell you that something is wrong.**

Second: the prescription does not survive. At the low end the repair is modest — raise `u` from 0.0687 to 0.3446, a five-fold increase in represented difference. At the centre it is a nine-fold increase to 0.6271. At the high end there is no value of `u` that works at all, because `c/eta = 2.566667` exceeds `ln 12 = 2.484907` and `F` cannot exceed `ln N`. The prescription flips from *tune this* to *restructure this*, and the flip happens inside the analyst's own error bar.

Third: the two prescriptions are not merely different in degree. They are different in kind. "Increase the difference between you" and "twelve categories are not enough to describe what you are doing, find at least fourteen" are not two settings of one dial. They are different projects, with different costs, aimed at different objects. An instrument that cannot tell them apart is not a weak instrument; it is a misleading one, because it returns a confident number either way.

The same case with the apparatus present

With `c` and `eta` known as magnitudes — which, in a constructed case, they are by construction — the reading is unambiguous, the ceiling comparison is decisive, and the lever analysis is determinate. Chapter 4's Worked Example runs all four levers on this case and finds that no single lever is comfortable: `u` must rise nine-fold, `eta` would have to exceed `1.101077` which is outside its admissible range, `c` must fall 72.8%, and `N` would have to reach 102. **The honest answer is a combination, and the arc that Chapter 4 carries — fix the numerator, then lower the loss — is what that combination looks like when it is ordered correctly.**

The intervention the unaudited book would have recommended

There is one more thing worth running, because it is the intervention a reader reaches for unprompted, and because it is the sharpest available demonstration that the apparatus was doing work.

The analyst, seeing a decaying bond, proposes building trust. Trust in this framework acts on `delta`: it reduces the rate at which what has been built leaks away. Book Two Chapter 49 gives

the mechanism as `delta_eff = delta_0 + (1−a)(1−v)`. Suppose the intervention succeeds completely and moves `delta` from 0.09 to 0.05.

```
before:  Q_inf = (0.30 * 0.563085 − 0.62) / 0.09 = −5.0119
after:   Q_inf = (0.30 * 0.563085 − 0.62) / 0.05 = −9.0215
```

The successful intervention made the destination 80% worse. The numerator `eta F − c = −0.451075` is negative; dividing a negative number by a smaller positive denominator makes it more negative. Trust did not add order. It amplified the sign the system already had, and the system's sign was negative.

And here is the part that belongs specifically to this chapter. The octave deficit did not move at all. It was `+4.338420` before the intervention and `+4.338420` after, because the deficit — as Chapter 4 §4.8 proves — contains no `delta` term whatsoever:

```
deficit = 2.885390 * (c/eta) − log2(f/k)        no delta anywhere
```

An instrument that reads unchanged while a well-intentioned intervention doubles the rate of loss is not a broken instrument. It is the instrument telling you that you have not touched the thing that is wrong. A reader with the join and without R15 would have read a stationary deficit as evidence that the intervention needed more time. A reader with both reads it correctly the first time: the numerator is the problem, and nothing done to the denominator will ever address it.

The one-line moral, with its number

```
d ln f_min / d(c/eta) = 2
```

A formula that exponentiates its inputs at a rate of two nats per unit cannot be published above the level of audit available for those inputs. The join is three lines. The apparatus that makes those three lines mean anything is a volume long. The order was not caution. It was arithmetic.

Chapter 2 restates both halves in full, with their derivations and every honesty clause, so that this book stands alone. Chapter 3 performs the join step by step and verifies it at twelve parameter sets. Chapter 4 owns the result.

Chapter 2 . The Two Halves

This chapter exists so that this book stands alone. Everything Chapter 3 joins and Chapter 4 owns is stated here in full: the two halves themselves, the constraint that binds their rendering, and — in every case — what each law is *proved* to do and what it does *not* do.

That last clause is not a formality. **A law's honesty clauses are part of the law.** A theorem detached from the conditions under which it was proved is not a compressed theorem; it is a different and false statement. So each of the four laws below is presented in three registers: the statement, the derivation or provenance, and the boundary. The boundary sections are as long as the statement sections, and that ratio is deliberate.

Four laws are needed.

```
   THE ESSENCE LAW      f = k n^2                       an index has a pitch
   THE SURPLUS LAW      F = ln(1 + (N−1)u)              a pair has a surplus
  THE STORAGE LAW    Q_{t+1} = (1−d)Q_t + eta S − C  surplus becomes structure, and leaks
  THE GABOR LIMIT    dt ∗ df >= 1/(4 pi)          what may be rendered, and how fast
```

The first two multiply to give the essence bridge. The third gives the fixed point. Chapter 3 multiplies the bridge by the fixed point. The fourth binds what can be done with the answer.

The Law

2.1 THE ESSENCE LAW — statement

```
f(n) = k ∗ n^2

k = (9/8)^2 / phi^2 = (81/64)/(phi + 1) = 0.483425732988414 Hz

n = the ESSENCE INDEX: the ordinal position of an essence type within its
    discipline's ordered category set
```

`phi` is the golden ratio, `(1 + sqrt(5))/2 = 1.618033988749895`. Because `phi^2 = phi + 1` exactly — verified here to machine zero, `phi^2 − phi − 1 = 0.000e+00` in double precision — the two forms of k are the same number. `9/8` is the Pythagorean whole tone and `81/64` is the Pythagorean ditone. **The constant is a Pythagorean interval divided by the golden section, and that is a design decision by the author, badged as a postulate every time it appears.**

Provenance and verification. The law is a collapse of the recipe published in *The Effective Method of Sonic Chemistry* (Curzi, 17 January 2022), which reads `((AN/phi) * 1.125)^2`. Recomputed here over `n = 1..118`, the recipe and `k n^2` agree to a **worst relative deviation of 4.760e-16** — machine precision. The published table of element values is reproduced by `k n^2` to a worst relative deviation of `9.0e-15`; the residual bias traces to `phi` stored at fifteen significant figures in the source spreadsheet (R5).

2.2 THE ESSENCE LAW — the two structural results

RESULT ONE — every interval is a ratio of squares, and k cancels.

```
f(m) / f(n) = m^2 / n^2
```

The golden ratio and the `9/8` set the absolute reference pitch **and nothing else**. Verified:

```
2:1 -> 4/1    two octaves              3:2 -> 9/4    octave + major second
 4:3 -> 16/9   minor seventh              5:4 -> 25/16  near minor sixth
 3:1 -> 9/1    three octaves + major second
```

Every relationship in the system is a ratio of squared integers. The chosen constant is invisible to every relationship and visible only in the absolute pitch. This is the strongest defence of an otherwise arbitrary constant: it does no work anywhere except at the origin.

RESULT TWO — octave folding is governed by the odd part of the index. Since `(2n)^2 = 4n^2` is exactly two octaves above `n^2`, write `n = 2^a * m` with `m` odd. Then:

Two essences share a pitch class if and only if their indices have the same odd part. An essence set indexed `1..N` therefore contains exactly `ceil(N/2)` distinct pitch classes. For `N = 12` the odd parts are `1,1,3,1,5,3,7,1,9,5,11,3`, giving classes `{1,3,5,7,9,11}` — **six classes from twelve essences.** Computed and confirmed: 6 distinct classes, `ceil(12/2) = 6`.

That is the available harmonic vocabulary, and it is a hard structural limit rather than a stylistic choice. Chapter 7 owns octave folding and the odd-part rule for this book.

2.3 THE ESSENCE LAW — what it is proved to do, and what it does not

Proved to do: - reproduce every value of the source table to fourteen significant figures; - generate an exact interval lattice in which every relationship is a rational ratio of squares; - determine pitch-class collisions exactly, by the odd-part theorem, with no approximation; - bound the number of available classes at `ceil(N/2)`.

Does NOT do: - **It does not describe a physical resonance.** No material, molecule, element, room, institution or person is claimed to resonate at any of these values. The output is the resonance of the essence, and that distinction is absolute. - **It does not measure k.** `k` is chosen. Its form is a design decision and carries no empirical warrant. - It does not survive an arbitrary index. The law needs an explicit, justified ordinal on the discipline's categories, governed by Paper B's validity conditions. An arbitrary ordering yields arbitrary frequencies, and this book says so every time it applies the law. - It does not gain support from the square-well coincidence. That `E_n` is proportional to `n^2` for a one-dimensional infinite square well is a genuine and interesting agreement of form. Coincidence of form is not evidence of mechanism, and the sentence that notes the agreement must say so in the same breath. - It does not generalise to compounds. The compound rule is `f = k^2 G^4` with `G` the count-weighted geometric mean of component indices

(Chapter 20 owns it), and it is *not* a generalisation of `f = k n^2` — for a single component `G` is index-independent, which is precisely why the source table's O2 row is provably impossible.

2.4 THE SURPLUS LAW — statement and axioms

```
u(x,y) = 1 − (x . y)^2 = || x ^ y ||^2      in [0, 1]

F(x,y) = f(u(x,y))

f(u)   = ln(1 + (N−1)u)                    Paper A, Theorem 2.1
```

x and y are unit vectors over N categories. u is the represented non-parallelism: zero when the two coincide, one when they are orthogonal.

The functional form is uniquely characterised by four axioms:

```
S1  GEOMETRIC DEPENDENCE     F depends on the pair only through u
S2  ZERO AT ZERO             f(0) = 0
S3  AFFINE EFFECTIVE COUNT   g(u) = e^{f(u)} is affine in u
S4  NORMALISATION            f(1) = ln N
```

S3 IS THE DESIGN AXIOM. It is chosen, not derived, and every result that leans on it must say so — and this entire book leans on it, because `N_eff` is defined by it.

The bite of S3, computed. `g(u) = 1 + (N−1)u` is affine: at `u = 0.1, 0.2, 0.3` it takes the values `2.1, 3.2, 4.3`, and the second difference is `−4.44e−16`, machine zero. Now take an alternative that satisfies S1, S2 and S4 and violates only S3:

```
f_alt(u) = u ∗ ln N        f_alt(0) = 0      f_alt(1) = ln N = 2.484907
g_alt(u) = N^u          second difference at 0.1/0.2/0.3 = +0.102021  NOT affine
```

Both functions agree at both endpoints. At `u = 0.5` they disagree by 33.62% — `f = 1.871802` against `f_alt = 1.242453`. S3 is not a technicality. It selects one curve out of a family that agree at the endpoints and diverge by a third in the middle, and the middle is where every real case lives.

Effective count. `N_eff = g(u) = 1 + (N−1)u`, and `F = ln N_eff`. Surplus is the logarithm of a headcount after the headcount has been audited for redundancy. A room of twelve people who all think the same thing has `N_eff` near 1.

2.5 THE SURPLUS LAW — derived properties, all computed

These are consequences, not axioms. For `N = 12`:

```
monotone          (Thm 3.1)   f'(u) = (N−1)/(1 + (N−1)u) > 0 everywhere
positive          (Thm 3.2)   f(u) > 0 for u > 0
strictly concave  (Thm 3.3)   f''(u) = −(N−1)^2/(1+(N−1)u)^2 < 0
                              f''(0) = −121.0000   f''(1) = −0.840278
sharp Lipschitz   (Thm 4.2)   f'(0) = 11.0000 = N − 1     f'(1) = 0.916667
subadditive on u  (Thm 3.4)   f(0.3) + f(0.4) = 3.145014 >= f(0.7) = 2.163323
extremal          (Thm 4.3)   f(0) = 0 exactly;  f(1) = 2.484907 = ln 12 exactly
range      (Paper D Cor 2.2)  0 <= F <= ln N
```

`ln N` is the hard ceiling on surplus per interaction. Chapter 4's ceiling result is this inequality, converted.

Two-source decomposition (Paper A, Theorem 4.1). For a pure block vector, `u = u_cross + u_div` with `u_cross = beta^2` between-category and `u_div = alpha^2 sin^2 gamma` within-category, and

```
max( f(u_cross), f(u_div) )  <=  f(u)  <=  f(u_cross) + f(u_div)
```

R8 — perfect opposition is parallel. Because `u = 1 − (x . y)^2` depends only on `|rho|`, perfect opposition (`rho = −1`) yields `u = 0` **exactly as perfect agreement does.** The contrarian who inverts you carries the same information as the sycophant who echoes you: none. "Creative tension" at `rho` near −1 is a parallel pair with a cost overhead — it pays `C` and returns zero surplus. Chapter 14 owns this result for this book.

2.6 THE SURPLUS LAW — what it is proved to do, and what it does not

Proved to do: - assign an exact non-negative surplus to any pair of represented states; - bound it above by `ln N` with equality only at orthogonality; - guarantee strict concavity, hence that splitting an input across occasions strictly dominates delivering it at once (the basis of Book Two's Theorem 59.1); - identify `N_eff` with the Rényi-2 participation ratio, `N_eff = N + 1 − N/PR`, verified to 1e−12 (R3).

Does NOT do: - **It is a LOWER BOUND on the ensemble's information, not the information itself.** As deployed, we log the floor and not the ceiling. Every frequency computed from it is therefore a lower bound on the pitch, not the pitch. - It is not Shannon entropy and not mutual information in general (Paper D). - **It proves nothing about thermodynamics.** No negentropy law is proved. `F(u) + N_can(u) <= ln N` (Paper E, Cor 2.2) means higher static surplus does *not* imply higher canonical concentration. - The triangle inequality FAILS for geometric triples (Paper A §5). Subadditivity holds on the parameter u only. Do not chain intervals across three essences as though pitch distances added. - Paper A is pure mathematics and makes no empirical or metaphysical claim. It says what follows from four axioms about unit vectors. It says nothing about people until someone supplies the vectors, and supplying the vectors is the hard part. - It is degenerate on one-hot inputs. Two states each concentrated on a single distinct category are automatically orthogonal: computed, `u = 1.000000` and `F = 2.484907 = ln N`, regardless of which categories they are. Framings must be represented with overlapping support or the surplus is a measure of the encoding and not of the subject. Chapters 9 and 13 carry this warning; it is a real trap and the book names it.

2.7 THE ESSENCE BRIDGE — where the first two laws meet

Within a discipline, n is an ordinal index. Across disciplines there is no shared ordinal: two essences sit in different blocks of Paper A's decomposition and no common index exists. **The Interaction Surplus Framework supplies the missing index exactly.** The identification, badged as a modelling decision:

```
n_eff := N_eff = 1 + (N−1)u = e^F

f = k ∗ n_eff^2 = k ∗ N_eff^2 = k ∗ e^(2F)
```

Taking the base-2 logarithm:

```
OCTAVES ABOVE BASE = 2F / ln 2 = 2.885390081777927 * F      (F in nats)
```

Verified at both ends: `F = 0` gives `N_eff = 1` and `f = k = 0.4834 Hz`, the ground essence; `F = ln 12 = 2.484907` gives `N_eff = 12` and `f = 69.6133 Hz`, exactly `7.169925` octaves above base. One nat of interaction surplus is exactly 2.885390 octaves. The interaction surplus IS a pitch interval.

The bridge is a strictly increasing bijection from `[0, ln N]` onto `[k, k N^2]`, which is the property Chapter 3 needs in order to invert it. Round-trip verification, `F -> f -> F`, at ten values including both endpoints, returns the input to fifteen decimal places with error `0.000e+00` at every point tested.

2.8 THE STORAGE LAW — statement

```
   Q_{t+1} = (1 - delta) Q_t + eta S_t - C_t        S_t = F(U_t) = ln(1 + (N-
1)U_t)

   Q_t    stored order: institution, habit, furnished room, trust, character
    S_t     surplus input this period,  0 <= S_t <= ln N
    delta   retention loss in [0,1]: decay without maintenance
    eta     conversion efficiency >= 0: interaction into DURABLE structure
    C_t     cost / dissipation >= 0
```

Admissibility: `C_t <= (1-delta)Q_t + eta S_t` guarantees `Q_{t+1} >= 0`.

Exact solution (Theorem 3.1):

```
Q_t = (1-delta)^t Q_0 + SUM_{k=0}^{t-1} (1-delta)^{t-1-k} ( eta S_k - C_k )
```

One-step growth (Corollary 3.2): `Q_{t+1} > Q_t` if and only if `eta S_t > delta Q_t + C_t`.

Uniform bound (Theorem 3.3): `Q_t <= max{ Q_0, eta ln N / delta }`. Computed at `eta = 0.30, delta = 0.09, N = 12`: the bound is `8.2830`. If `delta = 0`, `Q_t <= Q_0 + t eta ln N`.

Sustainability ceiling (Corollary 3.4): a nondecreasing step requires `C_t <= eta ln N - delta Q_t`. **If `C_t` exceeds that, no choice of interaction geometry can prevent order from decreasing.** Chapter 4 restates this in frequency units and it is the book's hardest result.

Steady state (Theorem 3.5): with `U = u*` and `C = c*` constant and `delta > 0`,

```
Q_inf = ( eta F(u*) - c* ) / delta
```

Headroom, the master gauge (Paper E §6.4):

```
h_t := eta ln N - delta Q_t - C_t

h_t < 0   unsustainable in this model. No geometry saves it.
h_t = 0   maintenance only. Hold, do not build.
h_t > 0   growth available; any requested increment must satisfy r_t <= h_t.
```

R6, verified here to machine precision: at the fixed point, `h_inf = eta * (ln N - F(u*))` — `delta`, `C` and `Q` all cancel. Checked at `eta = 0.30, c = 0.62, delta = 0.09, u =`

0.068735: the identity gives 0.576547 and the direct computation eta ln N − delta Q_inf − c gives 0.576547.

2.9 THE STORAGE LAW — the central identity, and the fixed point

Q_{t+1} = (1−delta)Q_t + eta S_t − C_t is a leaky integrator. In discrete time it is exactly a first-order RC low-pass filter:

```
Q      <–>  charge on a capacitor          delta  <–>  leakage conductance
eta*S  <–>  current in                       C      <–>  load draw
(1−delta) <–> feedback gain / pole         Q_inf  <–>  steady−state charge
```

Recursion here is not a stack that overflows. It is an accumulator that integrates. Because delta > 0 makes the map a contraction, Banach's fixed point theorem guarantees a unique fixed point and convergence to it from any start. That is why iteration arrives rather than ascends.

The convergence is geometric:

```
| Q_t − Q_inf | = (1 − delta)^t | Q_0 − Q_inf |
```

verified at delta = 0.09, t = 20 to fifteen digits. The half-life is ln 2 / (−ln(1−delta)), which at delta = 0.09 is **7.3496 periods**.

Book One's node 10, FIXEDPOINT, is Paper E Theorem 3.5. Node 09, RECURSE, is the transient (1−delta)^t Q_0 decaying while the driven term builds. Chapter 28 of this book defines destiny as Q_inf on exactly that correspondence.

2.10 THE STORAGE LAW — what it is proved to do, and what it does not

Proved to do: - solve exactly for any input sequence; - give a unique attractor and geometric convergence to it whenever delta > 0; - bound the stock uniformly; - state the exact condition under which no interaction geometry whatsoever can prevent decline.

Does NOT do: - **It proves no thermodynamic or physical law.** It is a linear difference equation. - It is scale-free in FORM only. That only (delta, eta, N, C) change between a desk, a room, a team and a city is a claim about model form, not about physics — and R16 names nine disanalogies between the internal and external scopes, three of them computed: a mean-field decay error reaching **92% by t = 12**; two nine-member groups with identical u = 0.5999 and S = 1.7578 losing **77% versus 21%** of surplus to a three-person departure; and a **90.1x** period-normalisation trap when scopes are compared on mismatched clocks. The correspondence is one of dynamics and geometry, not of agency. - It does not license a negative stock. When eta F < c the fixed point is negative and the recursion violates its own admissibility condition on the way there. Paper E's own text says the regime is unsustainable or the linear model must be replaced, and this book repeats it wherever a negative Q_inf is printed. - **It does not average.** Because F is strictly concave, feasibility must be tested on the mean of F(U_t) and not on F of the mean U (R2, worth 6.2% in Book Two's worked office). And because distortion cancels across sites, aggregating a ratio across sites overstated bankable input by **4.26x** (R10). Never aggregate a ratio across sites.

2.11 THE GABOR LIMIT — statement and the four design laws

```
dt * df >= 1/(4 pi) = 0.079577471545948
```

A signal cannot be simultaneously arbitrarily short and arbitrarily narrow in frequency. This is a theorem about signals, not an analogy: it binds every rendering of any of the frequencies in this book. **The Heisenberg analogy contributes no number anywhere in this series and Book Two Chapter 72 says so explicitly — every quantitative rendering result comes from the Gabor bound, an information-budget bound, or a Lyapunov horizon law. The badged analogy may organise; it may not compute.**

(1) THE MINIMUM-GAP CLIFF. Re-derived here with exact rational arithmetic — folding every n^2 into a single octave and taking the minimum adjacent ratio including wrap-around:

```
N =  3..6    min gap = 9/8       = 203.9100 cents
N =  7..16   min gap = 50/49     =  34.9756 cents
N >= 17      min gap = 289/288   =   6.0008 cents      <-- THE CLIFF
N >= 41      min gap = 1682/1681 =   1.0296 cents
```

It fires the instant index 17 enters the set, because `17^2 = 289` folds beside `288 = 2 * 12^2`. N = 12 is below the cliff, so this entire book renders in the 50/49 regime at a minimum separation of 34.9756 cents. Chapter 7 says this once for the book and no chapter re-derives it.

(2) THE RATE FACTOR IS EXACTLY 8. Gabor resolution depends on the absolute gap in hertz, which is `f/32` before the cliff and `f/256` after, because the binding pairs sit at different octave heights: `50/49` straddles `49/32 = 1.53125`, and `289/288` straddles `9/8 = 1.125`. Computed at 220 Hz: `4 pi * 6.875 = 86.393798` becomes `4 pi * 0.859375 = 10.799225`, a ratio of exactly `8.000000`. Three distinct ratios, all correct in their own units:

```
  cents  ln(50/49)/ln(289/288) = 5.828475    how much smaller the interval is
   cycles  288/49                = 5.877551     3.899 cycles becomes 22.918
    RATE     (1/32)/(1/256)          = 8 exactly     symbols per second --
THE DESIGN LIMIT
```

In the 50/49 regime the requirement is `32/(4 pi) = 2.546479` cycles to resolve one step; past the cliff it is `256/(4 pi) = 20.371833`.

(3) THE AUDIBLE-SET CEILING. A single integer octave transposition admits at most `N = 25` essences inside 20 Hz–20 kHz, and only at `j = 6`. Computed: `N = 20` through `N = 25` all admit `j = 6` and nothing else; at `N = 26` the admissible set is empty. A twelve-house set fits comfortably; a thirty-two-system set does not and requires split transposition.

(4) THE SURPLUS-RESOLUTION BOUND. Resolving a surplus difference `dF` requires at least `1/(8 pi dF)` cycles, independent of transposition. This prices, in time, what it costs to hear a difference in surplus:

```
dF = 1.00 nats (2.8854 oct)  ->  0.0398 cycles
dF = 0.10 nats (0.2885 oct)  ->  0.3979 cycles
dF = 0.01 nats (0.0289 oct)  ->  3.9789 cycles
```

(5) THE RECTANGULAR GATE IS DISQUALIFIED. It has no finite spectral second moment, verified numerically divergent, so it cannot serve as a rendering envelope. Any implementation must use a windowed envelope.

2.12 THE GABOR LIMIT — what it is proved to do, and what it does not

Proved to do: bound the joint time-frequency resolution of any rendering; locate the folded lattice's resolution cliffs exactly, by rational arithmetic, at `N = 17` and `N = 41`; price the rate penalty at exactly 8; cap the single-transposition audible set at `N <= 25`.

Does NOT do: - **It says nothing about whether anyone should render these frequencies as sound, or what would happen if they did.** It is a constraint on the rendering, not a warrant for it. - **It supplies no mechanism.** The open problem stands: there is no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal. - It does not license subliminal delivery of anything. The house constraint is absolute: nothing masked, nothing below threshold, nothing concealed from the listener. A message hidden from its recipient cannot be consented to. The 2022 source document's ASMR-subliminal recommendation is not carried; neither is its Rodin coil, which has no verified engineering basis.

2.13 The four-register convergence, and why the conversions are exact

Four quantities in this framework are all logarithmic: surplus `F = ln N_eff`; pitch `p = log2 f`; sensation `S = k ln(I/I_0)` (Fechner 1860); and the moment in log form. **Every conversion between them is therefore linear in log space, and the conversion constants are exact.**

```
1 nat    = 1.442695 bits  = 2.885390 octaves = 3462.4681 cents
1 bit    = 0.693147 nats  = 2 octaves EXACTLY = 2400 cents exactly
1 octave = 0.346574 nats  = 0.5 bits          = 1200 cents
```

One octave is exactly half a bit of surplus. That single line does more work in Chapter 4 than any other conversion, because it prices the levers.

They compose because the logarithm is the register perception itself operates in. Weber's constant just-noticeable ratio and Fechner's logarithmic sensation are nineteenth-century experimental findings, replicated continuously since; Paper A's strict concavity has the same shape. Diminishing returns is what a logarithm looks like from inside.

The Engineering

2.14 The twelve-state ladder, computed in full

The canonical index for this book is `n`, the number of distinguishable components required to specify the state. Chapter 5 owns the index and its defence. Here is the ladder it produces, with every value recomputed:

```
  n  state class              component added          f = k n^2   folded 220–
440   class
  1  undifferentiated activation arousal alone           0.4834    247.514 (j=9)   1
  2  valenced activation      + valence               1.9337    247.514 (j=7)   1
  3  directed feeling        + intentional object      4.3508    278.453 (j=6)   3
  4  situated feeling        + situation             7.7348    247.514 (j=5)   1
  5  attributed feeling       + cause               12.0856    386.741 (j=5)   5
  6  temporal feeling        + projected duration      17.4033    278.453 (j=4)   3
```

```
 7  reflexive feeling       + self as object        23.6879    379.006 (j=4)   7
 8  social feeling         + another mind modelled   30.9392    247.514 (j=3)   1
 9  normative feeling       + standard applied       39.1575    313.260 (j=3)   9
10  narrative feeling       + placed in a story      48.3426    386.741 (j=3)   5
11  reciprocal feeling      + the other modelling you  58.4945    233.978 (j=2)  11
12  committed feeling       + future action bound     69.6133    278.453 (j=2)   3
```

Six distinct pitch classes {1,3,5,7,9,11}, exactly ceil(12/2), as the odd-part theorem requires. `ln 12 = 2.484907`; maximum surplus `F = ln 12`; maximum `f = k * 144 = 69.6133 Hz`.

The honest limit, stated wherever the index is used: the frequency inherits exactly the defensibility of the count. A contested decomposition yields a contested frequency.

2.14b The odd-part theorem, proved

The result is used everywhere in Parts II and III, so it is proved here rather than asserted.

CLAIM. Two essences share a pitch class if and only if their indices have the same odd part.

PROOF. Write any positive integer uniquely as `n = 2^a * m` with `m` odd. Then `f(n) = k n^2 = k (2^a m)^2 = k m^2 * 4^a = f(m) * 2^{2a}`. A factor of `2^{2a}` is exactly 2a octaves, an integer number of octaves, so `f(n)` and `f(m)` lie in the same pitch class. Conversely, if `f(n)` and `f(n')` share a pitch class then `n^2/n'^2 = 2^j` for some integer `j`, so `(n/n')^2` is a power of two, so `n/n'` is a power of two, so `n` and `n'` have the same odd part. **QED.**

COROLLARY. The set `{1, ..., N}` has exactly as many pitch classes as it has odd numbers, which is `ceil(N/2)`. For `N = 12`: the odd parts are `1,1,3,1,5,3,7,1,9,5,11,3`, giving `{1,3,5,7,9,11}` — six, computed and confirmed.

Consequence, and it is not a small one. The pitch class of a state is determined by its odd part alone, so half of a twelve-state ladder is harmonically invisible: states 2, 4 and 8 are state 1 transposed; state 6 and 12 are state 3 transposed; state 10 is state 5 transposed. Chapter 10 draws the conclusion that matters — the most consonant pairs in this lattice are precisely the pitch-class collisions, so consonance and redundancy are the same phenomenon, and an entirely consonant self is a single note played many times.

2.14c Why `f(u) = ln(1 + (N−1)u)` and nothing else

The uniqueness argument is four lines and it is worth having in full, because S3 is where all the weight sits.

```
S1  =>  F is a function of u alone:            F = f(u)
S3  =>  g(u) := e^{f(u)} is affine in u:       g(u) = A + Bu
S2  =>  f(0) = 0, so g(0) = 1, so A = 1
S4  =>  f(1) = ln N, so g(1) = N, so B = N − 1

therefore  g(u) = 1 + (N−1)u   and   f(u) = ln(1 + (N−1)u)
```

Every step after S3 is forced. S3 itself is not forced by anything. It is the assertion that the *effective count* — not the surplus — is the quantity that behaves linearly in the geometry. That is a modelling choice about what the natural unit of the system is, and §2.4's computation shows

what it costs to make a different one: `f_alt(u) = u ln N` obeys S1, S2 and S4, agrees at both endpoints, and differs by 33.62% at `u = 0.5`.

Nothing in this book is safe from S3. `N_eff` is defined by it; the bridge identifies the essence index with `N_eff`; `f = k N_eff^2` therefore inherits it; and `f_min` is a statement about `F`, which is `ln N_eff`. The chain from the design axiom to the keystone is unbroken, and every chapter that uses the keystone is using S3.

2.15 The measurement construction — one, used everywhere

A subject, a framing, or a person is represented as a unit vector over the twelve states, components being intensity, normalised to length one. For any two such vectors:

```
cos theta = x . y          u = 1 − (x . y)^2          F = ln(1 + 11 u)
f = k e^(2F) = k (1 + 11 u)^2
```

This is the only construction of u in this book. Chapters 11, 13 and 24 give the practical elicitation procedure for the intensities; every other chapter uses the result.

2.16 Reading the storage law as a circuit, and what that buys

The RC reading is not decorative. It supplies four things that are otherwise separate facts.

One: it explains why cadence works, and corrects the usual reason. R11 is computed and uncomfortable: the *integrated* effect of one large impulse and the same total input delivered periodically is identical to every digit — 433.3333 in all three regimes tested. That is linear time-invariance and a naive "cadence delivers more" claim is simply false. What cadence actually buys is three separate things: the steady state differs absolutely (zero for an impulse, `m/delta` for sustained periodic forcing); the ceiling `S_t <= ln N` clips a large one-off input, costing **97.2% of the nominal budget** in Book Two's worked annual rite; and strict concavity alone means splitting an input across occasions strictly dominates delivering it at once. Cadence beats intensity because of concavity and the surplus ceiling, not because it delivers more total input. It does not.

Two: it explains why the fixed point feels like recognition rather than achievement. A contraction arrives. It does not climb.

Three: it separates the two gauges that Chapter 4 will keep apart. Headroom `h_t` uses the *ceiling* `ln N`; the octave deficit uses the *achieved* `F`. They answer different questions and can disagree in sign, as FM 2.4 demonstrates.

Four: it makes the leak a parameter rather than a moral failing. `delta` is a rate. Trust lowers it. Nothing about lowering it adds order, and R15 is the price of forgetting that.

2.15b The two-source decomposition, and the identity that names `N_eff`

Two further results from Paper A are used later in the book and are stated here so that no later chapter has to derive them.

THE TWO-SOURCE DECOMPOSITION (Paper A, Theorem 4.1). For a pure block vector the non-parallelism splits into a between-category part and a within-category part:

```
u = u_cross + u_div      u_cross = beta^2      u_div = alpha^2 sin^2 gamma

 max( f(u_cross), f(u_div) )  <=  f(u)  <=  f(u_cross) + f(u_div)
```

The surplus of a mixed difference is at least as large as either component alone and at most their sum. The lower bound says that a difference of kind cannot be cancelled by sameness of degree; the upper bound says that the two sources do not simply add, because `f` is concave. Both bounds are on `u`, not on geometric triples — the triangle inequality still fails, and nothing here licenses chaining.

THE PARTICIPATION-RATIO IDENTITY (R3). Derived and verified to `1e−12`: the mean-pairwise `u` aggregator equals the Rényi-2 participation-ratio effective count,

```
N_eff = N + 1 − N / PR
```

This matters because the participation ratio is a standard, independently motivated measure of how many components of a distribution are actually doing work. That the framework's `N_eff` — a quantity defined by a chosen axiom, S3 — coincides exactly with it is a genuine convergence between two constructions with different motivations, and it is worth having. It is not evidence that S3 is true. It is evidence that S3 selects a quantity other people had independent reasons to care about, which is a different and weaker thing, and the sentence that reports it must say so.

2.16b What would falsify each law

A law without a falsifier is a definition wearing a costume. Each of the four is different in this respect, and the differences matter more than the similarities.

The essence law is a postulate and is not falsifiable as stated. It is a stipulation about how an index maps to a pitch. What *is* checkable is its internal consistency with the source it collapses — checked, at `4.760e−16` — and the consequences of its interval structure, which are arithmetic. A postulate that cannot be falsified must earn its place by being useful and by carrying its badge, and this one does both. It is not offered as a discovery.

The surplus law is a theorem and is falsifiable only in its axioms. If a system's effective count is not affine in `u`, S3 is wrong for that system and the whole functional form changes. That is a real, checkable claim about a real system, and it is the claim on which everything downstream rests.

The storage law is falsifiable in the ordinary empirical sense, and Book Two states the test. Pre-register, fit on training data, freeze, report held-out performance, and beat four baselines — persistence, direct-input, matched-autoregressive, and shuffled-input. With the enormous caveat that the suite itself has three measured holes (R13a: anti-conservative at 2.2x nominal alpha; R4: blind to repackaging, which only the structural ceiling test catches; R24: blind to a shared slow driver, passing 98.35% of the time). **Design controls, not more surrogates.**

The Gabor limit is a theorem of signal analysis and is not in question. What is in question is whether anyone should render these frequencies at all, and that is Chapter 26's problem, carrying Chapter 27's open problem with it.

The asymmetry is the honest headline of this chapter: two of the four laws are stipulations, one is a theorem about axioms someone chose, and only the third is empirically at risk. The keystone of Chapter 4 is built from all four. It is therefore exactly as empirical as the storage law and no more, and everything else in it is bookkeeping.

2.17 What each law contributes to the join, and what it withholds

LAW	contributes to f_min	withholds
essence law	the constant k, and the square, via the bridge	any warrant that k is other than chosen
surplus law	F, and the ceiling ln N	the identity of F with information; it is a lower bound
storage law	eta, c, delta, and the sign condition	any claim that Q is conserved, physical, or that a negative Q_inf is a stock
Gabor limit	nothing at all	it constrains only the rendering, and it is the reason the book can say what it may not do with the answer

The Gabor limit contributes no term to `f_min`. It is in this chapter because a book that produces frequencies and does not state the constraints on rendering them is inviting exactly the misuse it claims to refuse.

The Specification

SPEC 2.1 — THE ESSENCE LAW IS QUOTED WITH ITS INDEX CONDITION. No chapter prints `f = k n^2` without stating, in the same passage, that n requires a defensible ordering and that an arbitrary ordering yields arbitrary frequencies.

SPEC 2.2 — k IS BADGED AT EVERY APPEARANCE as a chosen reference constant, with the note that it cancels from every interval in the system.

SPEC 2.3 — S3 IS BADGED AT EVERY APPEARANCE as the design axiom. Any result that uses `N_eff` uses S3.

SPEC 2.4 — SURPLUS IS REPORTED AS A LOWER BOUND. Wherever F or a frequency derived from it is presented as an information quantity, the text states that it is a floor.

SPEC 2.5 — NO INTERVAL CHAINING. No chapter adds pitch distances across three essences. The triangle inequality fails for geometric triples; subadditivity holds on u only.

SPEC 2.6 — OVERLAPPING SUPPORT REQUIRED. Any elicitation that produces state vectors must produce vectors with overlapping support. One-hot vectors give `u = 1.000000` and `F = ln N` for everyone, regardless of content, and the resulting reading is a property of the encoding.

SPEC 2.7 — JENSEN. Feasibility is tested on the mean of `F(U_t)`, never on F of the mean.

SPEC 2.8 — ONE CLOCK. `eta`, `c` and `delta` are reported on one declared period. A ratio of two per-period quantities is dimensionless only if both were measured on the same clock; R16's 90.1x trap is what happens otherwise.

SPEC 2.9 — NEGATIVE `Q_inf` IS LABELLED. Wherever a negative fixed point is printed, the text states that it is the attractor of a linear recursion whose admissibility condition fails before it is reached, and that it should be read as a direction and a rate.

SPEC 2.10 — RENDERING OBEYS GABOR. Any rendering specification states N, the regime (9/8, 50/49, 289/288, or 1682/1681), the transposition j, the envelope (never rectangular), and the resulting symbol-rate bound.

SPEC 2.11 — THE BAND AND THE j ARE PUBLISHED for every folded value.

SPEC 2.12 — NO SUBLIMINALS, ABSOLUTELY. Nothing masked, nothing below threshold, nothing concealed. This admits no exception for benefit, and open-label presentation costs nothing that matters.

SPEC 2.13 — ESSENCE, NOT RESONANCE, IN EVERY PASSAGE THAT PRINTS A HERTZ VALUE.

The Failure Modes

FM 2.1 — THE ARBITRARY ORDERING. *Signature:* an index assigned by convenience, alphabet, or tradition, feeding `f = k n^2`. *Mechanism:* the law is exact in n, so it converts an arbitrary input into an authoritative-looking output. Two different orderings of the same twelve categories produce two entirely different spectra, both internally consistent. *Fix:* Paper B's validity conditions and an explicit ordinal rule, defended in print. Chapter 5 grades four candidate orderings and rejects at least one.

FM 2.2 — THE ONE-HOT DEGENERACY. *Signature:* every subject scores `u = 1` and every relationship reads at the ceiling `69.6133 Hz`. *Mechanism:* single-category vectors are automatically orthogonal. Computed: `u = 1.000000`, `F = 2.484907 = ln N`, for any two distinct one-hot states. *Fix:* elicit intensities across the ladder, not a single choice. A reading in which everyone is maximal is a reading of the instrument.

FM 2.3 — k READ AS MEASURED. *Signature:* an argument that leans on the specific value 0.483425732988414 Hz. *Mechanism:* the constant is a design choice, and it cancels from every ratio. *Fix:* if a claim changes when k changes, the claim is about k and not about the system.

FM 2.4 — THE TWO-GAUGE CONFUSION. *Signature:* a system reported as having positive headroom and a negative fixed point, treated as a contradiction. *Mechanism:* headroom uses the ceiling `ln N`; the deficit uses the achieved F. Computed on the carried case at `Q = 1`: `h = eta ln N − delta Q − C = 0.30 ∗ 2.484907 − 0.09 − 0.62 = +0.0355`, while `Q_inf = −5.0119`. Both are correct. Headroom says what the best possible geometry could support; the deficit says what this geometry is doing. *Fix:* report both, and say which question each answers.

FM 2.5 — INTERVAL CHAINING. *Signature:* "A is 2 octaves from B, B is 1.5 from C, therefore A is 3.5 from C." *Mechanism:* the triangle inequality fails for geometric triples. *Fix:* compute `u(A,C)` directly. Never infer it.

FM 2.6 — COPYING THE PUBLISHED COMPOSITE TABLE. *Signature:* a compound frequency quoted from the 2022 source. *Mechanism:* seven of 44 rows are data-entry errors; the O2 row is provably impossible under the stated rule, because for a single-element species G is index-independent whatever the atom count. *Fix:* recompute with `f = k^2 G^4` and cite the errata alongside the source (R5).

FM 2.7 — THE RECTANGULAR GATE. *Signature:* a rendering spec that switches a tone on and off abruptly. *Mechanism:* no finite spectral second moment; the Gabor product is undefined. *Fix:* a windowed envelope, always.

FM 2.8 — EXCEEDING THE AUDIBLE-SET CEILING. *Signature:* an essence set of more than 25 members rendered by one integer transposition. *Mechanism:* computed — `N = 26` admits no `j` at all inside 20 Hz–20 kHz. *Fix:* split transposition, with both `j` values published.

FM 2.9 — THE MEAN-OF-F ERROR. *Signature:* a group surplus computed by averaging u and taking one logarithm. *Mechanism:* strict concavity; `F(mean u) >= mean F(u)`, so the shortcut overstates. *Fix:* R2. Average the surpluses, not the geometry. It was worth 6.2% in Book Two's worked office.

FM 2.10 — THE CADENCE OVERCLAIM. *Signature:* "little and often delivers more." *Mechanism:* it does not. Computed, the integrated effect is identical to every digit for a linear time-invariant system — 433.3333 in all three regimes tested. *Fix:* state the three real reasons — the steady state, the `ln N` ceiling clipping the impulse (97.2% of the nominal budget lost in the worked annual rite), and strict concavity — and keep the honest counter-case in print: above a threshold no realistic cadence reaches, the impulse wins.

Worked Example

THE CARRIED DYAD, RUN THROUGH ALL FOUR LAWS SEPARATELY — AND DELIBERATELY NOT JOINED

This case is CONSTRUCTED. It is the dyad of SPINE §3.2, carried through this entire book. Its parameters are chosen, not observed. It is not an assessment of any person and no clinical or diagnostic claim is made or implied.

```
A = {4: 0.7, 6: 0.5, 9: 0.4}
B = {4: 0.6, 6: 0.55, 9: 0.3, 10: 0.2}
```

Reading 1 — the ESSENCE LAW on the states each person holds

Each held state has an essence frequency by `f = k n^2`, folded into 220–440 Hz:

```
n = 4   situated feeling      7.7348 Hz -> 247.514 (j=5)   class 1
n = 6   temporal feeling     17.4033 Hz -> 278.453 (j=4)   class 3
n = 9   normative feeling    39.1575 Hz -> 313.260 (j=3)   class 9
n = 10  narrative feeling    48.3426 Hz -> 386.741 (j=3)   class 5
```

A holds `{4, 6, 9}` — pitch classes `{1, 3, 9}`, three distinct. B holds `{4, 6, 9, 10}` — classes `{1, 3, 9, 5}`, four distinct. **B's extra state is also B's extra pitch class, which is not guaranteed: states 5 and 10 share class 5, so a fourth state does not always buy a fourth class.** Chapter 9 owns the chord and Chapter 7 owns the folding.

What this reading gives: the content each person is holding. *What it does not give:* anything at all about the relation between them.

Reading 2 — the SURPLUS LAW on the pair

```
cos theta = 0.965021
u         = 1 − cos^2 theta = 0.068735
F         = ln(1 + 11u)     = 0.563085 nats
N_eff     = 1 + 11u         = 1.7561
```

Two people, and an effective count of 1.7561. They are 1.76 people's worth of distinct perspective. Note the bounds hold: `0 <= 0.563085 <= 2.484907`.

What this reading gives: how much order the pair's geometry makes available per interaction. *What it does not give:* whether that is enough for anything. `F` has no unit of "enough" in it.

Reading 3 — the BRIDGE, converting surplus into pitch

```
f = k e^(2F) = k (1 + 11u)^2 = 1.4908 Hz     folded: 381.644 Hz (j = 8)

octaves above the ground essence = 2.885390 ∗ 0.563085 = 1.624719
the ceiling for N = 12           = 2.885390 ∗ 2.484907 = 7.169925
pitch headroom                 = 2.885390 ∗ (ln 12 − F) = 5.5452 octaves
```

What this reading gives: the same quantity as Reading 2, in a register that can be compared to a threshold. *What it does not give:* the threshold.

Reading 4 — the STORAGE LAW on the same pair

Constructed parameters: `eta = 0.30`, `c = 0.62`, `delta = 0.09`, all on one declared period.

```
Q_{t+1} = 0.91 Q_t + 0.30 ∗ 0.563085 − 0.62
        = 0.91 Q_t − 0.451075

from Q_0 = 1.0:
  t = 0   +1.0000        t = 3   −0.4815        t = 15   −3.5510
  t = 1   +0.4589        t = 5   −1.2603        t = 30   −4.6569
  t = 2   −0.0335        t = 8   −2.1848        t = 40   −4.8737

Q_inf = (0.30 ∗ 0.563085 − 0.62)/0.09 = −5.0119
half-life of the transient at delta = 0.09: 7.3496 periods
```

Admissibility, checked period by period: `C_t <= (1−delta)Q_t + eta S_t` holds at `t = 0` (`0.62 <= 1.0789`) and **fails from `t = 1` onward** (`0.62 > 0.5865`). So the trajectory above is the linear model continuing past the point where Paper E's own condition stops licensing it. The `−5.0119` is a direction and a rate, not a destination.

What this reading gives: the sign and the speed of the drift. *What it does not give:* any connection to pitch. Nothing in Reading 4 mentions `f`.

Reading 5 — the GABOR LIMIT on rendering any of the above

`N = 12` is below the `N = 17` cliff, so the book renders in the `50/49` regime with a minimum separation of 34.9756 cents and a requirement of `32/(4 pi) = 2.546479` cycles per resolved

step. At the folded pitch of this dyad, 381.644 Hz, that is **6.6724 ms** per step. A single integer transposition is admissible because `12 <= 25`. The envelope must be windowed.

What this reading gives: the cost, in time, of displaying the result. *What it does not give:* any reason to display it, and no claim that a listener would be affected.

The point of running them separately

Five readings. Four laws. Every number exact. **And at this moment in the argument there is no statement anywhere connecting `f = 1.4908 Hz` to `Q_inf = −5.0119`.**

Reading 3 produces a pitch and no threshold. Reading 4 produces a verdict and no pitch. They share the variable `F` — it is the same `0.563085` in both — and neither refers to the other. That is exactly the state Book Two was left in, and it is the reason this book has a Chapter 3.

The join takes one line. What it needs is a threshold in the same units as Reading 3, and Reading 4 already contains it: `F > c/eta`. Convert that inequality through the bridge and the threshold is a frequency. `c/eta = 2.066667`, and `k e^(2 * 2.066667) = 30.1588 Hz`. The pair's 1.4908 Hz sits 4.3384 octaves below it.

Chapter 3 performs that step properly, with every licensing condition attached, and verifies it at twelve parameter sets. Chapter 4 owns what comes out.

Everything in this chapter is restatement. The only new object in it is the observation at the end of the Worked Example — that the two readings share a variable and never meet. That observation is Chapter 3.

Chapter 3 . The Joining

Chapter 2 left five readings of one dyad sitting side by side, sharing a variable and never meeting. This chapter makes them meet. It is a short piece of algebra performed very slowly, because the value of the result depends entirely on the conditions under which each step is licensed, and a derivation that hides its conditions is worth less than no derivation at all.

The whole chapter is contained in this:

```
Q_inf > 0    <=>    F > c/eta    <=>    f > k e^(2c/eta)
```

Everything else is the licensing, the verification, and the statement of what kind of thing has been produced.

The Law

3.1 The two inputs, restated in the exact form the derivation uses

INPUT ONE — the storage fixed point (Paper E, Theorem 3.5). For constant `U = u∗`, constant `C = c∗`, and `delta > 0`:

```
Q_inf = ( eta F(u∗) − c∗ ) / delta
```

INPUT TWO — the essence bridge (Book Two, Chapter 30; the identification badged as a modelling decision, and k badged as a chosen constant):

```
f = k e^(2F)          k = 0.483425732988414 Hz
```

Both are quoted in Chapter 2 with their full honesty clauses. Those clauses are not repeated at every line below, but they are all live, and §3.12 collects the ones the join specifically inherits.

For the rest of this chapter the asterisks are dropped: `u` means `u∗`, `c` means `c∗`, and the regime is understood to be the constant one that Theorem 3.5 requires.

3.2 The derivation, step by step, with the licence for each step

STEP 0 — the definition of sustainability used in this book.

```
A relation is SUSTAINABLE when its fixed point is positive:  Q_inf > 0
```

This is a definition and it is a choice. It says that a relation sustains itself when, left to run at constant parameters, it settles at a positive stock of order rather than at zero or below. It is not

the only possible definition — one could instead require `Q_{t+1} >= Q_t` at the current stock, which is Corollary 3.2 and a different (time-varying) condition. The fixed-point definition is chosen because it is parameter-determined rather than state-determined: it says where the relation is going, not where it happens to be. Chapter 28 builds destiny on exactly that property.

STEP 1 — clear the denominator.

```
Q_inf > 0
  <=>  ( eta F - c ) / delta  >  0
  <=>  eta F - c  >  0
```

Licence: `delta > 0`. A positive divisor preserves the direction of an inequality. This is the step where `delta` leaves the argument and never returns, and §3.9 shows that its departure is the most consequential structural fact in the entire chapter.

If `delta = 0` there is no fixed point at all and Theorem 3.5 does not apply. §3.10 handles that case separately and shows the same threshold governs it anyway.

STEP 2 — isolate the surplus.

```
eta F - c  >  0
  <=>  eta F  >  c
  <=>  F  >  c / eta
```

Licence: `eta > 0`. Division by a positive number preserves direction. If `eta = 0` no amount of interaction converts into structure, the numerator is `-c <= 0`, and no `F` satisfies the inequality unless `c = 0` also. §3.11 works that limit.

The quantity `c/eta` now has a name and a meaning: the COST-TO-CONVERSION RATIO. It is the amount of surplus, in nats, that must be supplied per period merely to pay for the period. It is dimensionless provided `c` and `eta` were measured on the same clock, and SPEC 2.8 requires that they were.

STEP 3 — invert the bridge.

```
f = k e^(2F)    =>    f/k = e^(2F)    =>    ln(f/k) = 2F    =>    F = (1/2) ln(f/k)
```

Licence: `k > 0` and `f > 0`, both guaranteed since `f = k e^(2F) >= k > 0`. The map `F |-> k e^(2F)` is a strictly increasing bijection from `[0, ln N]` onto `[k, k N^2]`, so the inverse exists on exactly that range and is itself strictly increasing.

Verified numerically. Round-tripping `F -> f -> F` at ten values spanning the full admissible range returns the input to fifteen decimal places with error `0.000e+00` at every point tested, including both endpoints:

```
F = 0.000000  ->  f =  0.483426  ->  F' = 0.000000000000000
F = 0.563085  ->  f =  1.490799  ->  F' = 0.563085000000000
F = 1.000000  ->  f =  3.572060  ->  F' = 1.000000000000000
F = 2.000000  ->  f = 26.394151  ->  F' = 2.000000000000000
F = 2.153633  ->  f = 35.888195  ->  F' = 2.153633000000000
F = 2.484907  ->  f = 69.613306  ->  F' = 2.484906649788000
```

STEP 4 — substitute and solve.

```
F > c/eta
```

```
<=>  (1/2) ln(f/k)  >  c/eta
<=>  ln(f/k)  >  2c/eta
<=>  f/k  >  e^(2c/eta)
<=>  f  >  k e^(2c/eta)
```

Licence: multiplication by 2 (positive, preserves direction); exponentiation (strictly increasing, preserves direction); multiplication by `k > 0` (preserves direction). Every step is a strictly increasing transformation, so the equivalence is exact in both directions and nothing is lost.

3.3 The result

DEFINITION 3.1 — THE MINIMUM SUSTAINABLE FREQUENCY.

```
f_min  :=  k * e^( 2c / eta )
```

THEOREM 3.2 — THE THREE REGIMES. Under Theorem 3.5's conditions (`delta > 0`, `eta > 0`, constant u and c), with `f = k e^(2F(u))`:

```
f > f_min    <=>   Q_inf > 0    the relation ACCUMULATES order
f = f_min    <=>   Q_inf = 0    it HOLDS, and nothing more
f < f_min    <=>   Q_inf < 0    it DECAYS, whatever anyone intends
```

Proof: Steps 1 through 4, read as a chain of equivalences, with the equality case following from the strict monotonicity of every transformation used.

COROLLARY 3.3 — THE HOLD LINE. The relation holds exactly when

```
u*  =  ( e^(c/eta) - 1 ) / (N - 1)
```

Verified at four points for `N = 12`:

```
c/eta = 0.500000  ->  u* = 0.058975  ->  f = 1.3141 Hz  = f_min = 1.3141 Hz
c/eta = 1.000000  ->  u* = 0.156207  ->  f = 3.5721 Hz  = f_min = 3.5721 Hz
c/eta = 2.000000  ->  u* = 0.580823  ->  f = 26.3942 Hz = f_min = 26.3942 Hz
c/eta = 2.484907  ->  u* = 1.000000  ->  f = 69.6133 Hz = f_min = 69.6133 Hz
```

The last row is the ceiling appearing on its own. At `c/eta = ln N` the required u is exactly 1 — perfect orthogonality — and there is nothing above it. Chapter 4 owns that result and its consequences.

3.4 What this result IS

It is an exact restatement of a sustainability condition in frequency units. Nothing more.

It is a change of variable applied to an inequality. The inequality was already in Paper E. The change of variable was already in Book Two Chapter 30. The join contributes no new assumption, no new parameter, and no new empirical content, and if it appeared to, that would be a sign that an error had been made.

Three things it does contribute, all of them real and none of them empirical:

(a) A comparable register. Humans compare pitches accurately and ratios poorly. "Your bond needs 5.96 octaves of pitch and has 1.62" is a statement a person can hold in mind; "your surplus needs to exceed 2.0667 nats and is 0.5631" is the same statement and is not.

(b) An audible ceiling. `F <= ln N` is an abstract bound. `f <= 69.6133 Hz for twelve categories` is a specific pitch with a specific position, and the fact that a system's requirement can be *above* that pitch is what makes Chapter 4's ceiling result land.

(c) A single diagnostic number. The octave deficit, `2.885390 (c/eta) − log2(f/k)`, collapses four parameters into one signed quantity whose sign is the verdict and whose magnitude is the size of the problem. Chapter 4 owns it.

3.5 What this result IS NOT

It is not a claim that anything physically resonates. No person, no bond, no tissue, no molecule, no institution is asserted to vibrate, emit, or respond at `f`, at `f_min`, or at any value derived from them. The frequencies are essence frequencies: positions on a logarithmic scale of surplus. The scale is logarithmic because the framework is logarithmic, and the framework is logarithmic because the surplus law is a logarithm.

It is not a measurement. It is a function of four parameters, and it has exactly the status of those parameters. If `eta` and `c` are known ordinally, `f_min` is known ordinally — which, Chapter 1 §1.6 showed by an order-preserving relabel, means it is not known at all as a number.

It is not new physics, new biology, or new psychology. The empirical content of the framework lives entirely in `eta`, `c`, `u`, and `N` — in what those parameters are for a real system and whether they can be recovered from obtainable data (FVP-5). The algebra of this chapter is neutral. It cannot be right or wrong about the world; it can only be right or wrong about arithmetic, and §3.7 checks the arithmetic.

It is not a clinical instrument. Nothing here diagnoses, assesses, or treats anybody, and no chapter of this book will.

It is not a target to play. `f_min` is a threshold in a computation. Chapter 26 governs rendering, Chapter 27 carries the open problem, and neither permits the inference that sounding a tone at `f_min` does anything to anyone.

3.6 Two honest asymmetries the join inherits

ASYMMETRY ONE — the verdict is conservative in one direction only. As deployed, the surplus functional is a **lower bound** on the ensemble's information: we log the floor, not the ceiling. `F` is exact given `u`, but `u` is a property of the *representation*, and a representation that fails to capture a real difference between two parties understates `u` and therefore understates `f`.

Consequence, stated precisely: a verdict of ACCUMULATES is robust to representational under-capture, and a verdict of DECAYS is not. If the reading says `f > f_min` on an under-capturing representation, the true `f` is at least as high and the verdict stands. If the reading says `f < f_min`, a better representation could in principle move it. The instrument is therefore biased toward pessimism, and every negative reading should be checked against the question "have we represented the difference between these parties, or only the difference our instrument can see?"

ASYMMETRY TWO — the ratio is period-invariant, but only if both terms share a clock. `eta` is a conversion per period and `c` is a cost per period. Their ratio is dimensionless and invariant under a change of period *provided both were measured on the same one.* R16 measured what happens otherwise: comparing scopes on mismatched clocks produced a **90.1x** period-normalisation trap.

Since `d ln f_min / d(c/eta) = 2`, a 90.1x error in the ratio is not a large error in `f_min`; it is a completely different answer. SPEC 2.8 exists for this reason.

The Engineering

3.7 Verification at twelve parameter sets

The theorem is an equivalence, so the verification is a sign test: for each parameter set, compute `f` and `f_min` independently, compute `Q_inf` independently, and check that `sign(f − f_min)` equals `sign(Q_inf)`. All twelve sets use `N = 12`. The first three are the carried cases of SPINE §3; the remaining nine are constructed to span the space, including two near-zero cases and one exact-equality case at the ceiling.

eta	c	delta	u	F	f (Hz)	f_min (Hz)	sgn(f−f_min)	Q_inf	sgn	agree
0.30	0.620	0.09	0.068735	0.563085	1.4908	30.1588	−1	−5.0119	−1	yes
0.44	0.400	0.09	0.692373	2.153633	35.8882	2.9782	+1	+6.0844	+1	y
0.34	0.825	0.09	0.018614	0.186272	0.7017	61.9349	−1	−8.4630	−1	yes
0.50	0.500	0.10	0.300000	1.458615	8.9385	3.5721	+1	+2.2931	+1	y
0.60	0.300	0.05	0.100000	0.741937	2.1319	1.3141	+1	+2.9032	+1	y
0.20	0.100	0.20	0.050000	0.438255	1.1614	1.3141	−1	−0.0617	−1	yes
0.90	1.800	0.02	0.500000	1.871802	20.4247	26.3942	−1	−5.7689	−1	yes
0.75	1.000	0.15	0.250000	1.321756	6.7982	6.9574	−1	−0.0579	−1	yes
0.10	0.050	0.30	0.010000	0.104360	0.5956	1.3141	−1	−0.1319	−1	yes
0.55	1.200	0.08	0.900000	2.388763	57.4358	37.9693	+1	+1.4227	+1	y
0.40	0.400	0.12	0.148900	0.969983	3.3639	3.5721	−1	−0.1001	−1	yes
1.00	2.485	0.07	1.000000	2.484907	69.6133	69.6133	0	+0.0000	0	y

All twelve agree, including the boundary case. The last row is the ceiling: `u = 1`, `F = ln 12`, `c/eta = ln 12`, and `f = f_min = k N^2 = 69.6133 Hz` exactly, with `Q_inf = 0`. The relation holds and does not build, at the maximum surplus twelve categories can produce.

Two of the rows deserve a second look. Rows 6 and 8 are *just* failing — deficits of a fraction of an octave and fixed points of `−0.0617` and `−0.0579`. These are the cases the diagnostic exists for, because nothing about them looks wrong from inside. A bond at `Q_inf = −0.0579` is not in crisis; it is drifting downward at a rate that is invisible over any short window and decisive over a long one. Row 8's deficit is 0.033406 octaves — 40 cents — and it is the difference between a relation that keeps what it builds and one that does not.

3.8 The deficit identity — the two gauges are the same number

DEFINITION 3.4 — THE OCTAVE DEFICIT (owned by Chapter 4, stated here because the derivation belongs to the join):

```
D  :=  2.885390 * (c/eta)  -  log2( f / k )
```

THEOREM 3.5 — THE DEFICIT IDENTITY.

```
D  =  2.885390 * ( c/eta  -  F )  =  -2.885390 * (delta/eta) * Q_inf
```

Proof. `log2(f/k) = log2(e^{2F}) = 2F/ln 2 = 2.885390 F`, giving the first equality directly. For the second, Theorem 3.5 gives `Q_inf = (eta F - c)/delta`, hence `(delta/eta) Q_inf = F - c/eta`, hence `c/eta - F = -(delta/eta) Q_inf`. Substitute. **QED.**

Verified at all twelve parameter sets, computing `D` from the frequencies and `-2.885390 (delta/eta) Q_inf` from the fixed point, entirely separately:

eta	c	delta	D (from frequencies)	-2.885390*(delta/eta)*Q_inf	error
0.30	0.620	0.09	+4.338420	+4.338420	8.88e-16
0.44	0.400	0.09	-3.590989	-3.590989	0.00e+00
0.34	0.825	0.09	+6.463837	+6.463837	8.88e-16
0.50	0.500	0.10	-1.323283	-1.323283	4.44e-16
0.60	0.300	0.05	-0.698084	-0.698084	2.22e-16
0.20	0.100	0.20	+0.178159	+0.178159	2.78e-17
0.90	1.800	0.02	+0.369901	+0.369901	5.00e-16
0.75	1.000	0.15	+0.033406	+0.033406	6.31e-16
0.10	0.050	0.30	+1.141576	+1.141576	0.00e+00
0.55	1.200	0.08	-0.597116	-0.597116	6.66e-16
0.40	0.400	0.12	+0.086610	+0.086610	5.55e-17
1.00	2.485	0.07	-0.000000	-0.000000	8.88e-16

The octave deficit and the fixed point are the same quantity in different units, related by the factor `-2.885390 (delta/eta)`. Chapter 4 draws the consequences; the one that matters most is already visible in the formula. The conversion factor between them contains `delta`. The deficit itself does not.

3.8b The join is an equivalence, and that is worth more than an implication

Theorem 3.2 is stated with <=> at every step, and this is not pedantry. **Because the chain is a chain of equivalences, either side may be measured and the other inferred.** That gives two independent routes to the same verdict, and the two routes have different measurement costs.

```
ROUTE A -- through the geometry
  elicit the state vectors -> compute u -> compute F -> compute f
  then measure eta and c   -> compute f_min
  compare

ROUTE B -- through the ledger
  measure the stock Q_t over several periods -> fit delta, eta, c
  compute Q_inf directly
  convert to a deficit via D = -2.885390 (delta/eta) Q_inf
```

Route A needs no history. Route B needs no elicitation. A new pair has no history, so Route A is the only one available to them. An institution with twenty years of records has plenty of history and a very hard elicitation problem, so Route B may be cheaper. And a system where both are available is a system where the framework can be checked against itself, which is the single most valuable thing an equivalence buys: the two routes share no measurement, so agreement between them is evidence and disagreement localises the fault.

This is also where FVP-5, identifiability, bites. Route B recovers `eta` and `c` only as the combination `eta F − c` unless `F` is independently known — a fitted trajectory cannot separate "converts well from a poor geometry" from "converts poorly from a good one." So Route B alone gives the verdict and not the prescription. It tells you the relation is decaying and cannot tell you which lever binds. Chapter 4's lever analysis requires Route A.

3.8c Reading the twelve-set table

Two features of §3.7's table are worth naming, because they generalise.

The near-boundary rows are the interesting ones. Rows 6, 8 and 11 have deficits of `+0.178159`, `+0.033406` and `+0.086610` octaves — 214, 40 and 104 cents respectively. A relation 40 cents below its threshold is failing in a way that no participant could plausibly notice, because the per-period loss is `0.0579 * 0.15 = 0.0087` units of stored order. Over one period it is nothing. Over a hundred periods it is the whole difference between a practice that persists and one that has to be rebuilt from scratch each generation.

This is the specific value the diagnostic adds over judgement. Large deficits are visible without arithmetic — everybody in a `+6.46` octave situation knows something is wrong, even if nobody can say what. Small deficits are invisible without arithmetic and decisive over time, and they are the reason to compute rather than to sense.

The sign flips are not symmetric in cost. Row 4 sits at `−1.323283` octaves: comfortably accumulating, with room to absorb a worsening. Row 8 sits at `+0.033406`: one small increase in cost from a much worse position, and one small decrease from safety. Proximity to zero is itself a reading, and Chapter 4 §4.19 gives the reporting rule — publish the deficit *and* the sensitivity of the deficit to the parameter you are least sure of.

3.9 `f_min` contains no `delta` and no `N` — and both absences are substantive

Look at what survived Step 1 and Step 2:

```
f_min = k e^(2c/eta)          contains: k, c, eta
                              does NOT contain: delta, N, u, Q_0, t
```

THE ABSENCE OF `delta`. `delta` is the leak — the rate at which what has been built drains away. Trust reduces it. Ritual, place, written record and institution all reduce it. **None of those things changes the threshold a relation must clear in order to be building anything at all.**

This is R15 in frequency form, and it is sharper here than in its original statement. R15 says trust amplifies whatever sign the system already has. The frequency form says something stronger and more useful: an intervention on `delta` does not move the reading at all. The deficit before and after is numerically identical, because `delta` is not in it. Chapter 4's Worked Example demonstrates

this on the carried case, where a successful trust intervention moves `Q_inf` from −5.0119 to −9.0215 while the deficit stays at +4.338420 to every digit.

A gauge that fails to move under an intervention is not a broken gauge. It is a gauge telling you that you have not touched the mechanism it measures.

THE ABSENCE OF `N`. `N` does not appear in `f_min` either — but it appears in the *achieved* `f`, through `F = ln(1 + (N−1)u)`, and it appears in the ceiling `c/eta <= ln N`. So `N` is not a lever on the threshold; it is a lever on the reachable range. Raising `N` does not lower the bar. It raises the highest bar that can be cleared. Chapter 4 makes this precise and prices it.

3.10 The `delta = 0` case: no fixed point, same threshold

Theorem 3.5 requires `delta > 0`. If `delta = 0` the map is not a contraction, there is no fixed point, and the exact solution reduces to a straight line:

```
Q_t = Q_0 + t ∗ ( eta F − c )
```

The stock grows without bound if the drift is positive, and falls without bound if it is negative. There is no `Q_inf` to be positive or negative — **and the threshold is unchanged**, because the sign of the drift is the sign of the same numerator.

Verified on two carried cases:

```
eta = 0.30, c = 0.62, u = 0.068735:  drift = −0.451075   f = 1.4908  < f_min = 30.1588  agre
eta = 0.44, c = 0.40, u = 0.692373:  drift = +0.547598   f = 35.8882 > f_min =  2.9782  agre
```

So `f_min` governs both regimes: it decides where a leaky system settles, and it decides which way a lossless system drifts. That is a stronger statement than Theorem 3.2 alone, and it is available precisely because `delta` fell out at Step 1.

3.11 The limiting cases — FVP-3, worked

```
c = 0                  f_min = k e^0 = 0.483426 Hz = k exactly.
              Any relation whatsoever sustains, because any u > 0 gives f > k.
               A costless relation is sustained by the smallest difference.

c > 0, eta -> 0+       f_min -> infinity. No pitch suffices.
                   Computed at eta = 0.01, c = 0.5:  f_min = 1.2995e+43 Hz.
              A system that converts nothing into structure cannot be saved by
              any arrangement of its parts. This is a real, nameable phenomenon:
              it is the meeting that generates insight and stores none of it.

c/eta = ln N       f_min = k N^2 = 69.6133 Hz for N = 12. The ceiling. Chapter 4.

c/eta > ln N         f_min > k N^2, which exceeds the maximum achievable f.
                     The condition is unsatisfiable. NO arrangement works.

u = 0                  f = k, the ground essence. Sustains only if c = 0.
              And note R8: u = 0 arises from perfect agreement AND from perfect
              opposition, since u depends only on |rho|. The sycophant and the
```

```
                          contrarian produce the same reading: none.

  u = 1          f = k N^2 = 69.6133 Hz. The maximum. Sustains iff c/eta < ln N.
```

Every divergence corresponds to a real, nameable phenomenon, which is what FVP-3 requires. A formula whose limits are nonsense is wrong even where it looks reasonable.

3.12 The clauses the join specifically inherits

Not all of Chapter 2's clauses bear on `f_min` equally. These five do, directly:

(i) k is chosen. `f_min` is proportional to `k`. **But so is `f`, and the deficit is a difference of logarithms, so k cancels from the deficit exactly.** The verdict does not depend on `k` at all; only the absolute hertz values do. This is Result One of the essence law doing its work: the constant sets the reference pitch and nothing else.

(ii) S3 is the design axiom. `F` is defined through `N_eff`, which is defined by S3. Everything here uses it.

(iii) Surplus is a lower bound. §3.6, Asymmetry One.

(iv) The index needs a defensible ordering. `u` is computed from vectors over an ordered category set. A contested decomposition yields a contested `u`, a contested `f`, and a contested verdict. Chapters 5 and 6 own this.

(v) R1's demotion travels. Any `eta`, `c`, or `u` estimated through `D/P` inherits the ordinal demotion, and the resulting `f_min` may be published as an ordering only.

3.13 What the join does to precision

One practical note that belongs here rather than in Chapter 4, because it is a property of the substitution rather than of the result.

```
  d ln f_min / d(c/eta) = 2                    exact
```

The join is an error amplifier by construction, and by a known, constant factor. This is not a defect — it is what exponentiation does — but it has a design consequence. Since the deficit is `2.885390 (c/eta − F)`, and since both terms carry error, the deficit's uncertainty is `2.885390` times the combined uncertainty in nats. In octaves, one nat of parameter uncertainty anywhere is 2.885390 octaves of uncertainty in the verdict. Chapter 4 §4.19 gives the reporting rule that follows.

3.13b Why this book joins at the dyad and not at the group

The join is scale-free in form, exactly as the storage law is. Nothing in Steps 0 through 4 mentions how many parties there are. **That is precisely why it must be applied carefully above two.**

For a set of `M` members there are `M(M−1)/2` pairs, each with its own `u`. The group's surplus is not the surplus of the mean geometry — R2 forbids that — and the group's cost is not the sum of pairwise costs unless the coordination structure happens to be a complete graph. Brooks' quadratic communication paths against Paper A's logarithmic surplus is the whole tension, and it is Chapter 16's subject.

Three specific hazards live above two, all of them already measured:

```
the Jensen gap   F(mean u) >= mean F(u)          6.2% in Book Two's worked office (R2)
the aggregation  distortion cancels across sites    4.26x overstatement (R10)
the structure    identical aggregates, different    77% vs 21% surplus lost to the same
                topologies                        three-person departure (R16)
```

So this chapter joins at the dyad, where u is a single number and the cost is a single number, and hands the group case to Part III with the hazards named. A keystone demonstrated at the smallest scale that exhibits the phenomenon is a keystone whose failure modes can be seen.

3.14 Where the linear model stops applying, computed

The fixed point of a decaying case is a formal object, and this section says exactly how formal.

Take the carried case as found: `eta = 0.30`, `c = 0.62`, `delta = 0.09`, `u = 0.068735`, so `eta F = 0.168926` and the per-period drift is `−0.451075`. Start at `Q_0 = 1.0` and check Paper E's admissibility condition `C_t <= (1−delta) Q_t + eta S_t` at each step:

```
t = 0:  Q_t = +1.0000   right-hand side = 1.0789   c = 0.62   ADMISSIBLE
t = 1:  Q_t = +0.4589   right-hand side = 0.5865   c = 0.62   FAILS
t = 2:  Q_t = −0.0335   right-hand side = 0.1385   c = 0.62   FAILS
t = 3:  Q_t = −0.4815   right-hand side = −0.2693  c = 0.62   FAILS
```

Admissibility fails at `t = 1`. The stock goes negative at `t = 2`. The fixed point `−5.0119` is never reached and was never going to be.

Therefore: a negative `Q_inf` is a direction and a rate, not a destination. It says the relation is losing order, it says how fast (`−0.451075` per period, decaying geometrically toward the formal attractor with a half-life of 7.3496 periods), and it says that the linear model will stop describing the system before the model's own answer arrives. What actually happens past `t = 1` is outside this model: the parties renegotiate, reduce cost, or leave. The model's job was to say that something must give, and it did that at `t = 1`.

This is Paper E behaving as designed. The theorem does not hide its failure mode, and neither does this book.

3.14b `f_min` through the Formula Validation Protocol, gate by gate

Book Two's rule is that no formula appears without passing all seven gates in print. **`f_min` is a formula and the rule applies to it.** Here is its row.

FVP-1 DIMENSIONS — PASS. k is in hertz; `e^(2c/eta)` is dimensionless because `c` and `eta` are both per-period quantities whose ratio is a pure number of nats. Both sides are hertz.

FVP-2 SCALE TYPE — CONDITIONAL. `c/eta` requires a ratio scale with a true zero on both terms. `c` has a natural zero (no cost) and `eta` has one (nothing converts), so the *quantities* admit ratio scaling — but whether a given estimate of them is ratio-scaled is a property of the instrument, not of the formula. This gate is passed by the user or not at all, and R1's demotion applies to any estimate obtained through `D/P`. A formula whose scale gate is conditional must say so at every use, and SPEC 3.4 requires it.

FVP-3 LIMITING CASES — PASS. §3.11 works all six limits and each names a real phenomenon: the costless relation, the non-converting group that generates insight and stores none, the ceiling, the unsatisfiable regime, the ground essence, and the orthogonal maximum.

FVP-4 FIXED POINTS — PASS. The invariant is `f = f_min`, equivalently `Q_inf = 0`, equivalently `u* = (e^(c/eta) − 1)/(N−1)` (Corollary 3.3). Its meaning is stated and is not flattering: the relation pays for itself exactly and banks nothing.

FVP-5 IDENTIFIABILITY — PARTIAL, AND THE FAILURE IS INFORMATIVE. From a fitted trajectory alone, `eta` and `F` are not separately recoverable — only the combination `eta F − c` is. §3.8b demonstrates two admissible decompositions of the carried case that are indistinguishable to any ledger. The verdict is identifiable; the prescription is not. Recovering the prescription requires the geometry, which means elicitation.

FVP-6 FALSIFICATION — INHERITED. `f_min` adds no empirical claim, so it has no falsifier of its own; it inherits the storage law's. The test is Paper E §5.3's template — preregister, fit, freeze, report held-out, beat four baselines — with all three measured holes in the baseline suite carried in full (R13a, R4, R24), and with design controls rather than more surrogates.

FVP-7 REPRODUCIBILITY — NOT ESTABLISHED, AND THIS IS THE GATE THAT DECIDES. Whether the same operationalisation of `u`, `eta` and `c` recovers the same parameters across people and across labs is unknown for these constructs. It is also the gate most often skipped, and Book Two's own research programme computed a 55.3% chance of dying at or before this stage under the author's badged priors. No chapter of this book may report FVP-7 as passed.

The row, compactly:

```
  FVP−1  dimensions        PASS
 FVP−2  scale type      CONDITIONAL −− on the user's instrument; R1 travels
  FVP−3  limiting cases    PASS
  FVP−4  fixed points      PASS
  FVP−5  identifiability   PARTIAL −− verdict yes, prescription no
FVP−6 falsification   INHERITED from the storage law, with all three baseline holes
  FVP−7  reproducibility   NOT ESTABLISHED
```

A keystone that publishes its own two open gates is more usable than one that claims seven passes. The two open gates are exactly where the work is, and Chapter 27 keeps the largest of them in view for the rest of the book.

3.15 The reading, as a procedure

Everything above collapses into eight steps. Chapter 4 turns them into the book's standing instrument; here they are stated in the order the derivation licenses.

```
1 FIX N.     Declare the category set and its ordering. State that the frequency
             inherits exactly the defensibility of the count.

  2  REPRESENT.     Elicit state vectors with OVERLAPPING SUPPORT. One-
hot vectors give
             u = 1.000000 for everyone and read the instrument, not the subject.

3 COMPUTE u.   u = 1 − (x . y)^2. One construction, used everywhere in this book.
```

```
    4  COMPUTE f.    F = ln(1 + (N-1)u);  f = k e^(2F) = k (1 + (N-1)u)^2.
                     Report the folded value and its j.

   5  MEASURE eta, c ON ONE CLOCK. Declare the period. Declare the scale status.

    6  CHECK THE CEILING FIRST.  Is c/eta <= ln N ?
              NO  -> the case is structural. Issue no tuning prescription. Chapter 4.
                     YES -> continue.

    7  COMPUTE f_min = k e^(2c/eta) AND THE DEFICIT D = 2.885390 (c/eta) -
log2(f/k).

      8   CROSS-CHECK.   Q_inf = (eta F - c)/delta, and verify D = -
2.885390 (delta/eta) Q_inf.
              If a negative Q_inf is reported, state the period at which admissibility
                     fails.
```

Step 6 is placed where it is deliberately. A ceiling check performed after the lever analysis is a ceiling check performed too late — the analyst has already produced a prescription, and prescriptions are hard to withdraw once they have been said out loud. The ceiling is cheap to test and it changes the kind of answer, so it goes first.

The Specification

SPEC 3.1 — DECLARE `delta > 0` AND `eta > 0` wherever Theorem 3.2 is invoked. If `delta = 0`, cite §3.10 and report the drift rather than the fixed point.

SPEC 3.2 — DECLARE THE CLOCK. `eta`, `c` and `delta` are reported on one stated period, and the period is named in the same passage as the ratio.

SPEC 3.3 — DECLARE `N`. Every reading states the number of categories used, because `N` sets both the achieved `F` and the ceiling.

SPEC 3.4 — DECLARE THE SCALE STATUS of `eta`, `c` and `u`. If any is ordinal, publish the ordering and not the frequency (SPEC 1.1).

SPEC 3.5 — REPORT `f`, `f_min`, AND THE DEFICIT TOGETHER. A frequency without its threshold is not a reading. A threshold without its frequency is not a reading either.

SPEC 3.6 — REPORT THE FOLDED VALUE WITH ITS `j` whenever a frequency is given in the audible band.

SPEC 3.7 — CHECK THE CEILING BEFORE PRESCRIBING. Compare `c/eta` to `ln N` first. If it exceeds `ln N`, the case is structural and no tuning prescription may be issued. Chapter 4 owns this test.

SPEC 3.8 — CHECK ADMISSIBILITY BEFORE REPORTING A NEGATIVE FIXED POINT, and state the period at which it fails.

SPEC 3.9 — DO NOT AGGREGATE. `f_min` is computed per relation, per site, per period. Ratios do not average (R10, 4.26x), and identical aggregate statistics hide structures with a 77%-versus-21% difference in outcome (R16).

SPEC 3.10 — USE THE MEAN OF `F`, NOT `F` OF THE MEAN when a reading covers more than one pair (R2).

SPEC 3.11 — RESTATE THE ESSENCE CLAUSE. Every passage printing a hertz value states that it is an essence frequency and claims no physical resonance.

SPEC 3.12 — NO CLINICAL LANGUAGE. A reading is not an assessment, a diagnosis, or a prognosis for any person.

The Failure Modes

FM 3.1 — DIVISION BY A ZERO LEAK. *Signature:* an infinite or undefined `Q_inf` reported for a system described as having no decay. *Mechanism:* Theorem 3.5 requires `delta > 0`; at `delta = 0` there is no fixed point. *Fix:* §3.10. Report the drift `eta F − c`, note that `f_min` still decides its sign, and say that the stock is unbounded in that direction.

FM 3.2 — THE ZERO-CONVERSION SYSTEM READ AS A TUNING PROBLEM. *Signature:* an enormous `f_min` — `1.2995e+43 Hz` at `eta = 0.01, c = 0.5` — treated as a target. *Mechanism:* as `eta -> 0`, `f_min -> infinity`. Nothing converts into structure, so nothing accumulates, and no geometry is relevant. *Fix:* recognise it as its own condition. The characteristic form is a group that reliably generates insight and reliably stores none of it: high `F`, high `c`, `eta` near zero. The remedy is a storage mechanism — a written record, a decision log, a durable artefact — not a better conversation.

FM 3.3 — THE MISMATCHED CLOCK. *Signature:* a `c/eta` computed from a weekly cost and an annual conversion rate. *Mechanism:* the ratio is dimensionless only on a shared clock. R16 measured a 90.1x period-normalisation trap; at two nats of amplification per unit, that is not an error bar. *Fix:* SPEC 3.2. Convert both to one period before dividing, and print the period.

FM 3.4 — `f_min` READ AS A TONE. *Signature:* any sentence proposing to play `f_min` to someone. *Mechanism:* the register slip of FM 1.9. `f_min` is a threshold in a computation. *Fix:* essence, not resonance, in the sentence. Rendering is Chapter 26's, under Chapter 27's open problem and the absolute no-subliminals rule.

FM 3.5 — THE FORGOTTEN CEILING. *Signature:* a tuning prescription issued for a system with `c/eta > ln N`. *Mechanism:* the prescription is unsatisfiable and the reader cannot tell, because the deficit is just a larger positive number and looks like more of the same problem. *Fix:* SPEC 3.7 — compare `c/eta` to `ln N` **first**, before any lever analysis. Chapter 4 §4.11.

FM 3.6 — THE JENSEN SHORTCUT. *Signature:* a group reading produced by averaging u across pairs and taking one logarithm. *Mechanism:* strict concavity means `F(mean u) >= mean F(u)`, so the shortcut inflates the surplus and therefore inflates `f`, and therefore under-reports the deficit. *Fix:* R2. Worth 6.2% in Book Two's worked office; more when the spread in u is larger.

FM 3.7 — THE AGGREGATED RATIO. *Signature:* one `c/eta` for a whole organisation. *Mechanism:* opposite-direction distortions cancel. R10 measured a **4.26x** overstatement of bankable

input from exactly this move. *Fix:* per relation, per site. Never aggregate a ratio across sites.

FM 3.8 — THE NEGATIVE STOCK TAKEN LITERALLY. *Signature:* a plan that anticipates arriving at `Q_inf = −5.0119`. *Mechanism:* admissibility fails first — at `t = 1` in the carried case — and the linear model stops applying. *Fix:* §3.14. Read the number as a direction and a rate.

FM 3.9 — REPRESENTATIONAL UNDER-CAPTURE READ AS A VERDICT. *Signature:* a confident DECAYS on a pair whose real differences were never elicited. *Mechanism:* u measures the represented difference. An instrument that cannot see a difference reports zero difference, which lowers `f`, which manufactures a deficit. *Fix:* §3.6, Asymmetry One. A negative verdict must be checked against the elicitation before it is acted on; a positive verdict need not be. And check the opposite trap too: one-hot framings give `u = 1.000000` for everyone and manufacture a surplus that is a property of the encoding.

FM 3.10 — THE ALGEBRA MISTAKEN FOR EVIDENCE. *Signature:* the join presented as a discovery about relationships. *Mechanism:* the derivation is impressive-looking and contentless. It is four strictly increasing transformations applied to an inequality. *Fix:* say what §3.4 says. The empirical content is in `eta`, `c`, u and `N`, and the algebra cannot supply any of it.

Worked Example

THE CARRIED DYAD, JOINED — BOTH DIRECTIONS, BOTH PHASES

This case is CONSTRUCTED. It is the dyad of SPINE §3.2 and it is used throughout this book. Its parameters are chosen, not observed. Nothing here is an assessment of any person and no clinical claim is made or implied.

Phase 2a — as found

```
A = {4: 0.7, 6: 0.5, 9: 0.4}
B = {4: 0.6, 6: 0.55, 9: 0.3, 10: 0.2}

eta = 0.30   c = 0.62   delta = 0.09      (one declared period, N = 12)
```

Direction one: from the geometry to the pitch.

```
cos theta = 0.965021
u         = 1 − 0.965021^2       = 0.068735
F         = ln(1 + 11 ∗ 0.068735) = 0.563085 nats
N_eff     = 1.7561
f        = k ∗ 1.7561^2         = 1.4908 Hz     folded 381.644 Hz (j = 8)
octaves present = log2(1.4908/k)  = 1.624719
```

Direction two: from the operating parameters to the threshold.

```
c/eta            = 0.62 / 0.30            = 2.066667
  check the ceiling: 2.066667 <= ln 12 = 2.484907          BELOW −−
a tuning problem, not structural
f_min        = k ∗ e^(2 ∗ 2.066667)  = 30.1588 Hz  folded 241.270 Hz (j = 3)
```

```
octaves required = 2.885390 * 2.066667    = 5.963140
```

The join.

```
f = 1.4908 Hz  <  f_min = 30.1588 Hz          =>  DECAYS

octave deficit D = 5.963140 − 1.624719 = +4.338420 octaves  ( = +5206.1 cents )
the ratio f_min/f = 20.229940, which is 4.3384 octaves. Same number, stated twice.
```

Cross-check through the fixed point, computed independently:

```
Q_inf = (0.30 * 0.563085 − 0.62) / 0.09 = −0.451075 / 0.09 = −5.0119        NEGATIVE
```

Cross-check through the deficit identity:

```
−2.885390 * (delta/eta) * Q_inf = −2.885390 * (0.09/0.30) * (−5.011940) = +4.338420
```

Three routes, one number. The frequencies say +4.338420. The fixed point says −5.0119. The identity converts one into the other exactly, with an error of 8.88e−16.

And the honest note that must travel with the negative fixed point: admissibility fails at t = 1 (§3.14). The pair does not arrive at −5.0119. They arrive at the point where something has to give, one period in.

What the reading says and does not say

It says: this bond is short by 4.34 octaves; the shortfall is structural in the sense of being a property of the geometry and the cost, not of anyone's intentions; and the case is *tunable*, because c/eta = 2.066667 is below ln 12 = 2.484907.

It does not say: that either person is deficient, that the bond should end, that anyone is at fault, or that anything about either person has been assessed. **The reading is of a configuration of four parameters. It is not a reading of a person, and this book never lets that elide.**

Phase 2b — after one intervention

CONSTRUCTED. B takes on two states A does not hold, and the pair adopts a decision log. The first change raises u; the second raises eta (interaction now sticks) and lowers c (coordination is cheaper when the record does the remembering). One realising vector, given for concreteness:

```
B' = {2: 0.95, 4: 0.60, 6: 0.55, 9: 0.30, 10: 0.20, 12: 0.8391}

cos theta = 0.554643     u = 0.692373      (SPINE carries this as 0.6924)

F     = ln(1 + 11 * 0.692373) = 2.153633 nats
N_eff = 8.6161
f     = k * 8.6161^2          = 35.8882 Hz     folded 287.106 Hz (j = 3)
octaves present = 6.214071

eta = 0.44      c = 0.40      delta = 0.09
c/eta            = 0.909091
```

```
 check the ceiling: 0.909091 <= 2.484907          BELOW, comfortably
f_min          = k e^(2 * 0.909091) = 2.9782 Hz   folded 381.212 Hz (j = 7)
 octaves required = 2.623082

D = 2.623082 - 6.214071 = -3.590989 octaves  ( = -4309.2 cents )    ACCUMULATES
```

Cross-checks:

```
 Q_inf = (0.44 * 2.153633 - 0.40)/0.09 = +6.0844
-2.885390 * (0.09/0.44) * 6.0844     = -3.590989     identity holds exactly
trajectory from Q_0 = 1.0:  t=1  +1.4576   t=5  +2.9116   t=20  +5.3134   t=40  +5.9675
 admissibility holds at every step, since the drift is positive
 convergence:  |Q_20 - Q_inf| = 0.7710275635960624
               (1-0.09)^20 * |Q_0 - Q_inf| = 0.7710275635960623
```

The bond has moved from 4.34 octaves short to 3.59 octaves in surplus — a swing of 7.93 octaves. In the fixed-point register the same swing is −5.0119 to +6.0844.

One structural note the join makes visible and neither half did

At phase 2b, `F = 2.153633` against a ceiling of `ln 12 = 2.484907`. The pitch headroom is

```
2.885390 * (2.484907 - 2.153633) = 0.9559 octaves
```

The repair worked, and it worked by spending most of the range that twelve categories make available. At 2a there were 5.5452 octaves of pitch headroom; at 2b there are 0.9559. Further gains of the same kind are not available from this category set at all — they would require raising N, which means finding genuinely distinct categories, not finer gradations of the twelve.

Neither half of the equation says this on its own. The bridge gives a pitch with no threshold; the storage law gives a verdict with no ceiling in view. Joined, the reading places the relation on a scale with a top, and says how much of the scale is left. That is what Chapter 4 is for.

The join checked against itself, both routes

§3.8b claimed two independent routes to the verdict. The carried case at phase 2b lets both be run.

Route A, through the geometry: elicit the vectors, get `u = 0.692373`, get `F = 2.153633`, get `f = 35.8882 Hz`; measure `eta = 0.44` and `c = 0.40`, get `f_min = 2.9782 Hz`; the deficit is `−3.590989`.

Route B, through the ledger: observe the stock over periods and fit. From `Q_0 = 1.0` the observed series is `1.0000, 1.4576, 1.8740, 2.2530, ...`, converging on +6.0844 with a transient half-life of 7.3496 periods. Fitting `delta = 0.09` from the decay of the transient and `Q_inf` from the asymptote, the deficit follows as `−2.885390 * (0.09/0.44) * 6.0844 = −3.590989`.

The two routes share no measurement and agree to 1e−15. Route A used no history at all; Route B used no elicitation at all.

And the asymmetry §3.8b predicted is visible: Route B cannot tell you what to do. From the ledger alone, `eta F − c = +0.547598` is one number, and there is no way to know which decomposition produced it. Two admissible ones, both consistent with every observation Route B can make:

```
eta = 0.44, F = 2.153633 (u = 0.692373)  -> f = 35.8882 Hz, pitch headroom 0.9559 oct
eta = 0.50, F = 1.895197 (u = 0.513987)  -> f = 21.4031 Hz, pitch headroom 1.7015 oct
```

Identical ledgers. Identical verdicts. Different amounts of room left, and therefore different next moves. The first pair has nearly exhausted what twelve categories can give them and must raise `N` to gain more; the second still has most of an octave and a half to buy in `u`. Route B says the bond accumulates. Only Route A says how much scale is left, which is the fact that determines what to do next.

The result now exists. Chapter 4 states it completely — the table, the octave form, the ceiling, the deficit, and the four levers — in the form that twenty-six later chapters will cite rather than re-derive.

Chapter 4 . The Minimum Sustainable Frequency

This is the chapter the book exists for, and it is written to be cited rather than re-derived. Twenty-six later chapters will use what is stated here. **They will not restate it. They will point at it.** So everything this book means by `f_min`, by the octave deficit, by the ceiling, and by the four levers is set out here completely, with every number computed and every boundary named.

The keystone, in one block:

```
f_min = k * e^( 2c / eta )                    k = 0.483425732988414 Hz

octaves required  = 2 (c/eta) / ln 2  =  2.885390081777927 * (c/eta)

octave deficit  D = 2.885390 * (c/eta)  -  log2( f / k )

    D <= 0    sustainable as it stands
    D >  0    short by that many octaves

CEILING:  c/eta <= ln N.  For N = 12: c/eta <= 2.484907, f_min <= 69.6133 Hz.
        Above it, NO arrangement of those categories is sustainable at all.

FOUR LEVERS:  raise u  .  raise eta  .  cut c  .  raise N.  And delta is NOT one of them.
```

The Law

4.1 Definition

DEFINITION 4.1 — THE MINIMUM SUSTAINABLE FREQUENCY. For a relation with cost c per period and conversion efficiency `eta > 0` measured on the same period,

```
f_min  :=  k * e^( 2c / eta )
```

`k = 0.483425732988414 Hz` is the ground essence, badged as a chosen reference constant (CANON_SONIC §6; Chapter 2 §2.1). `f_min` is an essence frequency. No person, bond, tissue, molecule, room or institution is claimed to resonate at it, or at any value in this chapter.

4.2 The three regimes

THEOREM 4.2. Under Paper E Theorem 3.5's conditions — `delta > 0`, `eta > 0`, constant `u` and `c` — and with `f = k e^(2F(u))` the relation's essence frequency:

```
f  >  f_min    <=>   Q_inf > 0     the relation ACCUMULATES order
f  =  f_min    <=>   Q_inf = 0     it HOLDS, and nothing more
f  <  f_min    <=>   Q_inf < 0     it DECAYS, whatever anyone intends
```

Proof: Chapter 3 §3.2, Steps 0–4, a chain of equivalences under strictly increasing transformations. *Verified* at twelve parameter sets in Chapter 3 §3.7, sign-agreement in all twelve including the exact-equality case.

The middle line is the one readers skip and the one that costs them. `f = f_min` is not a success condition. It is the condition under which a relation pays exactly for itself and banks nothing. A relation at its minimum sustainable frequency is not sustainable in any colloquial sense; it is stationary, and it has no reserve against any increase in cost.

4.3 The octave form

THEOREM 4.3. Taking the base-2 logarithm of Definition 4.1,

```
log2( f_min / k )  =  2 (c/eta) / ln 2  =  2.885390081777927 * (c/eta)
```

EVERY UNIT OF COST-TO-CONVERSION RATIO COSTS 2.885390 OCTAVES OF REQUIRED PITCH.

The constant is exact: `2/ln 2 = 2.885390081777927`. It is the same constant that converts one nat of surplus into pitch (Chapter 2 §2.7), and it appears here for exactly that reason — `c/eta` is measured in nats of surplus per period, and the conversion from nats to octaves is one number.

Equivalent statements, all exact:

```
1 nat of required surplus  =  2.885390 octaves  =  3462.4681 cents  =  1.442695 bits
 1 octave of pitch         =  0.346574 nats     =  0.5 bits EXACTLY
```

One octave is exactly half a bit. That line prices every lever in §4.13.

4.4 The table

Computed at `k = 0.483425732988414 Hz`, `N = 12`, folded into 220–440 Hz with the transposition `j` given:

c/eta	f_min (Hz)	octaves above k	N_eff required	folded 220–440
0.00	0.4834	0.000000	1.0000	247.514 (j=9)
0.10	0.5905	0.288539	1.1052	302.314 (j=9)
0.20	0.7212	0.577078	1.2214	369.247 (j=9)
0.30	0.8809	0.865617	1.3499	225.500 (j=8)
0.40	1.0759	1.154156	1.4918	275.426 (j=8)
0.50	1.3141	1.442695	1.6487	336.406 (j=8)
0.60	1.6050	1.731234	1.8221	410.888 (j=8)
0.70	1.9604	2.019773	2.0138	250.930 (j=7)
0.80	2.3944	2.308312	2.2255	306.486 (j=7)

```
   0.90          2.9246        2.596851           2.4596      374.343 (j=7)
   1.00          3.5721        2.885390           2.7183      228.612 (j=6)
   1.10          4.3629        3.173929           3.0042      279.227 (j=6)
   1.20          5.3289        3.462468           3.3201      341.049 (j=6)
   1.30          6.5087        3.751007           3.6693      416.558 (j=6)
   1.40          7.9498        4.039546           4.0552      254.392 (j=5)
   1.50          9.7099        4.328085           4.4817      310.716 (j=5)
   1.60         11.8597        4.616624           4.9530      379.509 (j=5)
   1.70         14.4854        4.905163           5.4739      231.767 (j=4)
   1.80         17.6925        5.193702           6.0496      283.080 (j=4)
   1.90         21.6097        5.482241           6.6859      345.755 (j=4)
   2.00         26.3942        5.770780           7.3891      422.306 (j=4)
   2.10         32.2379        6.059319           8.1662      257.903 (j=3)
   2.20         39.3754        6.347858           9.0250      315.004 (j=3)
   2.30         48.0933        6.636397           9.9742      384.746 (j=3)
   2.40         58.7413        6.924936          11.0232      234.965 (j=2)
  2.4849       69.6133       7.169925          12.0000     278.453 (j=2)   <-
- THE CEILING, N=12
   2.50         71.7467        7.213475          12.1825      286.987 (j=2)
   2.60         87.6317        7.502014          13.4637      350.527 (j=2)
   2.70        107.0336        7.790553          14.8797      428.134 (j=2)
   2.80        130.7311        8.079092          16.4446      261.462 (j=1)
   2.90        159.6753        8.367631          18.1741      319.351 (j=1)
   3.00        195.0279        8.656170          20.0855      390.056 (j=1)
```

The fourth column is the most useful and the least obvious. `N_eff required = e^(c/eta)` is the number of genuinely distinct perspectives the relation must field just to break even. At `c/eta = 1.00` it is 2.7183 — a two-person bond must be worth more than 2.7 people's worth of distinct view. At `c/eta = 2.00` it is 7.3891. At `c/eta = 2.4849` it is exactly 12, which is all of them, and that is the ceiling arriving.

4.5 The octave deficit

DEFINITION 4.4 — THE OCTAVE DEFICIT. For a relation with measured essence frequency `f` and measured cost ratio `c/eta`,

```
D  :=  2.885390 * (c/eta)  −  log2( f / k )

D <= 0    sustainable as it stands
D >  0    the relation is short by D octaves
```

This is the book's core instrument. Every practical chapter must be able to produce it, and every reading in this book is reported as a `D` with its sign, its magnitude, and the ceiling test that qualifies it.

THEOREM 4.5 — THE DEFICIT IDENTITY.

```
D  =  2.885390 * ( c/eta  −  F )  =  −2.885390 * (delta/eta) * Q_inf
```

Proof: Chapter 3 §3.8. *Verified* at twelve parameter sets to a worst error of `8.88e−16`.

Three readings of the same identity, and all three matter:

(i) `D = 2.885390 (c/eta − F)`. The deficit is the surplus shortfall, converted. A relation short by one nat of surplus is short by 2.885390 octaves. There is no other exchange rate.

(ii) `D = −2.885390 (delta/eta) Q_inf`. The deficit and the fixed point are the same quantity in different units. A pitch reading and a destiny reading are the same reading. Chapter 28 builds on this.

(iii) k cancels. `D` is a difference of logarithms in which the chosen constant appears in both terms. The verdict does not depend on k at all. Only the absolute hertz values do. This is the strongest available defence of the constant: it sets the reference pitch and is invisible to every verdict.

Deficit-to-fixed-point conversion, tabulated at `delta/eta = 0.30` (the carried case's ratio):

D (octaves)	f / f_min	Q_inf
−6.0	64.0000	+6.9315
−4.0	16.0000	+4.6210
−2.0	4.0000	+2.3105
−1.0	2.0000	+1.1552
−0.5	1.4142	+0.5776
0.0	1.0000	0.0000
+0.5	0.7071	−0.5776
+1.0	0.5000	−1.1552
+2.0	0.2500	−2.3105
+4.0	0.0625	−4.6210
+6.0	0.0156	−6.9315

A deficit of one octave means the relation is running at half the frequency it needs. A deficit of six octaves means it is running at 1.56% of what it needs, which is the situation Chapter 18 addresses and which no single adjustment repairs.

4.6 The ceiling — Paper E Corollary 3.4 in the frequency register

THEOREM 4.6 — THE CEILING. Since `F <= ln N` (Paper D, Corollary 2.2), the condition `F > c/eta` is satisfiable only if

```
c / eta  <=  ln N        equivalently        f_min  <=  k N^2
```

If `c/eta > ln N`, NO arrangement of those `N` categories is sustainable. Not a better pairing. Not a better schedule. Not a better conversation. Nothing.

Proof: `F > c/eta >= ln N >= F` is a contradiction. **QED.**

For the twelve-state ladder of this book:

```
  ln 12       = 2.484906649788
k * 12^2   = 69.61330555033166 Hz   -> folded 278.453 Hz (j = 2), pitch class 3
  max octaves = 2.885390 * ln 12 = 7.169925
```

For a twelve-category system, `c/eta` may not exceed 2.4849, and the corresponding `f_min` is 69.6133 Hz. Above that, the relation cannot be tuned. It must be restructured.

The ceiling by category count, computed:

N	ln N	f_ceiling = k N^2 (Hz)	max octaves above k
2	0.693147	1.9337	2.000000
3	1.098612	4.3508	3.169925
4	1.386294	7.7348	4.000000
5	1.609438	12.0856	4.643856
6	1.791759	17.4033	5.169925
7	1.945910	23.6879	5.614710
8	2.079442	30.9392	6.000000
9	2.197225	39.1575	6.339850
10	2.302585	48.3426	6.643856
12	2.484907	69.6133	7.169925
16	2.772589	123.7570	8.000000
17	2.833213	139.7100	8.174926
20	2.995732	193.3703	8.643856
25	3.218876	302.1411	9.287712
41	3.713572	812.6387	10.715104
64	4.158883	1980.1118	12.000000

COROLLARY 4.7 — THE MINIMUM CATEGORY COUNT. A cost ratio of `c/eta` requires at least `N >= e^(c/eta)` categories to be sustainable at all:

```
c/eta = 0.5000  ->  N >=  1.6487  ->  N_min =  2
c/eta = 1.0000  ->  N >=  2.7183  ->  N_min =  3
c/eta = 1.5000  ->  N >=  4.4817  ->  N_min =  5
c/eta = 2.0000  ->  N >=  7.3891  ->  N_min =  8
c/eta = 2.4849  ->  N >= 11.9999  ->  N_min = 12
c/eta = 2.5000  ->  N >= 12.1825  ->  N_min = 13
c/eta = 3.0000  ->  N >= 20.0855  ->  N_min = 21
c/eta = 3.5000  ->  N >= 33.1155  ->  N_min = 34
c/eta = 4.0000  ->  N >= 54.5982  ->  N_min = 55
```

The growth is exponential, and that is the whole content of the corollary. Going from `c/eta = 3` to `c/eta = 4` — a 33% increase in relative cost — raises the minimum viable category count from 21 to 55. Cost does not merely make a relation harder. Past a point it makes the relation's category set inadequate, and category sets are expensive to enlarge honestly.

4.7 The hold line

COROLLARY 4.8. The relation holds exactly (`D = 0`, `Q_inf = 0`) when

```
u*  =  ( e^(c/eta) − 1 ) / (N − 1)
```

Verified for `N = 12`:

```
c/eta = 0.500000  ->  u* = 0.058975   f = f_min =  1.3141 Hz
c/eta = 1.000000  ->  u* = 0.156207   f = f_min =  3.5721 Hz
c/eta = 2.000000  ->  u* = 0.580823   f = f_min = 26.3942 Hz
c/eta = 2.484907  ->  u* = 1.000000   f = f_min = 69.6133 Hz
```

The last row is the ceiling stated in the geometry rather than in the cost: at `c/eta = ln N` the required non-parallelism is exactly 1 — perfect orthogonality — and there is nothing above it.

4.8 The four levers

These are the only four. There is no fifth, and `delta` is not one of them.

```
    LEVER              acts on            partial derivative of D
    ------------------------------------------------------------------------
----------
   L1  raise u       the geometry      dD/du   = -2.885390 (N-1)/(1 + (N-1)u)
    L2  raise eta       the conversion     dD/deta  = -2.885390 c / eta^2
    L3  cut c           the cost           dD/dc    = +2.885390 / eta
   L4  raise N        the category set   dD/dN    = -2.885390 u / (1 + (N-1)u)
```

Each is derived directly from `D = 2.885390 (c/eta - F)` with `F = ln(1 + (N-1)u)`.

THEOREM 4.9 — `delta` IS NOT A LEVER ON THE DEFICIT.

```
    D = 2.885390 * (c/eta) - log2(f/k)          contains k, c, eta, N, u
                                                does NOT contain delta
```

Proof: by inspection of Definition 4.4 and of `f = k(1 + (N-1)u)^2`. **QED.**

Consequence, and it is the single most practically important statement in this chapter: an intervention that acts only on `delta` cannot change the deficit by any amount whatsoever. Trust acts on `delta`. Ritual, place, written record and institutional memory act on `delta`. All of them are valuable and none of them moves the reading.

What `delta` does do is scale `Q_inf`, through the identity `Q_inf = -D eta/(2.885390 delta)`. **It multiplies the existing sign.** In a relation with `D < 0` it makes a good destination better. In a relation with `D > 0` it makes a bad destination worse, and this is R15 in frequency form. Chapter 15 owns the order-of-operations rule that follows; the Worked Example below demonstrates it on the carried case.

4.9 Honesty clauses, restated because this is the chapter people will quote

(1) ESSENCE, NOT RESONANCE. Every frequency here is an essence frequency: a position on a logarithmic scale of surplus. **Nothing is claimed to vibrate.**

(2) NO CLINICAL CLAIM. The twelve states are not diagnoses; a reading is not an assessment; no procedure here is a treatment. Nothing in this chapter describes, evaluates, or prescribes for any person's health.

(3) THE READING INHERITS ITS INPUTS. `D` is exactly as defensible as `u`, `eta`, `c` and `N`. A contested decomposition yields a contested frequency. If `eta` or `c` is ordinal, publish the ordering and not the number (SPEC 1.1; R1's demotion travels).

(4) S3 IS THE DESIGN AXIOM. `F` is defined through `N_eff`, which is defined by the affine axiom. Everything in this chapter uses it.

(5) SURPLUS IS A LOWER BOUND. `f` is a floor. A verdict of ACCUMULATES is robust to representational under-capture; a verdict of DECAYS is not (Chapter 3 §3.6).

(6) CONSTRUCTED CASES. Every case in this book is constructed and labelled as such.

(7) CONSENT IS STRUCTURAL. No intervention derived from a reading may remove a person's ability to decline. Chapter 18 owns this and treats it as structure, not advice.

The Engineering

4.10 How to read the table

Three columns, three questions.

"How much pitch do I need?" — the second column, `f_min`. This is the threshold in hertz. It is the number to compare a measured `f` against, and it is the number to fold and publish with its `j` if the reading is to be displayed.

"How far is that?" — the third column, octaves above k. This is the threshold expressed as an interval from the ground essence, and it is the column to use when comparing two relations with different costs, because intervals are what compose.

"How many distinct views must this relation field?" — the fourth column, `N_eff = e^(c/eta)`. This is the column to hand to a person who does not want a frequency. It converts the whole apparatus into a sentence: *at your current costs, the two of you must be worth 7.4 genuinely different people to break even.*

4.11 The ceiling test comes first, and here is why

SPEC 4.1 (stated early because everything depends on it): compare `c/eta` to `ln N` before computing anything else.

The reason is that a deficit does not announce its own kind. A `+5.78` octave deficit at `c/eta = 2.566667` and a `+5.78` octave deficit at `c/eta = 2.100000` with a much lower `f` look identical in the reading and are entirely different problems. The first is unsatisfiable by any arrangement of twelve categories. The second is a large tuning problem.

```
if  c/eta <= ln N    the case is TUNABLE.      Some assignment of u closes it.
if  c/eta >  ln N    the case is STRUCTURAL.   No assignment of u closes it.
                                                 Only L2, L3 or L4 can help.
```

A prescription issued before this test may be unsatisfiable, and unsatisfiable prescriptions are worse than none — they consume the effort that restructuring needed.

4.12 The two gauges, and why they can disagree

Book Two's master gauge is headroom:

```
h_t = eta ln N − delta Q_t − C_t
```

This chapter's gauge is the deficit. **They answer different questions and can disagree in sign.**

Computed on the carried case as found, at `Q = 1`:

```
  h  = 0.30 ∗ 2.484907 − 0.09 ∗ 1 − 0.62  =  +0.0355      growth available
D  = +4.338420 octaves                       short by four and a third octaves
```

```
Q_inf = −5.0119                                                    decaying
```

Both are correct. Headroom uses the *ceiling* `ln N` — it asks what the best possible geometry could support at this cost. The deficit uses the *achieved* `F` — it asks what this geometry is doing. A system can have room to grow in principle and be shrinking in fact, and this case is exactly that: `h > 0` says the ceiling has not been hit, `D > 0` says the current arrangement is nowhere near it.

The bridge between them is R6's identity, `h_inf = eta (ln N − F(u*))`, verified here at `0.576547` by both routes. In pitch units:

```
PITCH HEADROOM = 2.885390 * ( ln N − F ) = octaves between the achieved f and the ceilin
```

For the carried case as found, that is **5.5452 octaves of unused range** against a requirement of 5.9631 and a present position of 1.6247. The relation needs 5.96 octaves, has 1.62, and the ceiling is at 7.17. The requirement fits under the ceiling with 1.21 octaves to spare, which is exactly why the case is tunable.

4.13 The price of an octave, and why the last one cannot be bought

```
1 octave of pitch = 0.346574 nats of surplus = exactly 0.5 bits
equivalently: N_eff must rise by a factor of sqrt(2) = 1.414214
```

Since `N_eff = 1 + (N−1)u`, buying an octave means multiplying `1 + 11u` by `sqrt(2)`. The cost in u is not constant. Computed:

```
from u = 0.068735   ->  u must reach 0.134861     (a rise of 0.066126)
from u = 0.300000   ->  u must reach 0.461920     (a rise of 0.161920)
from u = 0.500000   ->  u must reach 0.744763     (a rise of 0.244763)
from u = 0.800000   ->  u must reach 1.169027     IMPOSSIBLE: u <= 1
```

The octaves get more expensive as you climb, and the last one cannot be bought in u at all. Above `u = (12/sqrt(2) − 1)/11 = 0.680480`, no further octave is available from the geometry at all, no matter what anyone does. This is the ceiling appearing as a marginal cost rather than as a bound, and it is the honest answer to "why not just find more difference?"

4.14 LEVER 1 — RAISE u

What it is. Increase the represented non-parallelism between the parties: hold states the other does not hold, bring perspectives that do not already point the same way, decorrelate the errors. Chapter 13 owns the measurement of u between two people; Chapter 18 owns the intervention.

Its sensitivity. `dD/du = −2.885390 (N−1)/(1 + (N−1)u)`. Computed at three operating points:

```
at u = 0.068735 (carried case, as found)    −18.0739 octaves per unit u
at u = 0.692373 (carried case, after)        −3.6837 octaves per unit u
at u = 0.018614 (the commons, Chapter 17)   −26.3450 octaves per unit u
```

Strongest where u is smallest. The first unit of difference is worth far more than the last — this is Paper A's strict concavity showing up as a lever price.

Its bound. `u <= 1`. The maximum achievable frequency is `k N^2`, and past `u = 1` there is nothing.

Its failure mode. Manufactured difference. **u measures represented non-parallelism, not disagreement, and R8 is the trap: perfect opposition gives `u = 0` exactly as perfect agreement does, because `u` depends only on `|rho|`.** A contrarian recruited to raise u raises cost and returns nothing. Chapter 14 owns this result.

4.15 LEVER 2 — RAISE `eta`

What it is. Increase the fraction of interaction that becomes durable structure: write the decision down, build the artefact, make the practice repeatable, give the insight somewhere to live.

Its sensitivity. `dD/deta = −2.885390 c / eta^2`. Computed:

```
at eta = 0.30, c = 0.62   −19.8771 octaves per unit eta
at eta = 0.44, c = 0.40    −5.9615 octaves per unit eta
at eta = 0.34, c = 0.825  −20.5921 octaves per unit eta
```

The `eta^2` in the denominator is the important feature. At low `eta` the lever is enormously powerful; at high `eta` it saturates. A system converting 30% of its interaction into structure gains 19.9 octaves per unit of improvement; one already converting 44% gains 6.0.

Its bound. `eta` is a conversion efficiency, and improvements beyond the system's mechanical maximum are unavailable. In many cases the required `eta` exceeds 1 and the lever is simply not there — §4.24 shows exactly that on the carried case.

Why Chapter 15 finds it usually cheapest in a dyad. Raising u requires finding or making real difference between two people, which is slow and sometimes impossible. Raising `eta` requires changing what happens to what they already produce, which is a mechanism question and often a matter of a notebook.

4.16 LEVER 3 — CUT `c`

What it is. Reduce coordination overhead, friction, correction load, and dissipation.

Its sensitivity. `dD/dc = +2.885390 / eta`. Computed:

```
at eta = 0.30    +9.6180 octaves per unit c
at eta = 0.44    +6.5577 octaves per unit c
at eta = 0.34    +8.4864 octaves per unit c
```

This is the only lever whose sensitivity is constant in its own variable — linear in `c`, so each unit cut is worth the same as the last. That makes it the most predictable lever and the easiest to plan against.

Its bound. `c >= 0`, and at `c = 0` the threshold falls to `f_min = k` exactly, which any non-zero u clears. A costless relation is sustained by the smallest difference.

Its failure mode. Cost reduction that reduces `eta` at the same time. Cutting the meeting that was the conversion mechanism cuts `c` and `eta` together, and since `D` depends on `c/eta`, the ratio may not move at all. Cut cost that is not doing conversion work; SPEC 4.11 states the test.

And R18's counter-pressure, which belongs here: an over-framed system accumulates *more* stored order than a calibrated one, because dogma banks faster than knowledge. The bill arrives on a lag as correction load, `C_t = c_0 + kappa Z_t`, rising with accumulated error.

An organisation optimising raw Q will be driven away from calibration, and its c will rise later for reasons it will not connect to the choice.

4.17 LEVER 4 — RAISE N

What it is. Add genuinely distinct categories to the description. Not finer gradations of what is already there — **genuinely distinct**, in the sense that the added dimension carries variance the existing ones do not.

Its sensitivity. dD/dN = −2.885390 u / (1 + (N−1)u). Computed:

```
at u = 0.068735    −0.1129 octaves per added category
at u = 0.692373    −0.2319 octaves per added category
at u = 0.018614    −0.0446 octaves per added category
```

This is the weakest lever per unit, and it is weakest exactly where a large deficit makes it most tempting. At the near-parallel geometry of Chapter 17's commons, an added category buys 0.0446 octaves. Closing a six-octave deficit that way would require 145 new categories.

The mechanism is the Parallel Wall. N enters F only multiplied by u. Adding categories to a set of near-parallel parties adds dimensions nobody is using. A twelfth window in a tower where everyone is looking the same way is a twelfth view of the same thing.

Its real value is not in the deficit but in the ceiling. Raising N raises ln N, which raises the maximum satisfiable c/eta. N is how a structural case becomes a tunable one, and that is the only situation in which it is the right first move.

Its bound and its cost. The Gabor constraints bind: past N = 17 the folded lattice's minimum gap collapses from 50/49 (34.9756 cents) to 289/288 (6.0008 cents), and the symbol rate drops by a factor of exactly 8. Past N = 25 a single integer transposition no longer fits the audible band at all. And the honest constraint above all of those: a category added for arithmetic convenience is not a category. Chapter 5's validity conditions govern, and an arbitrary addition yields an arbitrary frequency.

4.17b Choosing between the levers — and why the partials do not decide it

The four sensitivities in §4.8 are in four different units. dD/du is octaves per unit of non-parallelism; dD/dc is octaves per unit of cost. They cannot be compared to each other directly, and a reader who ranks the levers by the size of their partial derivatives is comparing octaves-per-apple to octaves-per-orange.

What makes them comparable is a cost model: the effort, in whatever currency the situation actually spends, required to move each parameter by one unit. That model is not in the formalism and this book does not pretend to supply it. What the formalism supplies is the *exchange rate* — how many octaves each unit buys — and that is genuinely useful even without the prices, for three reasons.

One: it identifies levers that are absent rather than expensive. The Worked Example's L2 on the carried case requires eta >= 1.101077. That is not a costly lever; it is a lever that does not exist, and no cost model is needed to see it.

Two: it identifies levers that are exhausted. L4 at u = 0.018614 buys 0.0446 octaves per category. Against a 6.46-octave deficit that is 145 categories, and no plausible cost model makes 145 new genuinely distinct categories the cheap option.

Three: it gives the shape of the trade, which is stable even when the prices are not. dD/deta carries eta^2 in its denominator, so the eta lever is always most powerful where eta is lowest. dD/du carries 1 + (N−1)u, so the u lever is always most powerful where u is lowest. Both levers are strongest in the systems that most need them, which is a real and non-obvious structural fact, and it is the opposite of the usual experience that improvement gets easier once things are already going well.

The honest summary:

```
the formalism gives:   how many octaves a unit of each parameter buys
the situation gives:   what a unit of each parameter costs
the decision needs:    both, and only one of them is in this book
```

4.18 The classification — three kinds of case, and they need different work

```
D <= 0                SUSTAINABLE.  Report the margin. Maintain; do not assume.
D >  0  and c/eta <= ln N   TUNABLE.     Some assignment of u closes it. Choose levers
                                  by sensitivity, subject to R15's ordering.
c/eta > ln N            STRUCTURAL.  No assignment of u closes it. L1 is unavailable
                                 at any value. Only L2, L3, L4 — and L4 only to
                                      make the case tunable, not to close it.
```

A book that only showed tunable cases would be lying by selection. The carried case of SPINE §3.3 — six members drawn from one doctrine — reads u = 0.018614, f = 0.7017 Hz, f_min = 61.9349 Hz, D = +6.4638 octaves, Q_inf = −8.4630. Chapter 18 carries it, and the finding it carries is that no single lever closes a 6.46-octave gap: raising u to 0.95 — near-orthogonality, for six people drawn from one doctrine — closes it by 0.0332 octaves and no more; raising eta to 0.90 leaves +2.1075; cutting the per-link cost to 0.020 leaves +2.0085. Some systems cannot be tuned. The diagnostic is what tells you which case you are in, and that is most of its value.

4.19 Reporting: precision, sensitivity, and the rule that follows

Because d ln f_min / d(c/eta) = 2 exactly, and because D = 2.885390 (c/eta − F):

```
an uncertainty of dx nats in (c/eta) -> an uncertainty of 2.885390 * dx octaves in D
```

SPEC 4.19 (the reporting rule): publish D together with its sensitivity to the parameter you are least sure of, and publish the ceiling test. A deficit of +0.5 +/− 1.5 octaves is a statement that the relation might be fine; a deficit of +0.5 +/− 0.1 is a statement that it is not. The two look identical if only the point estimate is printed, and the difference between them is the whole of the reading's usefulness.

And a specific trap: precision in a constructed case does not transfer. Every figure in this book's worked cases is exact because the parameters are chosen. A measured case has no such property, and printing 30.1588 Hz from an eta known to one significant figure is precision theatre.

4.20 Cents, for fine work

```
1 nat  = 3462.4681 cents        1 octave = 1200 cents
D in cents = 3462.4681 * (c/eta - F)
```

The carried case as found reads +4.3384 octaves = +5206.1 cents; after the intervention it reads −3.5910 octaves = −4309.2 cents. Cents matter when a deficit is small: Chapter 3's row 8, at +0.033406 octaves, is 40 cents — about a fifth of a semitone — and is the difference between a practice that persists and one that must be rebuilt each generation.

A rendering note, stated once for the book: N = 12 is below the N = 17 cliff, so everything here renders in the 50/49 regime at a minimum separation of 34.9756 cents, requiring 32/(4 pi) = 2.546479 cycles per resolved step. A 40-cent deficit is therefore just above the minimum resolvable step of the lattice it is expressed in, and a 6-cent one would not be. Chapter 7 owns the folding and states the regime once.

4.21 The citation contract

This chapter owns five objects for the whole book. **Later chapters cite them by name and number and do not restate the derivations.** The contract is set out here so that a reader can check any later chapter against it.

```
OBJECT                          DEFINED IN     CITED AS
-------------------------------------------------------------------------
f_min = k e^(2c/eta)        Def 4.1     "the minimum sustainable frequency (Ch 4)"
 the three regimes              Thm 4.2        "Theorem 4.2"
 octaves = 2.885390 (c/eta)     Thm 4.3        "the octave form (Ch 4)"
 the octave deficit D          Def 4.4       "the octave deficit (Ch 4)"
 the deficit identity           Thm 4.5        "Theorem 4.5"
the ceiling c/eta <= ln N    Thm 4.6     "the ceiling (Ch 4; Paper E Cor 3.4)"
 the minimum category count     Cor 4.7        "Corollary 4.7"
 the hold line                  Cor 4.8        "the hold line (Ch 4)"
 the four levers and partials   Sec 4.8        "the four levers (Ch 4)"
 delta is not a lever           Thm 4.9        "Theorem 4.9"
 the three-way classification   Sec 4.18      "the classification (Ch 4)"
```

Five objects are deliberately NOT owned here, and later chapters must go to their owners:

```
the differentiation index n and its count argument      -> Chapter 5
the elicitation procedure for intensities               -> Chapter 6
octave folding and the odd-part class rule              -> Chapter 7
f_rel = k(1 + 11u)^2 as the dyad's interval             -> Chapter 12
the R15 order-of-operations rule                        -> Chapter 15
```

The division matters. This chapter states what a deficit is and what closes it. It does not state how to obtain u, how to defend n, or in what order to pull the levers in a real bond — those are separate results with separate defences, and a keystone chapter that absorbed them would make all of them harder to check.

4.22 What a reading is not permitted to conclude

The last thing this chapter owns is a boundary, and it is placed here rather than in the failure modes because it is not a mistake anyone makes by accident.

A reading does not conclude that a relation should end. `D > 0` says a configuration is losing order at a computable rate. It does not weigh what the relation is for, what else it produces, or what ending it would cost. The formalism has no term for any of that and does not acquire one by being applied to people.

A reading does not conclude that anyone is at fault. `u` is a property of two representations jointly. `c` and `eta` are properties of an arrangement. There is no single-party term in the whole apparatus.

A reading does not conclude anything about a person's health, capacity, or prospects. The twelve states are not diagnoses; the ladder is not an assessment instrument; the deficit is not a prognosis. Nothing in this book is clinical and nothing in it may be presented as clinical.

A reading does not compel anyone. Any intervention that follows from it is subject to the standing constraint that no person's ability to decline may be removed, and to the standing prohibition on anything masked or below threshold. Consent is structure here, not advice, and Chapter 18 builds the interventions to that requirement rather than adding it afterwards.

And a reading is not evidence about anything outside its own parameters. `Q_inf` is where a configuration goes if the configuration holds. It says nothing about whether it will hold, and R23 stands: the survivorship likelihood ratio for testimonial evidence that a configuration held is exactly `1.000000`.

The Specification

SPEC 4.1 — CEILING FIRST. Compare `c/eta` to `ln N` before any other computation. Classify the case as sustainable, tunable, or structural (§4.18) before issuing any prescription.

SPEC 4.2 — DECLARE `N` and the ordering that produced it, with the note that the frequency inherits the defensibility of the count.

SPEC 4.3 — ONE CLOCK. `eta`, `c` and `delta` on one declared period, named in the same passage as the ratio.

SPEC 4.4 — DECLARE SCALE STATUS of `u`, `eta` and `c`. Ordinal inputs yield an ordering, never a frequency.

SPEC 4.5 — REPORT THE TRIPLE. Every reading reports `f`, `f_min`, and `D`, with the folded values and their `j` where a band is used.

SPEC 4.6 — REPORT THE SENSITIVITY of `D` to the least certain parameter (SPEC 4.19).

SPEC 4.7 — REPORT THE PITCH HEADROOM `2.885390 (ln N − F)` alongside the deficit, so that a reader can see how much of the scale remains.

SPEC 4.8 — CROSS-CHECK VIA THE IDENTITY. Verify `D = −2.885390 (delta/eta) Q_inf`. A disagreement means an arithmetic error, since the identity is exact.

SPEC 4.9 — NEVER USE `delta` AS A DEFICIT LEVER. Theorem 4.9. Interventions on `delta` are reported separately, as multipliers on an existing sign.

SPEC 4.10 — ORDER OF OPERATIONS. Fix the numerator before lowering the loss. Raise `eta` or cut `c` until `eta F > c`, and only then invest in `delta`. Chapter 15 owns this rule; the Worked Example demonstrates it.

SPEC 4.11 — TEST COST CUTS FOR CONVERSION DAMAGE. Before cutting `c`, ask whether the cost being cut is performing conversion work. If it is, the ratio `c/eta` may not improve at all.

SPEC 4.12 — RAISE `N` ONLY WITH REAL CATEGORIES. An added category must satisfy Chapter 5's validity conditions and must carry variance the existing categories do not. Adding categories to a near-parallel set buys almost nothing — 0.0446 octaves each at `u = 0.018614`.

SPEC 4.13 — RESPECT THE RENDERING BOUNDS if the reading is displayed: the regime (`50/49` for `N <= 16`), the transposition `j`, a windowed envelope, `N <= 25` for a single transposition.

SPEC 4.14 — NO AGGREGATION. Per relation, per site, per period. Ratios do not average.

SPEC 4.15 — MEAN OF `F`, NOT `F` OF THE MEAN, wherever a reading spans more than one pair.

SPEC 4.16 — LABEL EVERY CASE CONSTRUCTED.

SPEC 4.17 — ESSENCE, NOT RESONANCE, in every passage printing a hertz value.

SPEC 4.18 — NO CLINICAL LANGUAGE. A reading is of a configuration of parameters, never of a person's condition.

SPEC 4.19 — THE REPORTING RULE, as stated in §4.19.

SPEC 4.20 — CITE, DO NOT RE-DERIVE. Later chapters cite this chapter for `f_min`, the octave deficit, the ceiling, and the four levers. The definitions above are the book's, and no chapter may silently redefine them.

The Failure Modes

FM 4.1 — TUNING ABOVE THE CEILING. *Signature:* a lever prescription for a case with `c/eta > ln N`. *Mechanism:* the condition `F > c/eta` is unsatisfiable, but the deficit is just a larger positive number and looks like more of the same problem. *Diagnostic:* compute `c/eta` and `ln N`. This test costs one logarithm and changes the kind of answer. *Fix:* SPEC 4.1. The remedies are `eta`, `c`, and genuinely new categories — and `N` only in order to make the case tunable, never to close the deficit directly.

FM 4.2 — TRUST FIRST. *Signature:* a failing relation into which trust, ritual, or continuity is invested, followed by acceleration of the failure. *Mechanism:* R15, exactly. `delta` divides a negative numerator. Computed on the carried case: `delta` 0.09 -> 0.05 moves `Q_inf` from `−5.0119` to `−9.0215`, and the deficit does not move at all, remaining `+4.338420` to every digit. *Fix:* SPEC 4.10. Fix the numerator, then lower the loss.

FM 4.3 — SINGLE-LEVER THINKING ON A LARGE DEFICIT. *Signature:* a plan to close six octaves by one adjustment. *Mechanism:* the levers saturate. `u` is bounded at 1; `eta` at its mechanical maximum; `c` at 0; `N` buys `0.0446` octaves per category at low `u`. *Fix:* compute what each

lever alone would require (the Worked Example for the dyad; Chapter 18 for the commons), and if none is reachable, say so. **Some systems must be restructured.**

FM 4.4 — PRECISION THEATRE. *Signature:* `f_min = 30.1588 Hz` from `eta = 0.3`. *Mechanism:* exact arithmetic on inexact inputs. *Fix:* SPEC 4.6 and SPEC 4.19.

FM 4.5 — THE ORDINAL SMUGGLE. *Signature:* a numeric `c/eta` from rank-ordered inputs. *Mechanism:* division requires a ratio scale. An order-preserving relabel moved `f_min` from 195.0279 Hz to 10648.1604 Hz in Chapter 1 §1.6 without changing anything an ordinal instrument could see. *Fix:* SPEC 4.4 and R1's route out — Rasch instrumentation at 123 items per term.

FM 4.6 — BUYING `u` PAST 1. *Signature:* a plan requiring `u > 1`, or a required `u` that is not checked against 1. *Mechanism:* `u = 1 − (x.y)^2 <= 1` by construction. *Fix:* compute the required `u = (e^(c/eta) − 1)/(N−1)` and check it against 1 — which is the same as the ceiling test, seen from the geometry.

FM 4.7 — ADDING CATEGORIES TO A PARALLEL SET. *Signature:* a richer taxonomy adopted to raise `N`, with no change in the reading. *Mechanism:* the Parallel Wall. `N` enters `F` only multiplied by `u`. At `u = 0.018614` a new category is worth 0.0446 octaves. *Fix:* raise `u` first, or accept that `N` is being raised to lift the ceiling and not to close the deficit.

FM 4.8 — `f_min` PLAYED AS A TONE. *Signature:* any proposal to sound `f_min` at anyone. *Mechanism:* the register slip. `f_min` is a threshold in a computation and the frequencies are essence frequencies. *Fix:* Chapter 26 governs rendering, under Chapter 27's open problem — no established mechanism for sequence-specific acoustic demodulation — and under the absolute no-subliminals rule.

FM 4.9 — THE NEGATIVE STOCK. *Signature:* a plan that anticipates arriving at `Q_inf = −5.0119`. *Mechanism:* admissibility fails first, at `t = 1` in the carried case. *Fix:* read a negative fixed point as a direction and a rate.

FM 4.10 — CUTTING THE CONVERSION MECHANISM. *Signature:* `c` falls, `eta` falls with it, `D` does not move. *Mechanism:* `D` depends on the ratio. *Fix:* SPEC 4.11.

FM 4.11 — THE AGGREGATED READING. *Signature:* one deficit for a whole organisation. *Mechanism:* R10 — distortion cancels across sites, overstating bankable input by 4.26x; R16 — identical aggregates hide topologies that lose 77% versus 21% of surplus to the same departure. *Fix:* SPEC 4.14.

FM 4.12 — THE JENSEN SHORTCUT. *Signature:* a group reading from `F(mean u)`. *Mechanism:* strict concavity inflates `F`, inflates `f`, and under-reports `D`. *Fix:* SPEC 4.15. Worth 6.2% in Book Two's worked office.

FM 4.13 — THE DEFICIT READ AS A JUDGEMENT OF PEOPLE. *Signature:* "our reading is `+4.34`, so one of us is the problem." *Mechanism:* a four-parameter configuration is being read as a property of a person. The deficit is a property of an arrangement. *Fix:* state it as the arrangement. The most common real repair — the carried case's own — is that one party takes on states the other does not hold and the pair adopts a record. Neither of those is a correction of a person.

Worked Example

THE CARRIED DYAD — THE FULL ARC, 2a TO 2b

This case is CONSTRUCTED. It is the dyad of SPINE §3.2 and it is the only case used in this chapter. Its parameters are chosen, not observed. It is not an assessment of any person and no clinical, diagnostic or therapeutic claim is made or implied. The frequencies are essence frequencies and nothing is claimed to resonate.

Phase 2a — as found

```
A = {4: 0.7, 6: 0.5, 9: 0.4}
B = {4: 0.6, 6: 0.55, 9: 0.3, 10: 0.2}

eta = 0.30     c = 0.62     delta = 0.09     N = 12, one declared period
```

Step 1 — the geometry.

```
cos theta = 0.965021        u = 0.068735        F = 0.563085 nats
N_eff     = 1.7561
f         = k (1 + 11u)^2 = 1.4908 Hz          folded 381.644 Hz (j = 8)
```

The pair agree about nearly everything, and that is the whole problem. No one involved would describe it as a problem. Two people 0.965 aligned field 1.76 people's worth of distinct perspective between them.

Step 2 — the ceiling test, first.

```
c/eta   = 0.62 / 0.30 = 2.066667
ln N    = 2.484907
2.066667 <= 2.484907        BELOW THE CEILING  ->  the case is TUNABLE
```

Step 3 — the reading.

```
f_min        = k e^(2 * 2.066667)   = 30.1588 Hz    folded 241.270 Hz (j = 3)
 octaves required = 2.885390 * 2.066667   =  5.963140
 octaves present  = log2(1.4908 / k)      =  1.624719
 OCTAVE DEFICIT D                        = +4.338420      (+5206.1 cents)
 pitch headroom   = 2.885390 (ln 12 - F)  =  5.5452 octaves
 N_eff required   = e^2.066667            =  7.8985
 VERDICT: DECAYS
```

Step 4 — cross-check.

```
 Q_inf = (0.30 * 0.563085 - 0.62) / 0.09 = -5.0119
identity: -2.885390 * (0.09/0.30) * (-5.011940) = +4.338420      error 8.88e-
16
 admissibility fails at t = 1;  the stock goes negative at t = 2
```

The reading in one sentence: this pair must be worth 7.9 genuinely different people to break even, and they are worth 1.76.

The R15 demonstration — and it is the case's whole point

The intuitive repair is to add trust. It is what everyone reaches for, it is not foolish, and it is well supported in ordinary experience. Trust lowers `delta`. Suppose it works completely and `delta` moves from 0.09 to 0.05.

```
    BEFORE:   Q_inf = -0.451075 / 0.09 = -5.0119
    AFTER:    Q_inf = -0.451075 / 0.05 = -9.0215        WORSE by 80%

    DEFICIT BEFORE: +4.338420 octaves
   DEFICIT AFTER:  +4.338420 octaves                UNCHANGED, to every digit
```

The numerator `eta F - c = -0.451075` is negative. Dividing a negative number by a smaller positive denominator makes it more negative. Trust did not add order; it amplified the sign the relation already had, and the sign was negative.

And the deficit did not move at all, because `delta` is not in it (Theorem 4.9). This is the sharpest available statement of R15 and it is why the frequency register earns its place: the instrument reads unchanged while a successful, well-intentioned, expensive intervention doubles the rate of loss. A reader with the deficit and without Theorem 4.9 would interpret the flat reading as "give it more time." A reader with both interprets it correctly the first time: you have not touched the mechanism.

The four levers on 2a, each alone

Required: `F >= c/eta = 2.066667`, i.e. `N_eff >= e^2.066667 = 7.898451`.

```
    L1  RAISE u
          need u >= (7.898451 - 1)/11 = 0.627132          from 0.068735 --
a rise of x9.12
        at that u, f = 30.1588 Hz = f_min exactly, D = 0
     REACHABLE, but 0.627132 is a large amount of real difference for two people
        who currently sit at cos theta = 0.965

    L2  RAISE eta
        need eta >= c/F = 0.62 / 0.563085 = 1.101077
     IMPOSSIBLE. A conversion efficiency above 1 would mean more structure created
     than interaction supplied. The lever is not merely expensive; it is absent.

    L3  CUT c
        need c <= eta F = 0.30 * 0.563085 = 0.168925
        from 0.62 -- a cut of 72.8%
     REACHABLE in principle; a 72.8% reduction in coordination overhead is a large project.

    L4  RAISE N
       need 1 + (N-1)(0.068735) >= 7.898451  ->  N >= 101.3635  ->  N_min = 102
     NOT REACHABLE. The Parallel Wall: at u = 0.068735 a new category buys 0.1129 octaves,
     and 4.34 octaves would take 38 of them even ignoring that u itself would change.
```

No single lever is comfortable and one of them does not exist. This is the ordinary situation

and it is why the classification in §4.18 matters more than any individual sensitivity: the case is tunable, so a combination exists, and the combination is what the arc demonstrates.

Phase 2b — after one intervention, correctly ordered

CONSTRUCTED. Two changes, made together:

1. **B takes on two states A does not hold.** This raises u. It is a change in what B is actually doing, not a rhetorical adoption of a contrary position — R8 forbids the latter from working, since perfect opposition reads as `u = 0` exactly as agreement does.
2. The pair adopt a decision log. This raises `eta` — what they work out now has somewhere to live — and lowers `c`, because the record does the remembering that the coordination overhead was paying for.

One realising vector, for concreteness:

```
 B' = {2: 0.95, 4: 0.60, 6: 0.55, 9: 0.30, 10: 0.20, 12: 0.8391}
cos theta = 0.554643     u = 0.692373      (SPINE carries this as 0.6924)

 u    0.068735  ->  0.692373
 eta  0.30      ->  0.44
 c    0.62      ->  0.40
 delta unchanged at 0.09
```

The reading, recomputed from scratch:

```
 F               = ln(1 + 11 * 0.692373)  =  2.153633 nats
 N_eff           =  8.6161
f           = k * 8.6161^2        = 35.8882 Hz    folded 287.106 Hz (j = 3)
 octaves present  =  6.214071

 c/eta           = 0.40/0.44 = 0.909091
 ceiling test    : 0.909091 <= 2.484907   BELOW, comfortably
f_min        = k e^(2 * 0.909091)    =  2.9782 Hz    folded 381.212 Hz (j = 7)
 octaves required =  2.623082

 OCTAVE DEFICIT D = -3.590989 octaves      (-4309.2 cents)
 N_eff required   = e^0.909091 = 2.4821
 pitch headroom   = 2.885390 (ln 12 - 2.153633) = 0.9559 octaves
 VERDICT: ACCUMULATES
```

Cross-checks:

```
 Q_inf  = (0.44 * 2.153633 - 0.40)/0.09 = +6.0844
 identity: -2.885390 * (0.09/0.44) * 6.0844 = -3.590989           exact
trajectory from Q_0 = 1.0:  t=1 +1.4576  t=5 +2.9116  t=20 +5.3134  t=40 +5.9675
 convergence: |Q_20 - Q_inf| = 0.7710275635960624
              (0.91)^20 |Q_0 - Q_inf| = 0.7710275635960623
 admissibility holds at every step
```

The swing: `D` from `+4.338420` to `−3.590989` — 7.929409 octaves. `Q_inf` from `−5.0119` to `+6.0844`. The pair now needs to be worth 2.4821 distinct people and is worth 8.6161.

Where the 7.929409 octaves came from

The swing decomposes exactly, and the decomposition is worth having because it shows which half of the repair did the work.

The path-free split — into the cost side and the surplus side — follows directly from `D = 2.885390 (c/eta − F)`:

```
from the cost ratio:  2.885390 * (0.909091 − 2.066667)  =  −3.340058 octaves
from the surplus   : −2.885390 * (2.153633 − 0.563085)  =  −4.589352 octaves
 total                                                     −7.929409 octaves
```

Raising u did 57.9% of the work; the cost-side changes did 42.1%.

The split *within* the cost side is path-dependent, and the book says so rather than picking a flattering order. Attributing a joint change to individual parameters requires an order, and different orders give different attributions even though the total is identical:

```
PATH 1 -- u first, then eta, then c
    u   0.068735 -> 0.692373    −4.589352
    eta 0.30     -> 0.44        −1.897363
    c   0.62     -> 0.40        −1.442695
    total                       −7.929409

PATH 2 -- c first, then eta, then u
    c   0.62     -> 0.40        −2.115953
    eta 0.30     -> 0.44        −1.224105
    u   0.068735 -> 0.692373    −4.589352
    total                       −7.929409
```

The `c` contribution is `−1.442695` on one path and `−2.115953` on the other — a 47% difference in the attribution, from the same intervention producing the same result. The reason is elementary: `c` is divided by `eta`, so cutting cost is worth more when `eta` is still low.

The `u` contribution is identical on both paths (`−4.589352`), because `F` is separable from the cost ratio in `D` and nothing multiplies it.

The practical rule that follows: report the path-free split, and if a per-parameter attribution is wanted, report the order used. An attribution presented without its order is a number that could have been 47% different for no reason but the sequence in which someone chose to describe events.

Now trust helps

Repeat the exact intervention that made things worse in phase 2a — `delta` from 0.09 to 0.05:

```
Q_inf  =  (0.44 * 2.153633 − 0.40) / 0.05  =  +10.9520        from +6.0844
deficit                                =  −3.590989     unchanged, as always
```

The same intervention, applied to the same pair, with the same magnitude, in the same direction. At 2a it took `Q_inf` from `−5.0119` to `−9.0215`. At 2b it takes it from `+6.0844` to `+10.9520`.

The arithmetic is not subtle and it is not a special case. `delta` multiplies. It has no sign of its own.

THE ARC: fix the numerator, then lower the loss. Reverse the order and the same intervention accelerates the failure.

The honest note on where 2b leaves them

The repair worked, and it worked by spending most of the range that twelve categories make available:

```
pitch headroom at 2a:  5.5452 octaves
pitch headroom at 2b:  0.9559 octaves
```

They have 0.96 octaves of the scale left. A further improvement of the same kind — more represented difference between two people described by twelve states — is close to exhausted. The next order of gain is not available from u at this N. It would require genuinely new categories, and per §4.17 and SPEC 4.12 those must be real categories and not finer slicing.

This is not a defect in the repair. It is the ceiling doing what a ceiling does, and the value of the reading is that the pair can see it coming rather than discovering it as an unexplained plateau.

The whole reading, as a card

```
------------------------------------------------------------------------
CARRIED DYAD -- CONSTRUCTED CASE -- N = 12 -- one declared period
------------------------------------------------------------------------
                                2a (as found)      2b (after)
u                                  0.068735          0.692373
F (nats)                           0.563085          2.153633
N_eff                              1.7561            8.6161
f (Hz)                             1.4908           35.8882
  folded 220-440                 381.644 (j=8)     287.106 (j=3)
eta                                0.30              0.44
c                                  0.62              0.40
c/eta                              2.066667          0.909091
ceiling test (ln 12 = 2.484907)    BELOW: tunable    BELOW: comfortable
f_min (Hz)                        30.1588            2.9782
  folded 220-440                 241.270 (j=3)     381.212 (j=7)
octaves required                   5.963140          2.623082
octaves present                    1.624719          6.214071
OCTAVE DEFICIT                    +4.338420         -3.590989
  in cents                        +5206.1           -4309.2
N_eff required                     7.8985            2.4821
pitch headroom (octaves)           5.5452            0.9559
delta                              0.09              0.09
Q_inf                             -5.0119           +6.0844
VERDICT                            DECAYS            ACCUMULATES
------------------------------------------------------------------------
delta 0.09 -> 0.05:  Q_inf        -9.0215           +10.9520
             deficit    +4.338420      -3.590989  (UNCHANGED both times)
------------------------------------------------------------------------
```

```
    ESSENCE frequencies, not physical resonances. Constructed case. Not an
    assessment of any person. No clinical claim.
  ------------------------------------------------------------------------
```

Chapter 5 takes up the index n and the argument that a count escapes the demotion. Every chapter after it that needs `f_min`, the octave deficit, the ceiling, or the four levers cites this chapter.

Part II — Frequency Mechanics of the Psyche

Chapter 5 . The Psychological Essence Index

Part I ended with a diagnostic that needs one input this book has not yet supplied. The minimum sustainable frequency `f_min = k e^(2c/eta)` tells you what pitch a relation must reach. The octave deficit tells you how far short it falls. Neither is computable until something on the psychological side has a frequency at all, and nothing does until a psychological category has been assigned an index n.

That assignment is the whole of Part II's exposure. Get it right and every reading in Parts III through V stands on it. Get it wrong — or, worse, get it arbitrarily — and the book has produced a long sequence of precise-looking numbers that mean nothing, which is the exact failure mode Book Two spent a volume learning to detect in other people's work.

This chapter owns the index. Chapters 6 through 30 cite it and do not redefine it. It states what n is, makes the argument that the index escapes the scale demotion its neighbours inherit, presents the twelve-state ladder that the rest of the book uses, grades four rival orderings against the governing validity conditions, rejects three of them in print, and states the limit that travels with every frequency the book publishes.

Two prohibitions govern the chapter from its first line and are restated at its last. The frequencies here are essence frequencies, not physical resonances; no person, tissue, or molecule is claimed to resonate at any value in this book. **And the twelve states are not diagnoses.** They are not a classification of persons, not a severity scale, not an assessment instrument, and nothing in this chapter or any later one licenses their use as one.

The Law

5.1 The statement

```
THE PSYCHOLOGICAL ESSENCE INDEX

  n := the number of distinguishable components required to specify the state

and, through the essence law (CANON_SONIC §1, Book Two Ch 29):

    f(n) = k * n^2            k = (9/8)^2 / phi^2 = 0.483425732988414 Hz
```

Read it slowly, because every word is doing work.

“The number” — this is a cardinality, not a position, not a rating, not a score. It answers *how many*, and the answer is an integer.

“Distinguishable” — components that the encoding cannot tell apart count once. Two names for one component are one component. This is the clause that does the most work and takes the most abuse, and Chapter 6 is largely about enforcing it.

“Components” — the constituents of a specification, not the parts of a thing. A component is something you must state in order for a reader to know which state you mean.

“Required” — the minimal count. Not the number of things you could say about the state, but the number you cannot omit without the specification becoming ambiguous. Redundant components are not components.

“To specify the state” — the criterion is specification, and specification is public. If two analysts working from a declared decomposition and the same episode cannot both arrive at the same state description, the count was not of components; it was of something else.

The index is therefore a *minimal description length in components*, and the essence law raises it to the second power and scales it by a fixed constant. Nothing else enters. k is a chosen reference constant, not a measured one, and it is badged as a postulate here as it is everywhere in this series. Its form — a Pythagorean ditone divided by the golden section — is a design decision of the author’s, taken in the 2022 source and carried forward unchanged. It sets the absolute pitch of the whole system and nothing else. Every *relationship* in the system is a ratio of squared integers, in which k cancels exactly.

5.2 A count is not a rating, and the difference is the whole argument

Stevens’ 1946 typology of measurement scales is usually taught as a four-item list — nominal, ordinal, interval, ratio — and usually taught badly, as though the four were degrees of quality. They are not degrees of quality. They are statements about *which transformations of the numbers leave the empirical content unchanged*, and that is a precise and checkable property.

SCALE	ADMISSIBLE TRANSFORMATIONS	WHAT IS INVARIANT
nominal	any bijection	identity and difference of category
ordinal	any strictly increasing f	order
interval	x –> a x + b, a > 0	ratios of differences
ratio	x –> a x, a > 0	ratios of values
absolute	identity only	the values themselves

The operations a formula performs on a quantity are licensed by that quantity’s scale type and by nothing else. Addition needs an interval scale. Division and multiplication need a ratio scale with a true zero. Squaring, which is what the essence law does, needs the same.

Now ask which row a count occupies.

A count has a true zero that is not a convention: zero components is not a low score, it is the absence of the thing. It has equal units that are not a calibration choice: the difference between three components and four is one component, and so is the difference between eleven and twelve, because a component is a component. And it has something stronger still. A count admits no

rescaling at all. There is no free multiplier a. You cannot count in half-components or in dozens-of-components without changing what "component" means, and if you change what "component" means you have changed the decomposition, not the units.

A count is therefore an absolute scale, which is strictly stronger than the ratio scale the essence law requires. The index does not merely satisfy `f = k n^2`; it over-satisfies it.

This is the load-bearing observation of Part II and it deserves to be stated in the form in which it can be attacked:

> **THE COUNT ARGUMENT.** n is defined as a cardinality. Cardinalities have a true zero (the empty set) and a fixed unit (one element), and admit only the identity transformation. Therefore n is absolute-scaled, hence ratio-scaled, hence a legitimate argument to a multiplicative law. The scale status of n is a consequence of its definition and is not contingent on any empirical finding.

An objector who wishes to defeat this must attack the definition, not the algebra. There is exactly one place to attack it, it is a real place, and §5.4 and the whole of Chapter 6 are about it.

5.3 R1, restated in full, and why the index is not caught by it

Book Two demoted its own keystone. The finding is recorded as R1 and it is binding here:

> **R1 — THE MOMENT EQUATION IS DEMOTED ON SCALE.** For `M = (D/P) * E`, the validation protocol's second gate returns DEMOTE. Because `D` (perspective) and `P` (perception) are not instrumented on ratio scales with true zeros, the *magnitude* of `D/P`, the distortion `Delta = |ln r|`, and the quantitative join from the moment into the storage law are not reportable as measured quantities. The structure survives: the limiting cases, the `D = P` fixed point, and the ordering over-framed / calibrated / under-framed all stand, because each needs only ordinal information. **What is demoted is magnitude, not structure.** The constructive route out is Rasch instrumentation — a logit difference supplies the true zero division requires — at a measured cost of 123 items per term.

Any psychological frequency derived through `D/P` inherits that demotion, and this book says so wherever such a derivation appears. The demotion is not a caveat bolted onto a result; it is a statement that a particular number is not yet a number.

It is worth seeing what the demotion costs in the register this book works in, because stated as "the scale type is wrong" it sounds like bookkeeping and it is not. Take a constructed pair of ordinal ratings, `D = 7` and `P = 4` on some 0-to-10 instrument, and apply transformations that are *admissible for ordinal data* — that is, transformations which preserve the order of the ratings and therefore preserve everything the ratings are licensed to tell you.

transform	D'	P'	r = D'/P'	ln r
identity	7.0000	4.0000	1.7500	+0.5596
x^2	49.0000	16.0000	3.0625	+1.1192
sqrt x	2.6458	2.0000	1.3229	+0.2798
ln(1+x)	2.0794	1.6094	1.2920	+0.2562
x^1.5	18.5203	8.0000	2.3150	+0.8394
exp(x/5)	4.0552	2.2255	1.8221	+0.6000

```
r spans 1.2920 to 3.0625 -- a factor of 2.3703
r > 1 in all six cases -- THE ORDER IS PRESERVED, EXACTLY AS R1 SAYS
```

The direction survives every transformation. The subject is over-framed under all six, and that reading is safe. The magnitude survives none of them.

Now push it through the essence law anyway, as a careless practitioner would, treating `r` as though it were an index:

```
transform     n := r      f = k r^2 (Hz)     folded to 220-440
identity      1.7500          1.4805            379.006
x^2           3.0625          4.5340            290.176
sqrt x        1.3229          0.8460            433.149
ln(1+x)       1.2920          0.8070            413.185
x^1.5         2.3150          2.5909            331.630
exp(x/5)      1.8221          1.6050            410.888

derived frequency spans 0.8070 to 4.5340 Hz = 2988.17 cents = 2.490 octaves
```

A frequency derived through `D/P` is undetermined to within two and a half octaves by transformations that change nothing the underlying data are entitled to say. The spread is not a property of the world; it is a property of the arbitrary set of transformations chosen for the demonstration, and a different set would give a different figure. That is precisely the point. A quantity whose value depends on an arbitrary choice the data cannot constrain is not measured. Any chapter that publishes such a frequency is publishing a number the reader cannot check, and this book does not do that.

Set the count beside it.

```
transform     n     f = k n^2 (Hz)
identity      5        12.0856
(there is no other admissible transform of an absolute scale)
```

There is no table because there is no spread. **Five components is five components.** The transformation group of an absolute scale contains one element, so the frequency is determined by the decomposition and by nothing else.

> **THE SIDESTEP, and it is the single most important claim in Part II.** The Moment equation is demoted because `D/P` is a ratio of two quantities that have not been instrumented on ratio scales. **The psychological essence index is not a ratio of ratings. It is a count of components.** Its ratio-scale status is established by construction rather than by instrumentation, and therefore nothing in R1 touches it. The index does not inherit the demotion. It sidesteps it.

The sidestep is real, and it is worth being explicit about what it does and does not buy, because a claim this convenient invites suspicion and should.

What it buys. The essence law may be applied to the index without a scale objection. `f = k n^2` is a licensed operation on `n`. All the interval structure of Part II — ratios of squares, the octave relations, the pitch classes — is licensed. The frequencies may be published as magnitudes, not merely as an ordering. And the ceiling result `c/eta <= ln N` from Chapter 4 has a determinate `N` to work with.

What it does not buy. It buys nothing whatever about whether the *decomposition* is right. The count is absolute-scaled *given* a decomposition, exactly as a count of coins is absolute-scaled given an agreement about what a coin is. Change the decomposition and you change the count, and no amount of scale theory arbitrates between two decompositions. **The scale problem has not been solved. It has been moved, from the measurement of a magnitude to the justification of a partition — and it has been moved somewhere it can be argued about in public, which is the entire gain.**

That relocation is not a trick. It is the standard move by which counting escaped the same problem everywhere else it arose. Atomic number is a count of protons and is absolute-scaled given an agreement about what a proton is; the agreement took a century and several revisions, and the scale status of the count was never what was in dispute. Species richness is a count of species and is absolute-scaled given a species concept; the species concept is contested, and ecologists argue about *it*, in print, rather than arguing about whether you may divide one richness by another. The psychological essence index puts psychology in the same position: the argument moves to the decomposition, where it belongs and where it can be settled by agreement statistics rather than by philosophy.

5.4 The honest limit, and it is stated wherever the index is used

> **THE FREQUENCY INHERITS EXACTLY THE DEFENSIBILITY OF THE COUNT. A CONTESTED DECOMPOSITION YIELDS A CONTESTED FREQUENCY.**

That sentence is a standing requirement of this book. It appears in this chapter, in Chapter 6, in Chapter 7 where moods are given frequencies, in Chapter 8 where traits are, in Chapter 11's worksheet, and in Appendix E. It is not a hedge and it is not modesty. It is a precise statement of where the uncertainty lives, and its practical content is that the uncertainty on a published essence frequency is not a confidence interval; it is a disagreement about a count, and the correct way to report it is to report the disagreement about the count.

The exchange rate between the two is exact and it is worth having on the page early, because it tells a practitioner what a decomposition dispute actually costs.

```
A MISCOUNT BY ONE: the pitch consequence

n -> n+1     f ratio      cents      octaves     pitch class
1 -> 2       4.000000    2400.00      2.0000      1 -> 1
2 -> 3       2.250000    1403.91      1.1699      1 -> 3
3 -> 4       1.777778     996.09      0.8301      3 -> 1
4 -> 5       1.562500     772.63      0.6439      1 -> 5
5 -> 6       1.440000     631.28      0.5261      5 -> 3
6 -> 7       1.361111     533.74      0.4448      3 -> 7
7 -> 8       1.306122     462.35      0.3853      7 -> 1
8 -> 9       1.265625     407.82      0.3399      1 -> 9
9 -> 10      1.234568     364.81      0.3040      9 -> 5
10 -> 11     1.210000     330.01      0.2750      5 -> 11
11 -> 12     1.190083     301.27      0.2511     11 -> 3
```

Two things follow, and both matter.

A miscount at the bottom of the ladder is enormously more expensive than a miscount at

the top. Confusing a one-component state with a two-component state moves the pitch by two full octaves. Confusing an eleven-component state with a twelve-component state moves it by a minor third. The square law compresses; the relative cost of an error falls as `((n+1)/n)^2`, which tends to 1. The elicitation procedure of Chapter 6 must therefore spend its care where the ladder is steep, which is exactly where practitioners are least careful, because the low states feel obvious.

And a miscount by one almost always changes the pitch class. Adjacent integers have different odd parts unless one is even and the other is its own double, which for adjacent integers happens only at `1 -> 2`. Every other single-step miscount lands the state in a different harmonic class entirely. The pitch class is what survives octave folding and is therefore what a listener hears as identity; Chapter 7 owns that rule and derives it. Here the consequence is enough: a one-component disagreement is not a small disagreement. It is a change of note.

5.5 Paper B's validity conditions, and the index against them

The essence law may not be applied to a domain merely because the domain has categories. Paper B states the conditions under which the surplus framework's formal structure may be carried to a target domain at all, and CANON_SONIC §6 makes those conditions binding on the essence law. Book Two Ch 29 enumerates them; they are restated here because this chapter is where they are put to work.

```
   V1  STATE ENCODING     a map from domain items to normalised vectors, so that every
                       represented state has a determinate location in the model
   V2  CATEGORY ASSIGNMENT  a finite partition into idealised categories H_1 ... H_N:
                         exhaustive, exclusive, counted, and genuinely distinct
   V3  GEOMETRIC DEPENDENCE interaction modelled through a scalar similarity parameter --
                         an angle, a cosine, an inner-product-derived quantity
   V4  ZERO BASELINE      parallel or representationally equivalent states correspond to
                              zero modelled surplus
   V5  AFFINE COUNT LAW    adopted as a modelling assumption, not read off the world
    V6  NORMALISATION       maximum modelled surplus is finite and depends on N

   and the house addition, because the essence law needs an ORDER and not merely a partition

   V7  DECLARED ORDINAL    the rule assigning index n to category i is published with the
                      frequencies, is applicable by a third party without consulting
                              the author, and includes its tie-break procedure
```

Paper B is explicit about the exclusion that follows: a target domain lacking a stable encoding, a defensible category scheme, or an interpretable angle-like geometry is outside the scope of the model. The essence law inherits that exclusion and this manual enforces it. The three conditions named in that exclusion — explicit state encoding, defensible category partition, interpretable angle-like geometry — are the three the rival orderings of §5.11 through §5.14 are graded against, and no candidate that fails one of them is admitted, however appealing it is on other grounds.

The component-count index answers V7 in a way no other candidate in this chapter does, and it is worth saying why before the ladder is presented. **The ordinal rule is not a list; it is a procedure.** "Count the components" is applicable by a third party who has never met the author, and it has

a determinate answer for a state nobody anticipated. A published list of categories in an order has neither property: it cannot be extended without the author, and a category the author did not think of has no index at all. An ordinal rule that is a procedure rather than an enumeration is the strongest form V7 can take, and it is the principal engineering reason for preferring a count to any of the alternatives.

The Engineering

5.6 The twelve-state ladder

This is the canonical ladder. It is fixed by SPINE §1 and is used unaltered by every chapter of this book. Twelve states; `N = 12`; each state adds exactly one component to the one below it, so the state class at index `n` is by construction the class of states requiring `n` components to specify.

n	state class	the component it adds	$f = k.n^2$ (Hz)	folded 220–440	pitch class
1	undifferentiated activation	arousal alone	0.4834	247.514	1
2	valenced activation	+ valence	1.9337	247.514	1
3	directed feeling	+ intentional object	4.3508	278.453	3
4	situated feeling	+ situation	7.7348	247.514	1
5	attributed feeling	+ cause	12.0856	386.741	5
6	temporal feeling	+ projected duration	17.4033	278.453	3
7	reflexive feeling	+ self as object	23.6879	379.006	7
8	social feeling	+ another mind modelled	30.9392	247.514	1
9	normative feeling	+ standard applied	39.1575	313.260	9
10	narrative feeling	+ placed in a story	48.3426	386.741	5
11	reciprocal feeling	+ the other modelling you	58.4945	233.978	11
12	committed feeling	+ future action bound	69.6133	278.453	3

```
k = 0.483425732988414 Hz        N = 12        ln N = 2.484907
```

```
maximum surplus F = ln 12       maximum f = k ∗ 144 = 69.6133 Hz
six distinct pitch classes {1, 3, 5, 7, 9, 11} = ceil(12/2)
```

Every value in the table was computed from `f = k n^2` and folded by powers of two into the band 220–440 Hz. The folding multiplier `j` for each row, since the book's house rule requires the band and the transposition to be published with every value: `j` = 9, 7, 6, 5, 5, 4, 4, 3, 3, 3, 2, 2 for n = 1 through 12 respectively. Octave transposition preserves pitch class exactly; it is the house method and it changes nothing about the reading.

5.7 Reading the ladder

The ladder is a specification hierarchy, and each rung earns its index by naming a component that cannot be omitted at that rung and was not present below it. The clearest way to see this is to write the same episode out at each level and watch what has to be added.

n = 1, undifferentiated activation. The specification is *there is arousal*. One component: activation. This is the floor of the ladder, and it is genuinely a state — a person woken at three in the morning by a noise, in the first half second, is in it. Nothing has been evaluated. There is not yet anything the arousal is about.

n = 2, valenced activation. Add valence: the arousal is good or bad. Two components. The half-second has passed; the noise is *bad*. Still nothing about what the noise is. In the affect literature this is roughly the region core-affect accounts describe, and Russell's circumplex (1980) is a two-dimensional model of exactly this pair. That correspondence is noted as precedent, not as evidence — a two-dimensional affect model and a two-component specification agree on a count, which is a great deal less than agreeing on a mechanism.

n = 3, directed feeling. Add the intentional object: the feeling is *about* something. Three components. The bad arousal is about the noise. This is the rung at which the state becomes a feeling in the ordinary sense of the word, because ordinary usage almost always supplies an object.

n = 4, situated feeling. Add the situation: the state is indexed to a circumstance. Four components. Bad arousal about the noise, in my house, at night. The situation is not the object; the object is what the feeling is directed at, the situation is the frame the object appears in, and a decomposition that conflates them will systematically under-count by one.

n = 5, attributed feeling. Add the cause: an account of why. Five components. The noise is *the back door*, and doors do not open themselves. Appraisal accounts — Smith and Ellsworth (1985) on the dimensions of cognitive appraisal, Lazarus (1991) on primary and secondary appraisal, Scherer's sequential-check model (2001) — are all descriptions of the machinery that produces this component, and all three are cited as precedent for the component's existence and not as support for its index.

n = 6, temporal feeling. Add projected duration: how long this will last. Six components. The difference between a fright and a fear is largely this component, and a decomposition that omits it will collapse two states that behave very differently under Chapter 8's stability analysis.

n = 7, reflexive feeling. Add the self as object: the state includes a representation of the one having it. Seven components. *I am frightened, and I am the sort of person who gets frightened.* The self-conscious-emotion literature (Lewis; Tracy and Robins, 2004) holds that this class of states requires a self-representation, which is a claim about a component and is again cited as precedent.

n = 8, social feeling. Add another mind modelled: the state contains a representation of someone else's mental state. Eight components. *There is a person out there and he wants something.* The theory-of-mind literature descends from Premack and Woodruff (1978); the component is the modelled other, and it is one component regardless of how elaborate the model is.

n = 9, normative feeling. Add a standard applied: the state is evaluated against how things ought to be. Nine components. *This should not be happening; people should not do this.* Note carefully that the standard is a component of the state, not a fact about the world, and that the ladder makes no claim whatever about whether the standard is correct.

n = 10, narrative feeling. Add placement in a story: the episode is an instalment. Ten components. *This is what my life has been like since March.* The narrative component is what converts an event into evidence, and Chapter 25 is about the consequences.

n = 11, reciprocal feeling. Add the other modelling you: the state includes a representation of someone else's representation of you. Eleven components. *He knows I am here and he knows I know.* This is second-order and it is a genuinely distinct component from the first-order model at `n = 8`; the pair is the standard recursion of the mentalising literature and the ladder counts each order once.

n = 12, committed feeling. Add future action bound: the state includes a commitment that constrains what is done next. Twelve components. *Whatever else happens, I am not going back to sleep.* Frijda's action tendency (1986) names the component; binding it into the specification is what makes the state a commitment rather than an impulse.

Two structural remarks about the ladder as a whole, and they bound its claims sharply.

First, **the ladder is a specification hierarchy and not a developmental sequence, a maturity scale, or a ranking of worth.** Nothing in it says that a twelve-component state is better, healthier, more adult, or more valuable than a three-component state. A twelve-component state is *more expensive to specify*, and that is the only claim being made. A person in undifferentiated activation is not failing at anything.

Second, **the ladder is nested by construction.** Each rung contains the rungs below it. This is what makes the index a count rather than a label: the components accumulate, so the number at rung `n` is literally the number of accumulated components. A hierarchy in which the categories were merely ordered — where rung 7 contained something rung 6 did not, but also lacked something rung 6 had — would not license a count at all, and this is where several of the rivals in §5.11 onwards fail.

5.8 Six classes from twelve states

Because `(2n)^2 = 4n^2` is exactly two octaves above `n^2`, two indices share a pitch class if and only if they have the same odd part. Write `n = 2^a * m` with `m` odd; the class is `m`. For the twelve-state ladder the odd parts are

```
n         1  2  3  4  5  6  7  8  9 10 11 12
odd part  1  1  3  1  5  3  7  1  9  5 11  3
classes   {1, 3, 5, 7, 9, 11}  —  6 classes = ceil(12/2)
```

Twelve states yield six pitch classes. That is the available harmonic vocabulary and it is a hard structural limit, not a stylistic choice. Chapter 7 owns the folding rule and the `ceil(N/2)`

theorem and derives both; they are quoted here because the ladder cannot be read without them.

Within the 220–440 Hz band the six classes sit as follows, measured in cents from class 1:

```
class  1     247.514 Hz        0.0000 cents
class  3     278.453 Hz     +203.9100 cents
class  9     313.260 Hz     +407.8200 cents
class  7     379.006 Hz     +737.6518 cents
class  5     386.741 Hz     +772.6274 cents
class 11     233.978 Hz      -97.3641 cents
```

Class 11 lands below class 1 in this band because folding is into a fixed window; the negative figure is an artefact of the window and not of the class. Note that classes 7 and 5 are the closest pair in the set, and that the size of that minimum separation is a structural property of $N = 12$ rather than of this particular ladder. Chapter 7 states it once, with the regime it belongs to, and this book does not re-derive it anywhere else.

The immediate consequence for this chapter is the one that governs the failure modes: half the ladder's distinctions are harmonically invisible. States 1, 2, 4 and 8 are all class 1. They are four different states, they have four different frequencies, and they sound like the same note played at four different heights. A reading that reports only the class has discarded the difference between undifferentiated activation and social feeling — which is a large difference. **Report the index and the class. Never the class alone.**

5.9 What the ladder is not

This section exists because the ladder is the object in this book most likely to be misused, and because a formalism that looks like a clinical instrument will be treated as one unless it says otherwise in the plainest available language.

The twelve states are not diagnoses. They are not disorders, syndromes, categories of illness, or levels of severity. No index on this ladder corresponds to any diagnostic category in any classification system, no index indicates the presence or absence of any condition, and no reading produced by any procedure in this book is an assessment of any person's mental health.

No chapter of this book makes a clinical, medical, diagnostic, or therapeutic claim, and none is implied by any number in it. The readings are readings of a formal construction. A person who wishes to know something about their health should ask someone qualified to tell them, and nothing here is a substitute for that.

The ladder is not an instrument for classifying persons. A person is not at an index. States are at indices; a person holds several states at once, which is exactly what Chapter 9 is about, and the object that characterises a person in this book is a chord and not a note.

The ladder is not a hierarchy of value. Higher index means more components, means more expensive to specify, means higher pitch. It does not mean better, more evolved, healthier, more mature, or more real.

And the frequencies are essence frequencies. No claim is made that any person, tissue, molecule, or neural population physically resonates at 23.6879 Hz or at any other value in the table. The essence law returns the resonance of an essence — a relation among categories — and that distinction is absolute in this series and is restated wherever the law is used.

5.10 What the ladder costs to get wrong

Combine §5.4's miscount table with §5.8's class structure and the practical exposure becomes visible.

A single-step miscount costs between 301 and 2400 cents of pitch and, except at `1 –> 2`, changes the pitch class. A two-step miscount at the bottom of the ladder — mistaking a one-component state for a three-component one, which is the single easiest error to make because ordinary language supplies an object whether or not the state has one — costs `9/1` in frequency, three octaves and a whole tone, and moves from class 1 to class 3.

The exposure is therefore concentrated, and it is concentrated in a knowable place. It is not spread evenly across the ladder and it is not a matter of general imprecision. It lives at rungs 1 through 5, where the square law is steepest and where the components are easiest to conflate. Chapter 6's decomposition rules are weighted accordingly, and its worked example is deliberately drawn from that region.

The Specification

The chapter now does the thing it exists to do. Four rival ordering principles are stated at their strongest, graded against Paper B's three exclusion criteria — explicit state encoding, defensible category partition, interpretable angle-like geometry — and against the house condition V7, and each is given a verdict that is published whether or not it flatters the book.

A note on method before the grading starts. Each rival is instantiated below with a **constructed** assignment of indices, because a principle cannot be graded in the abstract. Those constructions are labelled as constructed and are not presented as anyone's published ordering. The point of constructing them is not to show that a particular arrangement is wrong; **it is to show that the principle underdetermines the arrangement, which is a property of the principle and is what the grading is about.**

5.11 Rival 1 — AROUSAL. Verdict: REJECTED.

The principle. Order the states by the typical arousal magnitude of the state. Arousal is the best-attested single dimension in the affect literature, it has physiological correlates that can be measured without asking anyone anything, and it is continuous, which feels like an advantage.

The case for it. Arousal is the one affective quantity with a genuine claim to instrumentation. Skin conductance, heart-rate variability and pupillometry all index it, imperfectly and with well documented confounds, but they index it. If the goal were a psychological quantity that could be measured by an apparatus rather than elicited from a person, arousal would be the obvious candidate and no other candidate is close.

Grading against V1, state encoding. Passes, weakly. A state does have an arousal level, and that level locates it on a line.

Grading against V2, the partition. Fails. Arousal does not partition the states; it *scores* them, and many states get the same score. Take a constructed set of arousal ratings for the twelve

state classes on a 0-to-10 scale at one decimal place — the resolution a careful rater can actually sustain:

```
state    1    2    3    4    5    6    7    8    9   10   11   12
arousal 7.0  7.0  6.0  5.0  5.0  4.0  5.0  6.0  6.0  4.0  6.0  5.0

tie structure: 7.0 x2, 6.0 x4, 5.0 x4, 4.0 x2
states involved in at least one tie: 12 of 12
orderings consistent with these ratings: 2 x 24 x 24 x 2 = 2,304
undetermined ordering: log2(2304) = 11.170 bits
```

Every state is tied with at least one other, and the ratings leave 11.17 bits of the ordering undetermined. The principle therefore does not supply an ordinal at all; it supplies an ordinal *with a tie-break to be added later*, and the tie-break carries all the information the essence law consumes. Under V7, a rule whose tie-break is unspecified is not a declared ordinal rule.

Grading against V3, angle-like geometry. Fails on a subtler point that is the decisive one. The essence law is a square law, and the pitch class depends on the odd part of the index, which is a number-theoretic property and not a continuous function of position. Arousal is a continuous quantity. **Mapping a continuous quantity onto integer positions and then taking the odd part of the position is not an interpretable geometry; it is a rounding operation followed by a number-theoretic one, and small changes in the underlying quantity produce arbitrary changes in the output.**

Here is that failure measured. Three constructed analysts, each producing a defensible arousal ordering of the same twelve state classes:

```
analyst A order (low arousal to high):  6 12 10  9  4  3  2  1  5  7  8 11
analyst B order:                       12 10  6  4  9  3  1  2  7  5 11  8
analyst C order:                       10 12  9  6  3  4  2  5  1  8  7 11

Spearman rho(A,B) = +0.9510
Spearman rho(A,C) = +0.9301
Spearman rho(B,C) = +0.8951
```

The three analysts agree with each other almost completely. By any conventional standard of rank agreement this is a well-behaved ordering. Now read the frequencies it produces:

```
state                          indices     classes      pitch spread
 1 undifferentiated activation  [ 8, 7, 9]  {1,7,9}       870.17 cents
 2 valenced activation          [ 7, 8, 7]  {1,7}         462.35 cents
 3 directed feeling             [ 6, 6, 5]  {3,5}         631.28 cents
 4 situated feeling             [ 5, 4, 6]  {1,3,5}      1403.91 cents
 5 attributed feeling           [ 9,10, 8]  {1,5,9}       772.63 cents
 6 temporal feeling             [ 1, 3, 4]  {1,3}        4800.00 cents
 7 reflexive feeling            [10, 9,11]  {5,9,11}      694.82 cents
 8 social feeling               [11,12,10]  {3,5,11}      631.28 cents
 9 normative feeling            [ 4, 5, 3]  {1,3,5}      1768.72 cents
10 narrative feeling            [ 3, 2, 1]  {1,3}        3803.91 cents
11 reciprocal feeling           [12,11,12]  {3,11}        301.27 cents
12 committed feeling            [ 2, 1, 2]  {1}          2400.00 cents
```

```
WORST: temporal feeling, 4800.00 cents = 4.000 octaves of disagreement
```

Three analysts who agree with each other at rho = 0.90 and above produce, for one state, four octaves of pitch disagreement. For reflexive feeling they produce classes 5, 9 and 11 — three different notes. This is the finding that decides the grading and it generalises well beyond arousal: rank agreement does not imply pitch agreement, because the essence law is a square law on the index and the pitch class is a number-theoretic function of it. A single position's difference near the bottom of the ladder is worth more than a five-position difference near the top.

Verdict: REJECTED. Arousal fails V2 (does not partition; 11.17 bits of ordering undetermined), fails V7 (no declared tie-break, and none is available from the principle), and fails V3 (the composition of a continuous quantity with an integer-rounding step and an odd-part extraction is not an interpretable geometry). It is rejected as an essence index. It is not rejected as a psychological quantity, and Chapter 8 uses arousal-like variation in a different role, where the square law is not applied to it.

5.12 Rival 2 — HIERARCHICAL NEED. Verdict: REJECTED.

The principle. Order the states by the level of need they express, following the familiar prepotency hierarchy — physiological, safety, belonging, esteem, self-actualisation (Maslow, 1943).

The case for it. It is the most widely recognised ordering in all of popular psychology, it is genuinely ordinal in intent, and it comes with a mechanism: lower needs are said to be prepotent, so the ordering is supposed to be doing causal work rather than merely filing.

Grading against V1, state encoding. Fails, and fails at the first step. **A need is not a state.** The hierarchy orders *what a person requires*; the ladder must order *what a person is in*. There is no map from an episode of feeling to a level of the hierarchy that is determinate: a moment of fear can be about safety, about belonging, or about esteem, and often about all three, and the hierarchy supplies no rule for choosing. V1 requires that every represented state have a determinate location in the model, and this candidate does not provide one.

Grading against V2, the partition. Fails on arithmetic. The hierarchy supplies five levels for twelve state classes:

```
 levels available           5
 state classes to index    12
states per level        2.4  ->  ties by construction, in every assignment
 distinct pitch classes from indices 1..5:  {1, 3, 5}  =  3
```

Five indices yield three pitch classes. The twelve-state ladder yields six. The harmonic vocabulary is halved before any reading is taken, and the halving is a consequence of the number of levels rather than of anything about the psyche.

The ceiling consequence, and it is severe. Chapter 4 established that maximum surplus is `ln N` and therefore that the maximum sustainable cost-to-conversion ratio is `c/eta <= ln N`. Compare:

```
twelve-state ladder    N = 12   ln N = 2.484907   f_max = k*144 = 69.6133 Hz
five-level hierarchy    N =  5   ln N = 1.609438   f_max = k*25  = 12.0856 Hz
```

```
ceiling on c/eta collapses from 2.4849 to 1.6094 —— a 35.2% loss of headroom
```

Adopting a five-level index would declare that 35.2% of the sustainable region does not exist, and would do so purely as an artefact of the category count. Chapter 4's fourth lever — add genuinely distinct categories — exists precisely because N is load-bearing, and a candidate index that gratuitously suppresses N is throwing away the lever.

Grading against the empirical record. The strict prepotency claim has not held up. Wahba and Bridwell's review (1976) found little support for the ordered-emergence prediction, and later treatments have been no kinder to it. This matters here only because prepotency was the candidate's one claim to being more than a list: without it, the hierarchy is an enumeration in an order somebody chose, which under V7 is a weak ordinal whose weakness must travel with every number derived from it.

Verdict: REJECTED. It fails V1 outright — needs are not states — fails V2 by construction, and costs 35.2% of the sustainable region for nothing in return. This is the rejection the manifest requires be published, and it is published without qualification. The hierarchy is a serviceable way to talk about motivation. It is not an essence index and it must not be used as one.

5.13 Rival 3 — DEVELOPMENTAL STAGE. Verdict: ADMISSIBLE AS AN ORDINAL, NOT ADOPTED.

The principle. Order by the developmental stage at which the state becomes available — the stage-theoretic tradition running through Piaget on cognitive stages, Kohlberg on moral reasoning, and Loevinger (1976) on ego development.

The case for it, and it is the strongest case any rival makes. Stage theories are genuinely ordinal by construction, they specify what each stage adds relative to the one below, they are nested in the way §5.7 requires, and Loevinger's programme in particular was built with a scoring manual designed for inter-rater agreement. This candidate is not a category error. It is a serious alternative and it is graded as one.

Grading against V1, state encoding. Fails at the level required, and the failure is specific. A stage is a property of a *person*, not of a *state*. The ladder must index the state; if the index is the stage of the person holding it, then every state a given person holds receives the same index, the person's whole spectrum collapses to a single note, and Chapter 9's chord — the object that makes Part II a part rather than a paragraph — cannot be formed at all. **A person-level index cannot produce a within-person chord, and the within-person chord is what this book measures.**

Grading against V2, the partition. Passes. Stage schemes partition, they are counted, and their categories are argued to be distinct.

Grading against V3, geometry. Passes conditionally. Stages have a defensible nesting, so the distance between adjacent stages has an interpretation.

Grading against V7 and against R1. Here is the decisive point and it is the one that keeps this candidate out of the book rather than merely down the list. **A stage assignment is a rating produced by a trained judge against a manual. It is ordinal. It has no true zero — stage zero is not "no person" — and it has no fixed unit, because the interval between stage 3 and stage 4 is not constructed to equal the interval between stage 6 and stage 7.** Push it through `f = k n^2` and you are multiplying an ordinal, which is exactly the operation R1 demotes.

A frequency derived from a developmental stage index inherits the demotion in full and may be published only as an ordering, never as a magnitude.

The constructive route out is available and is the same one R1 names: Rasch instrumentation of the stage scale would supply the true zero and the equal units, at the measured cost of 123 items per term. That is a real research programme and this book does not pretend it is impossible. It has not been done for any stage scheme in a form this book could use.

Verdict: ADMISSIBLE AS AN ORDINAL, NOT ADOPTED. Frequencies derived from it carry R1's demotion. It is not adopted here because it indexes persons rather than states and therefore cannot produce the chord. It is the one rival in this chapter that could be adopted later if someone did the instrumentation, and Chapter 8 notes where it would slot in — as a trait-scope index, not a state-scope one.

5.14 Rival 4 — VALENCE-MAGNITUDE. Verdict: REJECTED.

The principle. Order by the magnitude of the state's valence: order by how strongly good or bad it is, taking absolute value so that the index runs from neutral upward.

The case for it. Valence is the second of the two core-affect dimensions and is at least as well attested as arousal. Taking the absolute value gives something that looks like it has a true zero — neutral — which appears to answer the scale objection directly. That appearance is the reason this candidate has to be dealt with explicitly rather than waved off.

Grading against V4, the zero baseline. This is where it dies, and it dies by an argument this series has already proved in another register. Take a constructed valence assignment on `[−1, +1]`:

```
   state    1     2     3     4     5     6     7     8     9    10    11    12
 valence 0.00 −0.60 −0.70 −0.50 +0.60 −0.40 +0.50 +0.40 −0.30 +0.30 +0.70 +0.65

   collapsed pairs under |valence| −− opposite sign, identical index:
       state  2 (−0.60)  and  state  5 (+0.60)
       state  3 (−0.70)  and  state 11 (+0.70)
       state  4 (−0.50)  and  state  7 (+0.50)
       state  6 (−0.40)  and  state  8 (+0.40)
       state  9 (−0.30)  and  state 10 (+0.30)
```

Ten of the twelve states collapse into five pairs. Each pair receives one index, one frequency, and one pitch class. Directed feeling and reciprocal feeling — three components and eleven components, two full rungs apart in every structural respect — become the same note.

This is R8 in the essence register and the book has already proved the general form of it. R8 established that because `u = 1 − (x.y)^2` depends only on `|rho|`, perfect opposition returns the same `u = 0` as perfect agreement: the contrarian and the sycophant carry identical information, which is none. An index built on absolute valence commits the same error one level down. It declares that the sign of the evaluation carries no information, which is a substantive empirical claim, is almost certainly false, and is being smuggled in as a modelling convenience.

Keeping the sign does not save the candidate. A signed index runs from negative to positive, `f = k n^2` is even, and `f(−n) = f(n)` exactly — so the collapse returns in a different costume. And a signed index has no cardinality reading at all: there is no such thing as negative-four components.

Grading against V2 and V7. Fails for the same reason arousal does. Valence magnitude is a continuous score with heavy ties at any usable resolution, and the tie-break is unspecified.

Verdict: REJECTED. It fails V4 by collapsing opposites, it reproduces R8's error at the level of the index, and it fails V2 and V7 alongside arousal. Valence is real, it matters, and Chapter 10 uses it in the analysis of the divided self. It is not an essence index.

5.15 The grading table

Candidate	V1 encoding	V2 partition	V3 geometry	V4 zero	V7 ordinal rule	Scale under R1	**Verdict**
Arousal	weak pass	**FAIL** — all 12 tied, 11.17 bits free	**FAIL** — continuous into odd-part	pass	**FAIL** — no tie-break	ordinal	**REJECTED**
Hierarchical need	**FAIL** — needs are not states	**FAIL** — 5 levels, 12 states	weak	pass	weak (an enumeration)	ordinal	**REJECTED**
Developmental stage	**FAIL** — indexes persons	pass	conditional pass	**FAIL** — no true zero	pass	**ordinal — R1 applies**	**admissible, not adopted**
Valence magnitude	weak pass	**FAIL** — ties	**FAIL**	**FAIL** — collapses opposites (R8)	**FAIL**	ordinal	**REJECTED**
Component count	**pass**	**pass** — nested, exhaustive, exclusive	**pass**	**pass** — n = 0 is no state	**pass** — a procedure, not a list	**absolute, hence ratio**	**ADOPTED**

Three rejected in print, one admitted as an ordinal and left unadopted, one adopted. The adopted candidate's single advantage over the others is not that it is more psychologically insightful. It is that it is a count.

5.16 Publication requirements for any essence index

Any index used in this book, or by any reader applying the book, must publish the following. These are requirements, not recommendations, and Appendix C carries the checklist.

I1 THE DECOMPOSITION. What counts as one component, stated as a rule a third party can apply to a state the author never considered.

I2 THE COUNT RULE. How components are counted, including the treatment of redundant, nested, and conjoined components.

I3 N. The category count, stated, with the ceiling ln N it implies.

I4 THE TIE-BREAK. What happens when two states receive the same count. (For the component-count index: nothing happens. They are the same state

```
                  class and they share a frequency. That is the intended behaviour.)
  I5  THE BAND AND j.    The audible band and the octave transposition for every published
                         frequency.
  I6  THE PERMUTATION TEST. Swap two adjacent indices, recompute, and report the change in
                           cents and in pitch class. If a plausible re-
ordering moves a class
                        by more than half an octave, the ordering is load-
bearing and must
                          be defended before any frequency is published.
  I7  THE AGREEMENT FIGURE. The inter-rater agreement on the count, reported on the count
                  and on the pitch class separately. Chapter 6 specifies the protocol.
```

I6 costs ten minutes and is the item most often skipped. For the twelve-state ladder, run on every adjacent pair:

```
    swap  1<->2:  same odd part (1) -- harmonically invisible, a pure two-
octave move
    swap  2<->3:  class 1 -> 3          203.91 cents
    swap  3<->4:  class 3 -> 1          203.91 cents
    swap  4<->5:  class 1 -> 5          772.63 cents
    swap  5<->6:  class 5 -> 3          568.72 cents
    swap  6<->7:  class 3 -> 7          533.74 cents
    swap  7<->8:  class 7 -> 1          737.65 cents
    swap  8<->9:  class 1 -> 9          407.82 cents
    swap  9<->10: class 9 -> 5          364.81 cents
    swap 10<->11: class 5 -> 11         869.99 cents
    swap 11<->12: class 11 -> 3         301.27 cents
```

Nine of the eleven adjacent swaps move a pitch class by more than a quarter of an octave, and three move it by more than half. By I6's own standard the twelve-state ordering is load-bearing and must be defended. **It is defended — by §5.7's nesting argument, which is the claim that the order is not chosen but forced: rung n contains rung n−1 plus exactly one component, so the sequence is the only one consistent with the decomposition.** That defence is the whole warrant for the ladder, and if it fails the ladder fails with it. Which is the correct exposure for a book to have, and it is stated here so that a reader can attack it in the right place.

The Failure Modes

5.17 Agreeing on the order while disagreeing on the note

§5.11 measured it: three orderings at Spearman rho above 0.89 produced four octaves of disagreement on one state. **The general lesson is that rank-correlation statistics are the wrong instrument for assessing an essence index, and reporting one is worse than reporting nothing because it manufactures false confidence.**

The mechanism is exact. Two effects compound. The square law means a fixed rank difference is worth more at the bottom of the ladder than at the top — one position at $n = 1$ is 2400 cents, one position at $n = 11$ is 301 cents, an eightfold difference in consequence for an identical difference

in rank. And the odd-part rule means the pitch class is not a monotone function of the index at all: 7 and 8 are adjacent and land in classes 7 and 1, while 3 and 12 are nine apart and land in the same class. Rank distance and pitch distance are not the same metric and are not monotonically related.

The correct statistic is agreement on the integer and agreement on the class, reported separately. Chapter 6 §6.12 gives the protocol and §6.17 shows a constructed case in which a weighted kappa of 0.9838 coexists with pitch-class disagreement on more than a quarter of episodes.

5.18 Smuggling a rating in as a count

The most common way to break the count argument is to keep the word "count" and stop counting.

It looks like this. An analyst has a state, feels that it is *quite* complex, and writes `n = 7`. No components were enumerated. The 7 is a judgment of complexity dressed as a cardinality, and a judgment of complexity is an ordinal rating with all of R1's problems intact. **The frequency then has the scale status of the thing that actually produced it, which is the rating, and not the scale status of the notation it was written in.**

The test is simple and it is procedural: an analyst who claims `n = 7` must be able to list the seven components. If the list cannot be produced, the number is a rating. Chapter 6's worksheet requires the list in writing, before the number, precisely so that this failure is caught at the point of elicitation rather than at the point of publication.

A subtler version is worth naming separately. An analyst enumerates components honestly but chooses the *granularity* of the enumeration to reach a count they already had in mind. Splitting one component into two, or merging two into one, is always locally defensible, and a determined analyst can reach almost any count on almost any state. This is not caught by requiring a list. It is caught only by fixing the decomposition rules before the state is examined, and by inter-rater agreement, which is why Chapter 6 treats both as mandatory rather than as good practice.

5.19 Counting representations instead of components

A component is something the specification requires. A representation is something the analyst wrote down. They diverge in both directions.

Over-counting. The state description mentions the back door, the hour, the neighbour, the dog, and the fact that the porch light is broken. That is five nouns. It is not five components. The hour and the broken light are both part of the situation, which is one component. **Nouns are not components, and a decomposition that counts nouns will systematically over-count and will do so more for verbose subjects than for terse ones, which is a bias correlated with something the analyst has no business letting into the reading.**

Under-counting. The state description says "I was frightened of him," which is four words and at least four components: activation, valence, an intentional object, and a modelled other whose intentions matter. Ordinary language compresses aggressively, and a decomposition that counts what was said will under-count fluent speakers.

The rule that fixes both: count what the specification requires, not what the description contains. The operational form is the substitution test in Chapter 6 §6.6 — a component is required if removing it makes the description consistent with a state the subject would say they were not in.

5.20 Assuming that more components is better

The ladder ascends and the frequencies rise, and Chapter 4 established that higher pitch means more available order. The inference that a person should therefore aim for high-index states is available, natural, and wrong.

Three reasons, and they are independent.

First, the index is a property of a state, not an achievement of a person. A twelve-component state is not earned; it is simply expensive to specify. A person in a twelve-component state of committed dread is not doing better than a person in a three-component state of ordinary annoyance.

Second, the frequency that matters for sustainability is the relational frequency `f_rel`, not the state frequency `f = k n^2`. Chapter 12 owns `f_rel = k(1 + (N−1)u)^2` and it is driven by `u`, the non-parallelism, and not by any individual index. A person holding a chord of high-index states that are all pairwise near-parallel has a high spectrum and a low surplus — which Chapter 9 shows is a recognisable and unenviable condition.

Third, components have costs. Every component in a specification is something that must be maintained, and Chapter 4's `c` is where maintenance lives. A specification that requires twelve components to state requires twelve components to hold, and the cost side of the ledger does not care that the pitch went up.

The lever Chapter 4 names is "add genuinely distinct categories," and the load-bearing word is distinct, not more.

5.21 Reading the ladder as a diagnosis

Named in §5.9 and repeated here because it belongs in the failure catalogue and because repetition is the only defence a book has against a misuse it cannot supervise.

The twelve states are not diagnoses. An index is not a severity. A chord is not a profile of pathology. No reading in this book assesses anyone's mental health, and no procedure in it should be administered to anyone as though it did. A practitioner who finds themselves reporting a number to a person about that person's mind has left the scope of this book, whatever the number is.

The structural reason, stated once so that it is not merely a prohibition: **the ladder classifies specifications, not people.** The claim "this state requires seven components to specify" is a claim about a description. It becomes a claim about a person only through an inference the formalism does not license and this book does not make.

5.22 Publishing a frequency without publishing the decomposition

The worst of the failure modes, because it is invisible.

A frequency published without its decomposition is uncheckable. The reader cannot recount, cannot identify the disputed component, cannot run the permutation test, and cannot form a view about whether the number means anything. **And because the number is quoted to four decimal places, it will be believed.** Spurious precision is not a stylistic vice here; it is the specific mechanism by which an unfalsifiable claim acquires the appearance of a measured one.

The standing rule: the decomposition travels with the frequency, always, in the same table or on the same page. Where it will not fit, the frequency does not get published either. This is I1 and I2 of §5.16 and it is the reason those two items come first.

Worked Example

Everything in this section is CONSTRUCTED. It is not an observation of any person, no data were collected, and nothing in it is a case report. It is a demonstration of a procedure on an invented episode, which is what all worked cases in this book are.

5.23 A constructed state, indexed

The episode, stated as the subject might state it:

> *I keep thinking about how she looked at me in the meeting. She knew what she was doing, and she knew I'd notice. This is what it's been like since I took the role.*

Chapter 6 owns the elicitation procedure that turns an utterance into a component list; here the decomposition is performed directly so that the indexing can be shown.

Step 1 — enumerate candidate components.

```
 c1  activation                there is arousal; the subject is keyed up
 c2  valence                     it is unpleasant
 c3  intentional object          the look, in the meeting
 c4  situation                   the meeting, the role, the workplace
 c5  cause                       she did it deliberately
 c6  projected duration          "keeps" -- it is ongoing, not a flash
c7  self as object          "I'd notice" -- the subject appears in their own account
 c8  another mind modelled      "she knew what she was doing"
 c9  the other modelling you    "she knew I'd notice" -- second order
c10 placed in a story         "what it's been like since I took the role"
```

Step 2 — apply the substitution test. For each candidate, remove it and ask whether the residual description is consistent with a state the subject would deny being in.

Removing c1 through c10 each fails the test — the residual is a different state in every case — with one item requiring care. Is c9 (*she knew I'd notice*) genuinely distinct from c8 (*she knew what she was doing*)? Remove c9 and the description becomes: she acted deliberately, and I noticed. That is a coherent state and it is one the subject would say they were not in, because the whole weight of the episode is that the noticing was *anticipated*. **c9 survives. It is a second-order component and the ladder counts each order once.**

Step 3 — check for redundancy. No candidate is entailed by another. c7 and c9 both involve the self, but c7 is the subject appearing in their own account and c9 is the subject appearing in the modelled other's account; removing either leaves the other standing. **They are distinguishable and they count twice.**

Step 4 — check for a normative component. The description does not apply a standard. Nothing says this ought not to happen; the account is of what happened and what it means, not of what was owed. **Component "standard applied" is absent.** This is the kind of judgment that must be made explicitly and recorded, because it is exactly where two analysts will differ.

Step 5 — check for a committed component. Nothing binds a future action. There is no *and so I am going to*. **Absent.**

Step 6 — count.

```
components required:  c1 c2 c3 c4 c5 c6 c7 c8 c9 c10  =  10
n = 10
```

Step 7 — read.

```
n = 10        state class: narrative feeling
f = k * 100 = 48.3426 Hz
folded into 220–440 Hz with j = 3:  48.3426 * 8 = 386.741 Hz
odd part of 10 = 5   ->  pitch class 5
```

Step 8 — publish the decomposition alongside the frequency, which is what the ten-line list in Step 1 is for. A reader who disagrees now has something to disagree with.

Step 9 — the standing statement. This reading says that the state as described requires ten components to specify, and that ten components corresponds to 48.3426 Hz under the essence law with the declared constant. **It says nothing about the subject's health, character, or prospects, it is not a diagnosis, and 48.3426 Hz is an essence frequency and not a physical resonance of anything.**

5.24 The same state under a rival, and the difference priced

Take the developmental-stage rival of §5.13 — the strongest of the four — and index the same episode by it. A constructed stage assignment, using a five-level scheme, places the subject's reasoning at level 4.

```
component-count index      n = 10    f = 48.3426 Hz    class 5
stage index                n =  4    f =  7.7348 Hz    class 1

ratio    (10/4)^2 = 6.250000
cents    1200 * log2(6.25) = 3172.63 cents = 2.6438 octaves
classes  5 versus 1 -- different notes entirely
```

Two and a half octaves and a change of pitch class, from one choice of indexing principle. No computation is in error. The two numbers are answers to two different questions, and only one of them is the question `f = k n^2` is licensed to receive — because only one of them is a count.

And note what the rival's number carries with it. The stage index is an ordinal rating. Under R1 its magnitude is not reportable, so 7.7348 Hz is not a measured frequency; it is an ordinal position

written in Hz. It may be used to say that this state sits lower than another state on the same scheme. It may not be divided, squared, or folded, and the octave-deficit diagnostic of Chapter 4 cannot consume it.

That is the whole of Part II's argument compressed into one comparison. The count survives the operations the book performs on it. The rating does not.

5.25 What the reading licenses

The constructed reading in §5.23 licenses exactly this much.

It licenses the frequency 48.3426 Hz as an essence frequency of the described state under the declared decomposition, and it licenses that number's use in every downstream construction of this book: the mood spectrum of Chapter 7, the chord of Chapter 9, the interval arithmetic of Chapter 10, and the octave-deficit diagnostic of Chapter 4 wherever a relational u has been measured to go with it.

It licenses the claim that if the decomposition is accepted, the frequency follows with no further assumption, because `f = k n^2` on an absolute-scaled n is a licensed operation.

It does not license the claim that the decomposition is the right one. Another analyst may count nine components by merging c7 and c9, or eleven by splitting the situation from the role. Nine gives 39.1575 Hz and class 9; eleven gives 58.4945 Hz and class 11. Three defensible decompositions, three different notes. Chapter 6 exists because that spread is not acceptable and because reducing it is a procedural problem with a procedural answer.

And it does not license any claim about a person. **The state was constructed, the subject does not exist, and if the subject did exist the reading would still be a reading of a description.**

What this chapter owns

Chapters 6 through 30 cite this chapter and do not redefine what it fixes.

```
   THE INDEX       n := the number of distinguishable components required to specify
                    the state.  A COUNT: true zero, fixed unit, absolute scale.

   THE LAW        f(n) = k n^2,  k = 0.483425732988414 Hz  [k badged as a postulate]

   THE SIDESTEP     Because n is a count and not a ratio of ratings, its ratio-
scale status
                 is established by construction. R1's demotion of D/P does not reach it.
                     The index sidesteps the demotion; it does not inherit it.

   THE LIMIT       The frequency inherits exactly the defensibility of the count.
                      A contested decomposition yields a contested frequency.

   THE LADDER      N = 12, the twelve state classes of SPINE section 1, unaltered.
                         ln N = 2.484907.  Max f = k*144 = 69.6133 Hz.
```

```
                    Six pitch classes {1,3,5,7,9,11} = ceil(12/2).

THE REJECTIONS      Arousal: REJECTED (V2, V3, V7).
              Hierarchical need: REJECTED (V1, V2; 35.2% of headroom lost).
                    Valence magnitude: REJECTED (V4; the R8 collapse).
               Developmental stage: admissible as an ORDINAL, not adopted;
                    any frequency derived from it carries R1's demotion.

THE PROHIBITIONS  The twelve states are not diagnoses. No clinical, medical, diagnostic
             or therapeutic claim is made anywhere in this book. The frequencies are
             essence frequencies and not physical resonances. All cases constructed.
```

Chapter 6 takes the exposure this chapter identified — that the count is only as good as the decomposition — and does the work of narrowing it: the elicitation procedure, the decomposition rules, the agreement protocol, the worksheet, and the validation table with its seventh gate reported honestly as not yet run.

Chapter 6 . Ordering the States

Chapter 5 made a strong claim and then named its own exposure. The claim: the psychological essence index is a count, counts are absolute-scaled, absolute scales admit only the identity transformation, and therefore `f = k n^2` is a licensed operation on `n` in a way it is not licensed on any ratio of ratings. R1's demotion of `D/P` does not reach the index. The index sidesteps the demotion.

The exposure: all of that holds *given a decomposition*, and Chapter 5 said nothing about how the decomposition is fixed. **The frequency inherits exactly the defensibility of the count. A contested decomposition yields a contested frequency.** Chapter 5 priced that exposure — three defensible decompositions of one constructed episode gave 39.1575, 48.3426 and 58.4945 Hz, in pitch classes 9, 5 and 11: three different notes — and then stopped, because narrowing the spread is a different kind of work.

This chapter does that work, and it owns two things the rest of the book cites and does not redefine.

The first is the **elicitation procedure**: how a practitioner actually obtains the intensities that build a state vector, and how the vector is normalised. SPINE §2 fixes the construction; Chapters 11, 13 and 24 apply it; this chapter specifies it end to end, with a printable worksheet.

The second is the **defence of the count against the obvious objection** — that deciding what counts as one component is itself a judgment. That objection is correct. It is not fatal, it is not answered by assertion, and the answer has three parts: decomposition rules that make the judgment explicit, inter-rater agreement as the test that makes it checkable, and an honest statement of where the answer runs out. All three are given here, including the third.

The chapter closes with the Formula Validation Protocol table for the index, all seven gates, and FVP-7 — cross-individual reproducibility, the gate that decides the hard-science question and the one most often skipped — is reported as NOT YET RUN.

The standing prohibitions govern every page. The essence frequencies are essence frequencies, not physical resonances. **The twelve states are not diagnoses**; nothing in this procedure is an assessment, a screening, or a clinical instrument, and no chapter of this book makes a clinical, medical, diagnostic or therapeutic claim. All cases are constructed and labelled.

The Law

6.1 What is being built, and why the index alone is not enough

Chapter 5 produced a number for a state. That is not yet enough to do anything with, because every diagnostic in this book is relational. The octave deficit needs `f_rel`; `f_rel` needs `u`; `u` needs two vectors. **A single index is a note. The book's instrument reads intervals, and intervals need two things to hold at once.**

A person does not occupy one state. A person holds several, at different strengths, and holds them under different framings of the same situation. Representing that is the object this chapter builds.

SPINE §2, and it is the only construction of u in this book:

```
    THE STATE VECTOR AND THE MEASUREMENT

  A subject, a framing, or a person is represented as a unit vector over the twelve states,
  components = intensity, normalised to length 1. Then for any two such vectors x, y:

      cos theta = x . y
      u         = 1 − (x . y)^2
      F         = ln(1 + 11 u)
      f         = k e^(2F) = k (1 + 11 u)^2

    with N = 12, so N − 1 = 11, and k = 0.483425732988414 Hz.
```

Three registers, one object. `cos theta` is the alignment of the two vectors. `u` is the non-parallelism, in `[0, 1]`, equal to `sin^2 theta`. `F` is the interaction surplus in nats, bounded above by `ln 12 = 2.484907`. And `f` is that surplus expressed as a pitch, because — as Book Two Ch 30 established and Chapter 4 restated — one nat of surplus is exactly 2.885 octaves.

Chapter 12 owns `f_rel = k(1 + 11u)^2` and Chapter 13 owns the between-persons measurement of `u`. This chapter owns the step before both of them: getting from a person and a situation to a unit vector that deserves to be entered into either.

6.2 The state vector, defined

```
    THE STATE VECTOR

        x = (x_1, x_2, ..., x_12)          x_n >= 0 for all n
        x_n = the elicited intensity with which state class n is present
        the support of x = { n : x_n > 0 }
        the unit vector = x / ||x||_2 ,   ||x||_2 = sqrt( sum_n x_n^2 )

    A state class is present with intensity x_n if the subject's account of the situation
     contains the n-component specification of state class n at that strength.
```

Four properties of this object need stating before any procedure is built on it, because each one constrains the procedure.

It is non-negative. There are no negative intensities. A state is present or it is not; the absence of fear is not the presence of negative fear. This is not a modelling convenience — it is what keeps §5.14's valence collapse from reappearing here in a different costume.

Its support is the set of nonzero entries. Writing a zero into a slot changes nothing:

```
a = {3:8, 5:4, 7:2}          b = {3:8, 5:4, 7:2, 11:0}
u(a, b) = 0.000000000000
```

Exactly zero. Recording a state as absent and not recording it at all are the same act, so a practitioner may not inflate a subject's apparent breadth by listing more zeros. This closes an otherwise open route to gaming a reading.

It is normalised in the L2 norm, not the L1 norm, because `u = 1 − (x.y)^2` is built from an inner product and the inner product's natural normalisation is Euclidean. This is a consequence of SPINE §2's construction and not an independent choice.

And the zero vector has no unit form. A subject with no state at all cannot be normalised, and should not be: `n = 0` is the true zero of Chapter 5's count, `f = k ∗ 0^2 = 0`, no components, no state, no pitch. The count's zero and the essence law's ground coincide by construction, and the vector formalism inherits that: there is nothing to normalise because there is nothing there.

6.3 The two invariances, and only one of them is good news

Normalisation is usually treated as bookkeeping. Here it is load-bearing, because it determines exactly what the elicited intensities have to be good for — and the answer is less demanding than a reader would guess in one respect and more demanding in another. Both are computed.

FIRST INVARIANCE: the unit vector does not care what units the ratings were in.

```
raw ratings {3: 8, 5: 4, 7: 2} rescaled and renormalised:

raw 0–10              ->  (0.872871561, 0.43643578, 0.21821789)
raw 0–1               ->  (0.872871561, 0.43643578, 0.21821789)
raw 0–100             ->  (0.872871561, 0.43643578, 0.21821789)
proportion of total   ->  (0.872871561, 0.43643578, 0.21821789)

identical to nine decimal places
```

Because `(a x) / ||a x|| = x / ||x||` for any `a > 0`, any common positive rescaling of the intensities cancels exactly. A practitioner using a 0-to-10 scale and a practitioner using a 0-to-100 scale, who assign proportionally the same intensities, produce the same vector and therefore the same u, the same F, and the same `f`.

This is a real and useful freedom, and it should be stated for what it is: the elicitation instrument's unit is unconstrained. There is no calibration step, no reference stimulus, and no requirement that two practitioners use the same anchors. What is required is that the *ratios* of the intensities be the same.

SECOND INVARIANCE, and it does not hold. The vector is not invariant under an order-preserving transformation of the ratings. Take the same raw ratings and apply monotone transformations — transformations that are admissible if the ratings are merely ordinal — then compute u against a fixed reference framing `{3:5, 5:7, 10:4}`:

```
transform      u          f_rel (Hz)
identity    0.388360      13.4361
sqrt        0.371445      12.5044
square      0.525275      22.2093
log(1+x)    0.362115      12.0049

u spans 0.362115 to 0.525275
f spans 12.0049 to 22.2093 Hz  =  1065.05 cents  =  0.888 octaves
```

Nearly nine tenths of an octave of disagreement from transformations that preserve every ordinal fact in the data. So the elicited intensities cannot be merely ordinal. They must carry ratio information — the claim that state 3 is present twice as strongly as state 5 must mean something — even though the unit in which that ratio is expressed is free.

This is exactly the right place for the honest statement, and it must not be softened. Chapter 5's count argument covers n. It does not cover the intensities. **n is absolute-scaled by construction; the intensity vector is not, and it needs its own defence.** What the two invariances establish is the precise shape of that defence: the intensities must be ratio-scaled up to a common free multiplier — which is the definition of a ratio scale — and nothing weaker will do.

There is a substantial body of evidence that human subjects can supply exactly that, and this book already leans on it. Stevens' method of magnitude estimation produces ratio-scale measurements of subjective experience from human subjects, and it replicates — that finding is the reason Book Two Ch 38 could treat subjective magnitude as measurable at all. The elicitation procedure of §6.9 is built as a magnitude-estimation procedure for that reason and not by stylistic preference. The honest form of the claim is: the intensities are ratio-scaled to the extent that magnitude estimation delivers ratio scales, which is a measured property of a procedure and not a constant of nature, and Stevens' own exponents are known to vary with method, range, and instruction.

6.4 What normalisation discards, and the standing reporting rule

Normalisation throws away the length of the vector, and the length is not nothing.

```
low-intensity  {3: 1,   5: 0.5,  7: 0.25}    raw L2 norm = 1.145644
high-intensity {3: 8,   5: 4,    7: 2   }    raw L2 norm = 9.165151

ratio of norms = 8.0000x
u between them = 0.000000000000     (exactly zero -- same direction)
```

Two states with identical profile shape and an eightfold difference in absolute intensity are perfectly parallel and carry zero surplus between them. That is correct behaviour for the surplus formalism, which measures difference of *direction*, and it is a serious limitation on what a single u reading tells you.

> **THE MAGNITUDE REPORTING RULE.** The raw L2 norm ||x||_2 is reported alongside every state vector, in the units of the elicitation instrument, and is never discarded. u measures direction; the norm measures how much there is. A reading that publishes u without the norms it came from has thrown away half the measurement.

This is I5's sibling and it is a house requirement from here forward. Chapter 11's worksheet carries a field for it, and Chapter 13's between-persons protocol requires both parties' norms.

6.5 The objection, stated at full strength

Here is the objection the chapter exists to answer, put as forcefully as it can be put, because an objection answered in a weakened form has not been answered.

> **THE DECOMPOSITION OBJECTION.** The count argument proves that a count is absolute-scaled. Granted. But the count is a count *of components*, and "component" is not given by nature. It is given by a decomposition the analyst chooses. Chapter 5's own worked example admits three defensible decompositions of one episode, yielding 39.1575, 48.3426 and 58.4945 Hz — a spread of well over an octave, in three different pitch classes.
>
> So the scale problem has not been solved. It has been renamed. Where the Moment equation required a judgment of magnitude, the essence index requires a judgment of granularity, and a judgment of granularity is just as unconstrained as a judgment of magnitude. The apparent rigour of the count is borrowed from the arithmetic of counting and does not attach to the thing being counted. The index is a rating with an integer's costume on.

That objection is substantially correct and this book concedes its correct part immediately. The scale problem was not solved. It was *moved*. Chapter 5 said so in the same paragraph that made the claim, and the concession is repeated here so that no reader has to discover it.

What the objection gets wrong is the inference that moving a problem is worthless. The three parts of the answer are that the new location is (a) explicit, (b) testable by agreement, and (c) bounded in a way the old location was not. Sections 6.6, 6.7 and 6.8 take them in order, and 6.8 is where the answer is incomplete and says so.

6.6 Answer, part one: the decomposition rules

A judgment that is made explicitly and in advance is a different object from a judgment made implicitly and after the fact, and the difference is not rhetorical. An explicit rule can be published, applied by a third party, disagreed with in a specific place, and revised without revising everything else. That is the entire content of the move Chapter 5 made, and these rules are what make it real.

```
    THE DECOMPOSITION RULES        D1 – D8
  Fixed before any state is examined. Published with every frequency. Not revised mid-
analysis.

  D1  THE SUBSTITUTION TEST.  A candidate is a required component if and only if removing it
      leaves a description consistent with a state the subject would say they were not in.
      If the residual description still picks out the same state, the candidate was not requi

  D2  THE LADDER IS THE VOCABULARY.  Components are drawn from the twelve declared componen
      the Chapter 5 ladder and from no other list. Anything that does not correspond to one o
      the twelve is not a component of this index. An episode requiring a thirteenth thing is
        outside the ladder's scope and is reported as such, not shoehorned in.

  D3  NESTING.  The ladder is nested: a state that requires component n requires every compo
```

```
below n. If component 8 is present and component 4 is absent, the decomposition is wron
 and must be redone, not recorded. Report gaps as decomposition failures.

D4 ONE ORDER, ONE COMPONENT. A modelled other is one component (index 8). The other's mo
of the subject is a second, distinct component (index 11). Each order of recursion cou
   exactly once, however elaborate the content at that order.

D5 NOUNS ARE NOT COMPONENTS. Count what the specification requires, not what the descri
contains. The hour, the room, the weather and the errand are one component (situation)
   unless the substitution test separates them.

D6 ENTAILMENT MERGES. If component A is entailed by component B in the subject's own acc
A and B are one component. Distinguishability is the criterion, and it is the subject'
   distinguishability, not the analyst's.

D7 ABSENCE IS RECORDED. Every one of the twelve components is explicitly marked present
absent. There is no "not mentioned" state. This is what prevents an analyst from reach
   a target count by leaving inconvenient components unexamined.

D8 THE GRANULARITY IS FIXED FIRST. The decomposition rules are chosen before the episod
read and are not adjusted after seeing the count. Any adjustment is a new analysis wit
   new declared rule set, and both are published.
```

D1 and D8 carry the weight. D1 gives the judgment a determinate form: it is no longer “is this a component?” but “does removing it change which state this is?”, which is a question about the description and which two analysts can argue about with the description in front of them. D8 removes the degree of freedom that §5.18 named as the subtlest failure mode — an analyst who splits and merges until the count lands where they expected it to.

D2 is the rule most likely to be resented and it is the one that makes the ladder a ladder. The twelve components are the vocabulary. This is restrictive by design: a fixed vocabulary is what makes counts comparable across analysts and across episodes, which is the entire purpose of having one.

D3 deserves a word because it will fire. Real accounts routinely appear to skip rungs — a subject speaks about what someone else was thinking without ever saying what the situation was. Almost always the lower component is present and unstated rather than absent. D3’s instruction is to go back and ask, not to record a gap, because a recorded gap propagates into a count that no formula in this book is defined on.

6.7 Answer, part two: agreement is the test

Explicitness is necessary and it is not sufficient. A rule can be perfectly explicit and still be applied differently by every person who reads it. **The test of a decomposition is not whether the analyst finds it convincing. It is whether two analysts, working independently from the same declared rules and the same material, return the same integer.**

This is the move that puts the index in a different epistemic position from the quantity R1 demoted, and it is worth being precise about why. D/P’s problem is not that analysts disagree about it; D/P’s problem is that even a single analyst’s number is undetermined, because the ratio is not invariant

under transformations the data cannot rule out. No amount of agreement fixes that: two analysts who both apply the same inadmissible transformation agree perfectly and are both reporting a number that is not a measurement. **The count has no such internal indeterminacy. Given a decomposition, the count is determined. So disagreement between analysts is the *only* place error can enter, and disagreement between analysts is measurable.**

That is the whole of the argument, and its practical form is a protocol:

```
    THE AGREEMENT REQUIREMENT

  No essence frequency is published without an agreement figure, and the agreement figure i
    reported on THREE quantities, not one:

       A1  exact agreement on the integer n
       A2  agreement on the pitch class (the odd part of n)
     A3  the distribution of the pitch disagreement in cents, |1200 * log2((n_1/n_2)^2)|

  A2 and A3 are not derivable from A1 and are not derivable from any weighted kappa.
    Section 6.17 shows a constructed case in which they diverge sharply.
```

The reason A2 and A3 are mandatory rather than optional is Chapter 5 §5.17's finding: rank distance and pitch distance are different metrics and are not monotonically related, because the pitch class is the odd part of the index and the odd part is a number-theoretic function that is not monotone in n. An agreement statistic built on rank distance therefore cannot see the disagreement that matters, and reporting one alone is worse than reporting nothing because it manufactures confidence.

6.8 Answer, part three: where the answer is incomplete

Three concessions. None is rhetorical, and none of them is repaired later in this book.

CONCESSION ONE — agreement is not correctness. Two analysts trained on the same rules by the same person will agree with each other and may both be wrong in the same direction. Inter-rater agreement measures the reliability of a procedure and says nothing about its validity. This is a standard and well-understood limitation of every agreement statistic in psychometrics, and it is not special to this construction. What would be needed is a criterion external to the decomposition against which counts could be checked, and this book does not have one. The nearest available substitute is predictive: does the count predict something it was not fitted to? That is FVP-6's business, the design is specified in §6.14, and it has not been run.

CONCESSION TWO — D2 fixes the vocabulary and does not justify it. The twelve components are defended in Chapter 5 §5.7 by the nesting argument: each rung adds exactly one component to the rung below, so the order is forced rather than chosen. That defence establishes the *order* given the components. **It does not establish that these twelve are the right twelve, or that twelve is the right number.** Ashby's requisite variety and Chapter 4's ceiling `c/eta <= ln N` both make N consequential — raising N is one of only four levers — so the choice of twelve is not harmless. A thirteenth genuinely distinct component would raise the ceiling, and this book has no argument that no such component exists. It has only the claim that it could not find one that survived D1 and D6.

CONCESSION THREE — and this is the deepest one. The substitution test D1 appeals to what

the subject would say they were not in. That is a judgment made by a person about their own state, and this book has no independent access to it. The whole apparatus therefore rests on first-person report at exactly one point. **The count is absolute-scaled given a decomposition; the decomposition is explicit and testable; and underneath both there is a subject saying yes, that is a different state, and there is nothing underneath that.**

This is not a defect the book can engineer away, and it should not pretend to. It is the same foundation psychophysics rests on, and psychophysics has done a century of replicable work on it — which is a genuine reason for confidence and not a proof of anything. **The honest summary is that the essence index converts an unbounded judgment into a bounded, explicit, testable one, and does not eliminate judgment.** No procedure does. What this book claims is that the judgment has been moved somewhere it can be audited, and Chapter 5's claim about scale type is a claim about the arithmetic downstream of that judgment, not about the judgment itself.

The Engineering

6.9 The elicitation procedure

This is the procedure. Chapters 11, 13 and 24 cite it and do not restate it. It is written for a practitioner working with one subject on one situation, producing one or more framings. Steps 1 to 5 establish the frame; 6 to 11 elicit; 12 to 16 build and check the vector; 17 to 20 report.

A prohibition that governs the whole procedure and is repeated inside it: this is not an assessment. Nothing in the procedure evaluates the subject, ranks them, screens them, or produces anything that may be described to them or to anyone else as a finding about their mind. The procedure produces a description of a description. A practitioner who cannot hold that distinction should not run it.

```
STEP 1 -- DECLARE THE RULE SET BEFORE READING ANYTHING.
 Write down: the decomposition rules in force (D1–D8 or a declared variant), N = 12, the
 ladder in use, the intensity instrument and its range, and the audible band and folding
    convention. Date it. This page is published with the reading. (D8)

STEP 2 -- OBTAIN CONSENT, AND MAKE DECLINING STRUCTURAL.
 The subject is told what is being elicited, what will be computed, what it does and doe
 not mean, and that any question may be skipped without giving a reason and without the
 procedure being abandoned. **No intervention in this book may remove a person's abilit
 decline. Consent is structural, not advisory.** Nothing is masked, nothing is conceale
    and nothing is administered below the subject's awareness.

STEP 3 -- FIX THE SITUATION.
 One situation, named in the subject's own words, in one sentence, written down before
 anything else is asked. Everything downstream is indexed to this sentence.

STEP 4 -- FIX THE FRAMINGS TO BE ELICITED.
 Decide in advance how many framings of the situation will be elicited and what each is.
 Two is the minimum for any relational reading; three is the house default. Name them
```

```
        before eliciting any of them.

    STEP 5 -- SILENT WRITE-FIRST.
      The subject writes each framing out in full, independently, before any two are compare
      and before the practitioner comments on any of them. This is the personal-
scope form of
      the silent-write-first rule and it exists to stop the framings converging by conversat
        before they have been recorded.

    STEP 6 -- READ THE FIRST FRAMING AGAINST THE TWELVE COMPONENTS.
      For each of the twelve components in order, mark PRESENT or ABSENT. Every one of the
        twelve is marked. There is no blank. (D7)

    STEP 7 -- APPLY THE SUBSTITUTION TEST TO EVERY COMPONENT MARKED PRESENT.
      Remove it; read the residual description back to the subject; ask whether that is stil
      the state they were in. If yes, the component was not required: unmark it. (D1)

    STEP 8 -- CHECK NESTING.
      If any component is marked present above a component marked absent, go back and ask
        about the absent one. Do not record a gap. (D3)

    STEP 9 -- COUNT.
      n = the number of components marked present after steps 7 and 8. Record the count and,
      beside it, the list. **A count without its list is a rating and is rejected.** (Ch 5 §5

    STEP 10 -- ELICIT THE INTENSITY, BY MAGNITUDE ESTIMATION.
      Choose the state class the subject reports most strongly and assign it a reference valu
      -- 10 on a 0-10 instrument. Then, for each other state class present, ask: "compared wi
      that one, how strong is this one -- half as strong, a quarter, three times?" **The ques
      is asked in ratios because ratios are what the construction consumes** (§6.3). Record
        answers. Do not average across framings.

    STEP 11 -- ELICIT THE REMAINING FRAMINGS.
      Repeat steps 6-10 for each framing named in step 4, without showing the subject the
        earlier vectors.

    STEP 12 -- CHECK SUPPORT OVERLAP. THIS STEP IS NOT OPTIONAL.
       If two framings have disjoint support -- no state class present in both -
- then u = 1
      exactly, regardless of the intensities, and the reading is degenerate. Verified:

            five constructed pairs with disjoint support, random intensities:
             u = 1.000000000000 in every case

      A degenerate reading is not a high reading. It is no reading. **Framings must be
      represented with overlapping support.** Chapter 9 owns this trap and states it in full
      the procedural requirement here is to detect it before anything is computed and to
        return to step 5 if it fires.
```

```
    STEP 13 -- RECORD THE RAW NORM, THEN NORMALISE.
     Compute ||x||_2 in the instrument's units and write it down (§6.4). Then divide every
     component by it. The unit of the instrument cancels (§6.3); the norm does not, and is
        reported.

    STEP 14 -- COMPUTE THE PAIRWISE READINGS.
              For every pair of framings: cos theta = x.y, u = 1 -
(x.y)^2. Report all pairs
        individually. Do not report only the mean.

    STEP 15 -- AGGREGATE.
     Mean pairwise u over all pairs; then F = ln(1 + 11 u_bar), N_eff = 1 + 11 u_bar,
        f = k(1 + 11 u_bar)^2.

    STEP 16 -- COMPUTE THE SPECTRUM AND THE CHORD.
     For each state class in the union of the supports, f = k n^2, folded into the declared
     band, with its pitch class = odd part of n. Chapter 9 owns the chord; this step produces
        its inputs.

    STEP 17 -- RUN THE SECOND RATER.
     An independent analyst, working from the step 1 rule set and the step 5 written framing
        alone, repeats steps 6-13. Report A1, A2 and A3 (§6.7).

    STEP 18 -- RUN THE PERMUTATION TEST.
     Swap two adjacent indices in the ladder, recompute, report the change in cents and in
        pitch class. This is I6 of Chapter 5 §5.16 and it takes ten minutes.

    STEP 19 -- PUBLISH THE DECOMPOSITION WITH THE FREQUENCY.
     Same table, same page. Where it will not fit, the frequency is not published either.

    STEP 20 -- STATE THE BOUNDS IN THE REPORT ITSELF.
     Every report carries, in its own words: that the frequencies are essence frequencies a
     not physical resonances; that **the twelve states are not diagnoses and the reading is
     a clinical, medical or therapeutic assessment of anyone**; and that the frequency inhe
        exactly the defensibility of the count.
```

6.10 The worksheet

Printable. One sheet per framing, plus one summary sheet. Appendix C carries the same form.

```
  ================================================================================
   ESSENCE STATE VECTOR -- FRAMING SHEET                       sheet ___ of ___
  ================================================================================
    Rule set declared (date / version): _____________________   N = 12
  Ladder: the twelve-state ladder, Chapter 5 §5.6, unaltered  k = 0.483425732988414 Hz
    Intensity instrument and range: _______________  Audible band: _______ Hz
    Consent recorded, decline available at every step:  [ ]
```

```
    THE SITUATION, in the subject's words, one sentence:
   ____________________________________________________________________________

   THE FRAMING, name: _____________________  written independently before comparison [ ]

    ____________________________________________________________________________
---------
    n  state class              component added         PRES  SUBST  INTENSITY
    ____________________________________________________________________________
---------
    1  undifferentiated activation  arousal alone          [  ]  [  ]  _________
    2  valenced activation        + valence               [  ]  [  ]  _________
    3  directed feeling          + intentional object     [  ]  [  ]  _________
    4  situated feeling           + situation             [  ]  [  ]  _________
    5  attributed feeling         + cause                 [  ]  [  ]  _________
    6  temporal feeling          + projected duration     [  ]  [  ]  _________
    7  reflexive feeling          + self as object        [  ]  [  ]  _________
    8  social feeling            + another mind modelled   [  ]  [  ]  _________
    9  normative feeling          + standard applied       [  ]  [  ]  _________
   10  narrative feeling          + placed in a story      [  ]  [  ]  _________
   11  reciprocal feeling        + the other modelling you  [  ]  [  ]  _________
   12  committed feeling          + future action bound    [  ]  [  ]  _________
    ____________________________________________________________________________
---------
    PRES  = marked present or absent. EVERY row is marked. No blanks.  (D7)
    SUBST = survived the substitution test.                            (D1)
    Nesting check passed (no present-above-absent):  [ ]               (D3)

   COUNT  n = __________     COMPONENT LIST (required; a count without a list is void):
   ____________________________________________________________________________

   RAW NORM  ||x||_2 = __________ (instrument units)  <- reported, never discarded

    UNIT VECTOR (raw / norm), to 6 d.p.:
      x1 _____  x2 _____  x3 _____  x4 _____  x5 _____  x6 _____
      x7 _____  x8 _____  x9 _____  x10 _____ x11 _____ x12 _____
    Support = { _____________________ }
   ============================================================================

   ============================================================================
    ESSENCE STATE VECTOR -- SUMMARY SHEET
   ============================================================================
    Framings elicited: _____________________________________________
   SUPPORT OVERLAP CHECK (step 12): every pair shares at least one state class [ ]
     if any pair does not: u = 1 by construction, reading is DEGENERATE, return to step 5

    PAIRWISE, all pairs listed individually:
```

```
      pair _______________  cos theta = ________   u = ________
      pair _______________  cos theta = ________   u = ________
      pair _______________  cos theta = ________   u = ________

    mean pairwise u_bar = _________
    F = ln(1 + 11 u_bar) = _________ nats       N_eff = 1 + 11 u_bar = _________
    f = k (1 + 11 u_bar)^2 = _________ Hz       folded (j = ____) = _________ Hz

    SPECTRUM: states spanned { _______________ }
      n = ___  f = _________ Hz  folded _________  class ___
      n = ___  f = _________ Hz  folded _________  class ___
      n = ___  f = _________ Hz  folded _________  class ___
      n = ___  f = _________ Hz  folded _________  class ___
   distinct pitch classes: { _____________ }  count ____  of ceil(12/2) = 6 available

    AGREEMENT (step 17):  A1 exact-n ____/____   A2 pitch-class ____/____
                          A3 cents: mean ______ median ______ max ______
      PERMUTATION TEST (step 18): swapped ___ <-> ___ ;   class ___  -
> ___ ;  ______ cents

    BOUNDS, restated on every report:
      [ ] essence frequencies, not physical resonances
    [ ] the twelve states are NOT diagnoses; this is not a clinical, medical, diagnostic
          or therapeutic assessment of any person
      [ ] the frequency inherits exactly the defensibility of the count
      [ ] decomposition published with the frequency
   ================================================================================
```

6.11 A worked pass through the vector arithmetic

Before the full carried case, one framing taken end to end so the arithmetic is visible.

Raw intensities elicited by magnitude estimation, on a 0-to-10 instrument, for a framing with three state classes present:

```
state 3  (directed feeling)      8
state 5  (attributed feeling)    4
state 7  (reflexive feeling)     2
```

Step 13, the raw norm:

```
||x||_2 = sqrt(8^2 + 4^2 + 2^2) = sqrt(64 + 16 + 4) = sqrt(84) = 9.165151
```

Recorded, then divided out:

```
x_3 = 8 / 9.165151 = 0.872872
x_5 = 4 / 9.165151 = 0.436436
x_7 = 2 / 9.165151 = 0.218218

check: 0.872872^2 + 0.436436^2 + 0.218218^2 = 1.000000
```

And the first invariance in operation: had the same subject been elicited on a 0-to-1 instrument at 0.8, 0.4, 0.2, the norm would have been 0.916515 and the unit vector identical to nine decimals. The instrument's unit cancels. The ratios 4 : 2 : 1 are the entire content.

The Specification

6.12 The agreement protocol

```
S6.1  TWO RATERS MINIMUM.  Independent, working from the declared rule set and the written
        framings only. No consultation until both have completed step 13.

S6.2  BLIND TO EACH OTHER AND TO THE HYPOTHESIS.  The second rater does not see the first
     rater's count and is not told what reading the analysis is expected to produce.

S6.3  REPORT A1, A2 AND A3 SEPARATELY.  Exact agreement on n; agreement on pitch class;
     the distribution of cents disagreement. A weighted kappa may be reported in addition
        and never instead.

S6.4  THE THRESHOLD IS ON PITCH, NOT ON KAPPA.  Declare, before the analysis, the maximum
     pitch disagreement at which a frequency will still be published. Report the fraction
        of episodes exceeding it.

S6.5  DISAGREEMENTS ARE ADJUDICATED IN PRINT OR NOT AT ALL.  Where raters differ, publish
     both counts and both component lists. A consensus count with the disagreement erased
        is a fabricated agreement figure.

     S6.6   RATER DRIFT IS CHECKED.   A fixed set of episodes is re-
rated at intervals by the same
       rater. Within-rater agreement over time is reported alongside between-
rater agreement.
```

6.13 The reporting standard

Every published essence frequency in this book, and every one produced by a reader following it, carries: the decomposition (I1), the count rule (I2), N and the ceiling `ln N` (I3), the tie-break (I4), the band and the folding exponent `j` (I5), the permutation test result (I6), and the agreement figure (I7). Those are Chapter 5 §5.16's seven publication requirements and this chapter's procedure is built to produce all seven as a by-product of running it.

To which this chapter adds two:

```
I8  THE RAW NORMS.  ||x||_2 for every vector, in the instrument's units.  (§6.4)
I9  THE SUPPORT OVERLAP.  For every pair, the states shared. A pair sharing none is
   reported as DEGENERATE and its u is not published as a reading. (step 12, Ch 9)
```

6.14 The Formula Validation Protocol table for the index

No formula appears in this series without passing all seven gates in print. The protocol is Book Two Ch 52's and it is binding. This is the FVP table for the psychological essence index and the essence law applied to it.

F-06 . THE PSYCHOLOGICAL ESSENCE INDEX

```
    STATEMENT

      n := the number of distinguishable components required to specify the state
       f  =   k n^2,     k = 0.483425732988414 Hz,   N = 12,   n in {1, ..., 12}

   READS. How many things you must say before a listener knows which state you mean —
—
            squared, and scaled to a pitch.

   STATUS. The index is a DEFINITION. The essence law f = k n^2 is a POSTULATE about essence
        structure (CANON_SONIC §6), verified to collapse exactly from the 2022 source
        recipe and reproducing all 118 published element values to a worst relative
         deviation of 9.0e-15 (R5). k is a CHOSEN reference constant, badged.
```

Gate	Verdict	Content
FVP-1 DIMENSIONS	**PASS**	n is a dimensionless count. k carries Hz. `f = k n^2` therefore carries Hz on both sides. Every interval `f(m)/f(n) = m^2/n^2` is dimensionless and k cancels exactly, which is why the interval structure of Part II does not depend on the constant at all.

Gate	Verdict	Content
FVP-2 SCALE TYPE	**PASS — and this is the gate the chapter exists for**	n is a cardinality. It has a true zero (`n = 0`: no components, no state, `f = 0`) and a fixed unit (one component), and it admits only the identity transformation. **It is therefore ABSOLUTE-scaled, which is strictly stronger than the ratio scale that squaring requires.** Contrast R1: `D/P` returns DEMOTE at this gate because `D` and `P` are ordinal ratings whose ratio is not invariant under admissible transformations — measured spread of 2.490 octaves in Chapter 5 §5.3. The index is not a ratio of ratings and does not inherit that demotion; it sidesteps it. The gate is passed for n and is passed for n only. The intensities that build the state vector are a separate quantity with a separate answer: they must be ratio-scaled up to a free multiplier (§6.3), the free multiplier is genuinely free (first invariance, exact), and the ratio requirement is genuinely binding (second invariance fails, 1065.05 cents of spread under monotone transformation). The warrant for the intensities is Stevens' magnitude estimation, reported as a measured property of a procedure and not a constant of nature.

Gate	Verdict	Content
FVP-3 LIMITING CASES	**PASS**	`n -> 0`: `f -> 0`. No components, no state, no pitch — and the zero vector correctly has no unit form. `n = 1`: `f = k`, the ground essence itself; a one-component state sits at the system's floor, which is the right place for undifferentiated activation. `n -> N`: `f -> k N^2 = 69.6133 Hz`, the ceiling, and by Chapter 4 the maximum sustainable cost ratio `c/eta <= ln N = 2.484907`. No divergence anywhere in range. The unbounded limit `n -> infinity` is not reachable: the ladder is finite by D2, and an episode requiring a thirteenth component is out of scope and reported as out of scope rather than indexed.
FVP-4 FIXED POINTS	**PASS**	`f(n) = n` has the fixed point `n = 1/k = 2.06856`, which is not an integer and therefore not attained — the index and its frequency never coincide, and no reading should be constructed as though they do. The structurally meaningful invariant is different and is number-theoretic: **`f` is invariant in pitch class under `n -> 2n`**, since `(2n)^2 = 4n^2` is exactly two octaves. The fixed points of the class map are the odd integers, which is why the twelve-state ladder has exactly `ceil(12/2) = 6` classes. Chapter 7 owns the derivation.

Gate	Verdict	Content
FVP-5 IDENTIFIABILITY	**PARTIAL — declared**	n is recoverable from obtainable data: the component list is the data, and steps 6 to 9 recover it. **But the map from episode to count is not injective and not everywhere agreed.** Two decompositions of one episode can return different n, which is the decomposition objection of §6.5, conceded in §6.8. The identifiability that holds: given a declared rule set, two raters' counts are comparable and their disagreement is measurable, which is a stronger position than R1's `D/P`, where a single rater's number is undetermined. The identifiability that does not hold: there is no procedure in this book that identifies the *correct* decomposition, and §6.8 concession two states that the choice of twelve components is itself unjustified beyond a failed search for a thirteenth.

Gate	Verdict	Content
FVP-6 FALSIFICATION	**SPECIFIED — NOT RUN**	The essence law is a postulate about essence structure and makes no empirical claim on its own; what is falsifiable is the operationalisation. Design, following Paper E §5.3: preregister the rule set, the ladder, the elicitation script and the analysis; fit on training episodes; **FREEZE**; report held-out performance on whether the count predicts a criterion it was not fitted to. Beat four baselines — persistence, direct-input, matched autoregressive, and shuffled-input control. Kill condition, stated in advance: if counts elicited under the frozen protocol fail to beat the persistence baseline on held-out prediction, the operationalisation is dead and the ladder is withdrawn. Two standing corrections apply and must be carried into the design: **R13(a)** — the shuffled-input control is anti-conservative as ordinarily run, with measured false-rejection rates of 0.108 (iid permutation) and 0.120 (circular shift) against a nominal 0.05, so the surrogate must be calibrated by simulation on the study's own design before use; and **R13(b)** — no surrogate detects fabricated timing, so design controls (blinding, channel separation, prospective sampling, a yoked control) are mandatory and not substitutable. **This test has not been run.**

Gate	Verdict	Content
FVP-7 REPRODUCIBILITY	**NOT YET RUN — declared**	**The cross-individual test has not been performed for this index and this book does not claim it has.** The question is whether the same operationalisation, applied by different analysts to different subjects in different settings, recovers the same parameters — specifically whether the component count for a given class of episode is a property of the episode or a property of the analyst who counted it. This is the gate that decides the hard-science question and it is the one most often skipped. It is skipped here too, and it is reported rather than omitted. What exists instead is the constructed demonstration of §6.17, which is an illustration of the *protocol*, not a result: constructed data cannot reproduce anything. The route to running it is the same one R1 names for the Moment equation — Rasch instrumentation, which achieves separation of person and item parameters so that a measure does not depend on which items were used, at a measured cost of 123 items per term. R14's costed programme puts the cross-subject recovery stage at the point of highest mortality, with a 55.3% chance of the programme dying at or before it, and Book Two's §56 notes that this is not a coincidence: the gate that decides the question is the gate most likely to kill the answer. Until FVP-7 is run, every frequency in this book is a frequency *under a declared decomposition*, and that qualification is not decorative.

Overall verdict on F-06: passes gates 1 to 4 outright, gate 5 partially and with the shortfall named, gate 6 specified but unrun, gate 7 unrun. That is a formula in good standing on scale type and structure and unvalidated on reproducibility, and the correct way to use it is the way this book uses it: as an exact construction whose inputs are declared, whose disagreements are published, and whose empirical status is stated in the same breath as its numbers.

The Failure Modes

6.15 Leading the elicitation

The practitioner reads the ladder aloud, in order, asking after each rung whether that one is present. By rung nine the subject has learned the shape of the instrument and what a fuller answer looks like. **Counts elicited this way climb through a session, and they climb for every subject, which is the signature of the artefact.**

The defences are in the procedure and each is there for this reason. Step 5's silent write-first captures the framing before any prompting. Step 7's substitution test removes components the subject will not defend when the residual is read back to them. Step 17's second rater works from the written framing and never meets the subject, so a leading elicitation shows up as a systematic gap between the two raters' counts rather than as a shared error.

Detection: plot count against position in session. A rising trend is an artefact until shown otherwise.

6.16 Disjoint support, and why a high u can be a null result

Step 12 exists because of an exact and easily missed degeneracy. If two framings share no state class, their inner product is zero, so `u = 1` regardless of what the intensities are. Five constructed pairs with disjoint support and randomly drawn intensities give `u = 1.000000000000` every time.

A one-hot framing — a subject represented as holding exactly one state — is the extreme case, and one-hot framings are automatically orthogonal to each other. They score `u = 1` for everyone, on every topic, regardless of content, which means the reading has ceased to depend on the subject at all.

A degenerate reading is not a strong reading. It is the absence of a reading wearing the costume of the strongest possible one, and because `u = 1` is the top of the scale it looks like a triumph. This is a real trap and the book names it. Chapter 9 owns it and states it in full; Chapter 13 restates it for the between-persons case. The obligation here is procedural: check overlap at step 12, before anything is computed, and return to step 5 if the check fires.

6.17 Agreeing on kappa while disagreeing on the note

This is the failure mode most likely to defeat a careful practitioner, because every conventional instrument says the work is fine.

A **CONSTRUCTED** two-rater exercise. Fifteen constructed vignettes; two raters, working independently from the same declared rule set, each returning a count.

```
rater 1:  3  5  7  4  9  6 11  2  8  5 10  3  7 12  6
rater 2:  3  5  6  4  9  7 11  2  8  6 10  4  7 12  6
```

The agreement statistics:

```
exact agreement on n        11/15 = 0.7333
agreement within one         15/15 = 1.0000
mean |dn|                    0.2667

unweighted kappa             0.7044
linear-weighted kappa        0.9185
quadratic-weighted kappa     0.9838
```

By the conventional benchmarks — Landis and Koch's much-quoted table puts anything above 0.81 in the top band — a quadratic-weighted kappa of 0.9838 reads as very nearly perfect agreement. Every rater was within one category of the other on every single episode. A methods section reporting these figures would pass without comment.

Now report A2 and A3, which S6.3 requires:

```
agreement on PITCH CLASS     11/15 = 0.7333
pitch disagreement, cents:   mean 179.66   median 0.00   max 996.09
                             max = 0.8301 octaves

the four disagreements:
  ep 3   n = 7 vs 6    class 7 vs 3     533.74 cents
  ep 6   n = 6 vs 7    class 3 vs 7     533.74 cents
  ep10   n = 5 vs 6    class 5 vs 3     631.28 cents
  ep12   n = 3 vs 4    class 3 vs 1     996.09 cents
```

A quadratic-weighted kappa of 0.9838 coexists with pitch-class disagreement on more than a quarter of episodes and a worst-case disagreement of 996 cents — five sixths of an octave, from a one-component difference at the steep end of the ladder.

The mechanism is Chapter 5 §5.17's and it is exact. Weighted kappa penalises disagreement by rank distance. The essence law's consequence is a square law on the index, and the pitch class is the odd part, which is not monotone in `n`. On episode 12 the raters differ by one rank — the smallest possible disagreement, barely penalised by any weighting — and land 996 cents and two pitch classes apart, because at the bottom of the ladder one component is worth almost an octave.

The rule that follows is S6.3 and it is absolute: report agreement on the pitch, always, and never substitute a rank-based statistic for it. A weighted kappa is not wrong; it is answering a question about ranks, and this book does not consume ranks.

The same failure appears one level down, in the intensities. A **CONSTRUCTED** intensity exercise, ten items, two raters, on a 0-to-10 instrument, gives `ICC(2,1) = 0.9379` — again, by convention, excellent. Follow one such disagreement downstream:

```
rater 1 vector {3:8, 5:4, 7:2}  -> u = 0.388360  against a fixed reference framing
 rater 2 vector {3:7, 5:5, 7:2}   ->  u = 0.301994
                                      |du| = 0.086365
```

```
    f_rel:  13.4361 Hz  versus  9.0300 Hz  =  687.99 cents apart
```

Two raters differing by one point on two items, at an ICC of 0.9379, produce relationship frequencies 688 cents apart. Chapter 12 will show that `f_rel = k(1 + 11u)^2` is steep in u for precisely this reason. **The agreement figure that matters is the agreement on the quantity you intend to publish, and if you intend to publish a frequency then you must report agreement on the frequency.**

6.18 Retro-fitting the decomposition

An analyst expects a state to come out around eight, counts six, and revisits the substitution test on two borderline components with a rather more generous reading. The count is now eight. Nothing dishonest has been done at any single step.

D8 is the only defence and it is a procedural one: the rule set is fixed and dated before the episode is read, and any revision produces a second, separately published analysis rather than a correction to the first. Step 1 requires a dated rule-set page for exactly this reason. An analysis whose rule set has no date is not auditable and its frequencies should not be believed.

The second defence is step 17. A blinded second rater working from the original rule set will not reproduce the generous reading, and the disagreement lands in A1, A2 and A3 where it can be seen.

6.19 Running the procedure as an assessment

This is the failure mode with consequences outside the book, and it is prohibited rather than merely discouraged.

The procedure produces a number about a description. It does not produce a finding about a person. A practitioner who reports to a subject that their state “came out at nine,” or who compares two subjects’ counts, or who tracks a subject’s counts over time as though the trend meant something about them, has converted a formal construction into an assessment. **The twelve states are not diagnoses. The counts are not severities. The chord is not a profile. No reading in this book assesses anyone’s mental health, and no procedure in it may be administered as though it did.**

Step 2 and step 20 are the structural defences: the subject is told at the outset what the procedure does and does not mean, and every report restates it. **And step 2’s consent clause is not a formality — no intervention in this book may remove a person’s ability to decline, at any step, without explanation, and without the procedure being abandoned in consequence.** Nothing is masked, nothing is concealed, nothing operates below the subject’s awareness. A person cannot consent to what is hidden from them.

6.20 Rater drift

A single analyst’s decomposition shifts over weeks. Early counts and late counts are not comparable, and because the drift is smooth it will not show up as noise; it will show up as a trend, and a trend invites an interpretation.

S6.6 is the defence: a fixed set of episodes, re-rated at intervals by the same rater, with within-rater agreement reported alongside between-rater agreement. The statistic is the same triple — A1, A2, A3 — computed against the rater's own earlier counts.

Drift is not a minor operational nuisance in this construction. It is FVP-7's question asked within a single person, and a procedure that cannot reproduce its own counts across time has no prospect of reproducing them across people.

Worked Example

CONSTRUCTED throughout. No data were collected, no person was elicited, and nothing below is a case report or an observation. This is the book's carried subject, fixed by SPINE §3.1, and every chapter that needs this case uses this one and no other.

6.21 THE SUBJECT — three framings of one situation

Step 3, the situation, in the constructed subject's words: *A colleague did something in a meeting that I have not been able to stop thinking about.*

Step 4, the framings to be elicited. Three, named in advance:

```
THREAT   -- the situation read as something happening to me
STORY    -- the situation read as an instalment in a longer account
MIRROR   -- the situation read as being about what it says about me
```

Steps 5 to 9, the decomposition. Each framing written independently, then read against the twelve components, with the substitution test applied to every one marked present and the nesting check run. The three framings return these supports:

```
THREAT  states 3, 5, 7    directed feeling, attributed feeling, reflexive feeling
STORY   states 3, 5, 10   directed feeling, attributed feeling, narrative feeling
MIRROR  states 5, 7, 10    attributed feeling, reflexive feeling, narrative feeling
```

Step 12, the support overlap check, run before anything is computed:

```
THREAT & STORY   share {3, 5}      2 states   OK
THREAT & MIRROR  share {5, 7}      2 states   OK
STORY  & MIRROR  share {5, 10}     2 states   OK
```

Every pair overlaps. The reading is not degenerate. Had the three framings been one-hot, or had any pair been disjoint, u would have been 1 by construction and the analysis would have returned to step 5. This check is the reason the carried case is built with overlapping support, and it is the fix, not a convenience.

Step 10, magnitude estimation. Raw intensities, elicited by ratio comparison against the strongest state class in each framing, on a 0-to-1 instrument:

```
THREAT   {3: 0.8, 5: 0.4, 7: 0.2}
STORY    {3: 0.5, 5: 0.7, 10: 0.4}
MIRROR   {5: 0.3, 7: 0.8, 10: 0.5}
```

Step 13, raw norms recorded, then normalisation:

```
||threat||_2 = sqrt(0.64 + 0.16 + 0.04) = sqrt(0.84) = 0.916515
||story ||_2 = sqrt(0.25 + 0.49 + 0.16) = sqrt(0.90) = 0.948683
||mirror||_2 = sqrt(0.09 + 0.64 + 0.25) = sqrt(0.98) = 0.989949

threat unit  = (0.872872 at 3,  0.436436 at 5,  0.218218 at 7 )
story  unit  = (0.527046 at 3,  0.737865 at 5,  0.421637 at 10)
mirror unit  = (0.303046 at 5,  0.808122 at 7,  0.505076 at 10)
```

The norms are reported and not discarded (I8). They are close here — 0.9165, 0.9487, 0.9899 — which says the three framings are held at comparable overall intensity. Had one been an order of magnitude smaller, u would not have shown it, and the reading would have been of a framing the subject barely holds.

Step 14, the pairwise readings, all reported individually:

```
pair              cos theta      theta        u
mirror & story     0.436566    64.1150 deg   0.809410
mirror & threat    0.308607    72.0247 deg   0.904762
story  & threat    0.782074    38.5491 deg   0.388360
```

Step 15, aggregation:

```
mean pairwise u_int = (0.809410 + 0.904762 + 0.388360) / 3 = 0.700844

F_int = ln(1 + 11 * 0.700844) = ln(8.709284) = 2.164390 nats
N_eff = 1 + 11 * 0.700844 = 8.7093

f = k (1 + 11 u)^2 = 0.483425732988414 * 8.709284^2 = 36.6686 Hz
folded into 220–440 with j = 3:  36.6686 * 8 = 293.349 Hz
```

Step 16, the spectrum and the chord inputs:

```
states spanned: {3, 5, 7, 10}

n = 3   f =  4.3508 Hz   folded 278.453   class 3
n = 5   f = 12.0856 Hz   folded 386.741   class 5
n = 7   f = 23.6879 Hz   folded 379.006   class 7
n = 10  f = 48.3426 Hz   folded 386.741   class 5

pitch classes {3, 5, 7} -- THREE distinct, though FOUR states are held.
States 5 and 10 share odd part 5, so one of the four adds pitch but no new class.
```

That last line is the reading's most instructive feature and it is not an accident of this case. The subject holds four state classes and commands three notes. Adding the narrative component at `n = 10` raised the pitch by two octaves — `(10/5)^2 = 4` exactly — and added no new harmonic class, because 10 and 5 have the same odd part. **Chapter 7 owns the rule and Chapter 9 owns what it means for a chord; here it is enough to observe that the twelve-state ladder offers six classes** (`ceil(12/2)`) **and this subject occupies three of them.**

Steps 18 and 19. The permutation test, run on the pair that matters most for this subject — swapping indices 5 and 6 in the ladder:

```
class 5 -> class 3,  568.72 cents
```

Well over the quarter-octave threshold, so the ordering is load-bearing for this reading and is defended by the nesting argument of Chapter 5 §5.7 or the reading is not published. The decomposition — the three component lists behind the three supports — is published in the same table as the frequencies.

6.22 The agreement pass on the same subject

Step 17, run on the THREAT framing. A second analyst, blind to the first analyst's counts and to the intended reading, works from the subject's written framing and the dated rule set alone.

The first analyst marked states 3, 5 and 7 present with intensities 0.8, 0.4, 0.2. Suppose the second analyst agrees on the support and reads the intensities as 0.7, 0.5, 0.2 — a one-point difference on a 0-to-10 instrument, at two of three items. Section 6.17 computed exactly this case:

```
analyst 1  ->  u(threat, story) = 0.388360
analyst 2  ->  u(threat, story) = 0.301994
|du| = 0.086365

f_rel:  13.4361 Hz  versus  9.0300 Hz  =  687.99 cents apart
```

Perfect agreement on the count — A1 = 3/3, A2 = 3/3, A3 = 0.00 cents on the spectrum — and 688 cents of disagreement on the relational frequency. The two agreement questions are different questions and S6.3 requires both because neither predicts the other.

What a practitioner does with that. Not average the two. **S6.5: publish both, with both intensity sets.** The honest report of this framing's relational reading is that it lies between 9.03 and 13.44 Hz under two defensible elicitations, and that the interval — not the point — is the result. A reading published as 13.4361 Hz, with the second analyst's elicitation quietly discarded, would be a fabricated precision, and it is the precision, not the number, that would be the lie.

6.23 The reading, and its bounds

What the constructed case establishes. Under the declared rule set, the twelve-state ladder, and the elicitation of §6.9, this constructed subject holds three framings of one situation with mean pairwise non-parallelism `u_int = 0.700844`, internal surplus `F_int = 2.164390` nats, effective framing count `N_eff = 8.7093` of the twelve available, and a chord width of 36.6686 Hz, folded to 293.349 Hz. The subject spans four state classes and three pitch classes.

What it does not establish. Nothing about any person. The subject is constructed; the situation is invented; the intensities were chosen to demonstrate a procedure. **No observation is reported here and none is implied.**

What it licenses downstream. These exact numbers are the ones Chapters 9, 10 and 11 use. Chapter 9 builds the chord from the four states and the three classes and owns the one-hot degeneracy trap that step 12 was written to catch. Chapter 10 reads the intervals. Chapter 11 assembles the full personal worksheet around them. **No chapter re-elicits this subject and no chapter invents a different one.**

And the bounds, restated as step 20 requires them on every report.

36.6686 Hz is an essence frequency. **No claim is made that this constructed subject, or any person, or any tissue, or any molecule, physically resonates at that value or at any other value in this book.**

The twelve states are not diagnoses. This reading is not a clinical, medical, diagnostic or therapeutic assessment of anyone, the counts are not severities, and the chord is not a profile of anything.

And the limit that travels with every number this chapter produces: the frequency inherits exactly the defensibility of the count. A contested decomposition yields a contested frequency. Section 6.22's 688-cent spread is what that sentence looks like when it is measured instead of asserted, and **FVP-7 has not been run**, so every frequency in this book remains a frequency under a declared decomposition — which is a real thing to have, and is less than a measured one.

What this chapter owns

```
   THE ELICITATION PROCEDURE  Steps 1–20, §6.9. Cited by Ch 11, 13 and 24; not restated ther

   THE STATE VECTOR        Non-negative intensities over the twelve states, L2-
normalised.
                        Support = nonzero entries. Zero vector has no unit form.

   THE TWO INVARIANCES      The instrument's unit cancels exactly (verified to 9 d.p.).
                       Monotone transformation does NOT cancel (1065.05 cents of
                             spread). Therefore the intensities must be ratio-
scaled up to a
                       free multiplier, and Stevens' magnitude estimation is the warrant.

   THE MAGNITUDE RULE       ||x||_2 is reported with every vector and never discarded.
                       u measures direction; the norm measures how much there is.

   THE DEFENCE OF THE COUNT   D1–D8 make the granularity judgment explicit; A1/A2/A3 make it
                          testable; §6.8 concedes what remains: agreement is not
                       correctness, twelve is unjustified beyond a failed search for a
                          thirteenth, and the substitution test rests on first-
person
                                    report with nothing underneath it.

   THE AGREEMENT PROTOCOL    S6.1–S6.6. Report agreement on the pitch, always. A weighted
                                       kappa of 0.9838 coexisted with pitch-
class disagreement on 4 of
                          15 constructed episodes and a 996.09-cent worst case.

      THE FVP TABLE                        F-06, all seven gates. Gates 1-
4 PASS. Gate 5 PARTIAL, declared.
```

```
Gate 6 SPECIFIED, NOT RUN, with R13's two corrections carried.
       **Gate 7 NOT YET RUN**, and reported as not run.
```

Chapter 7 takes the index this chapter has learned to elicit and applies `f = k n^2` to transient states, where it owns octave folding, the odd-part class rule, and the `ceil(N/2)` ceiling that has been quoted twice already and derived neither time.

Chapter 7 . The Frequency of a Mood

A mood changes. Something is different at four o'clock that was not different at three. The ordinary vocabulary for this is a vocabulary of magnitude — it got worse, it lifted, it deepened — and that vocabulary has one job, which is to say that something moved. It cannot say *what kind of thing* moved. It cannot distinguish the case where the same feeling arrived at a higher resolution from the case where a different feeling replaced it. Those two events feel, from inside, remarkably alike. Both are experienced as change. Both are reported with the same words.

This chapter takes the essence law from Chapter 29 of Book Two, points it at the twelve-state differentiation ladder that Chapter 5 fixed, and produces a distinction that the ordinary vocabulary cannot: **some mood changes move the pitch and leave the pitch class exactly where it was, and some change the class.** The first kind is an octave leap. The second kind is not. And the line between them is not drawn by judgment, taste, or clinical convention. It is drawn by the odd part of an integer.

That is the chapter's real content, and it is worth saying at the outset that it was not designed in. Nobody sat down to build a formalism that would separate intensification from transformation. The square law was already there; the folding rule was already there; the ladder was already there. The distinction falls out of the arithmetic of `(2n)^2 = 4n^2` and refuses to be argued with.

Three pieces of ownership are settled here, and every later chapter cites this one rather than re-deriving:

1. **Octave folding** — how a sub-audible essence frequency is brought into a stated band, and why the operation is exact.
2. **The odd-part class rule** — that two states share a pitch class if and only if their indices share an odd part, with the corollary that an indexed set `1..N` holds exactly `ceil(N/2)` classes.
3. **The 50/49 regime** — that `N = 12` sits below the `N = 17` minimum-gap cliff, so this entire book renders with a minimum class separation of 34.9756 cents.

Chapters 9, 16 and 26 use all three. None of them restates them.

The Law

7.1 The essence law, and what it is being applied to

POSTULATE (the essence law; Book Two Ch 29, CANON_SONIC §1). An essence type occupying ordinal position n within its discipline's ordered category set has essence frequency

```
f(n) = k * n^2

k = (9/8)^2 / phi^2 = (81/64)/(phi + 1) = 0.483425732988414 Hz
```

computed:

```
phi                 = 1.618033988749895
(9/8)^2             = 1.265625
phi^2 = phi + 1     = 2.618033988749895
k                   = 0.48342573298841435 Hz
(81/64)/(phi+1)     = 0.48342573298841435 Hz    (identical, as required)
```

k is a **chosen reference constant, not a measured one** — a Pythagorean ditone divided by the golden section, and a design decision by the author. It is badged as a postulate every time it appears, and Result One of CANON_SONIC §2 explains why the badge costs so little: k cancels out of every ratio in the system. It sets the absolute reference pitch and nothing else.

The index this chapter feeds into the law is **not this chapter's to define.** SPINE §1 fixes it, Chapter 5 owns it, and it is the twelve-state differentiation ladder:

```
n := the number of distinguishable components required to specify the state.
```

Chapter 5 makes the argument that matters — that this is a **count**, with a true zero and equal units, and therefore ratio-scaled by construction, and therefore *not* subject to the demotion R1 imposes on D/P. Chapter 6 defends it against the obvious objection. **This chapter neither redefines a state, renames one, nor adds one.** It cites the ladder as fixed and computes on it.

It also carries forward the honest limit that SPINE §1 attaches to every use of the index:

> **The frequency inherits exactly the defensibility of the count. A contested decomposition yields a contested frequency.**

That sentence appears in this chapter because this chapter publishes numbers, and a number published without its warrant is worse than no number.

7.2 Octave folding

The raw essence frequencies of the twelve-state ladder run from 0.4834 Hz to 69.6133 Hz. The lower half is sub-audible; the whole range sits below the pitch region where human pitch discrimination is at its best. To be heard, compared, or rendered, the values must be transposed.

The house method is octave transposition and only octave transposition (CANON_SONIC §5):

```
f_audible = f * 2^j          j an integer, positive or negative

STATE THE BAND AND THE j FOR EVERY VALUE PUBLISHED.
```

This book's standing band is **220–440 Hz** — one octave, closed below and open above, so that `j` is unique for every `f`. The choice of band is arbitrary and stated; the choice of *operation* is not arbitrary at all.

Multiplication by `2^j` is exact in the sense that matters here: it moves a frequency by a whole number of octaves and therefore **preserves pitch class exactly.** No rounding, no approximation, no tempering. Whatever two states shared before folding, they share after. Whatever separated them before folding, separates them by the same interval in cents after. Folding is a change of register and never a change of relationship.

Two consequences follow immediately and are used throughout the book:

- **Folding cannot create a collision.** If two states fold to the same value, they were already at the same pitch class.
- **Folding cannot remove one.** If two states share a class, no choice of band separates them.

Section 7.8 turns the first into a theorem and Failure Mode F8 turns the second into a warning about band-shopping.

7.3 The odd-part theorem

THEOREM 7.1 (the odd-part class rule). Write any positive integer index in its unique form

```
n = 2^a * m          a >= 0 integer,  m odd
```

`m` is the **odd part** of `n`. Then the pitch class of `f(n) = k n^2` is determined entirely by `m`, and two indices share a pitch class if and only if they share an odd part.

Proof. Substitute:

```
f(n) = k * (2^a * m)^2 = k * 2^(2a) * m^2 = 4^a * (k m^2) = 2^(2a) * f(m)
```

`2^(2a)` is a power of two, so `f(n)` is exactly `2a` octaves above `f(m)`, and octave displacement is precisely the equivalence that defines pitch class. Hence `class(n) = class(m)`.

Conversely, suppose `class(n1) = class(n2)`, i.e. `f(n1)/f(n2) = 2^j` for some integer `j`. Then

```
(n1/n2)^2 = 2^j
```

Write `n1 = 2^a1 m1`, `n2 = 2^a2 m2` with `m1, m2` odd. Then

```
(n1/n2)^2 = 2^(2a1 − 2a2) * (m1/m2)^2 = 2^j
=>  (m1/m2)^2 = 2^(j − 2a1 + 2a2)
```

The left side is a ratio of odd squares in lowest terms, hence odd over odd; the right side is a power of two. The only odd power of two is `2^0 = 1`. Therefore `m1 = m2`. **The odd parts are equal.** □

The proof also delivers a second result for free, and it is used in §7.8:

COROLLARY 7.1a (no odd octaves). In the exponent equation above, `j − 2a1 + 2a2 = 0`, so `j = 2(a1 − a2)` is **even**. Two states of the same class are separated by an *even* number of octaves — never one octave, never three, never five. The smallest class-preserving interval in the system is exactly two octaves, 2400 cents.

7.4 The `ceil(N/2)` ceiling

COROLLARY 7.2 (the class-count ceiling). An essence set indexed `1..N` contains exactly `ceil(N/2)` distinct pitch classes.

Proof. By Theorem 7.1 the classes are in bijection with the set of odd parts realised by `1..N`. Every odd `m <= N` is realised (take `n = m` itself, `a = 0`). No odd `m > N` is realised, since `n = 2^a m >= m > N`. So the class set is exactly the odd numbers in `1..N`, and the count of odd numbers in `1..N` is `ceil(N/2)`. []

Verified computationally for every N from 1 to 25 — the range that CANON3 §5's single-transposition audible capacity allows:

```
   N :  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
 cls :  1  1  2  2  3  3  4  4  5  5  6  6  7  7  8  8  9  9 10 10 11 11 12 12 13
ceil :  1  1  2  2  3  3  4  4  5  5  6  6  7  7  8  8  9  9 10 10 11 11 12 12 13
   all match: True
```

Twelve states yield six pitch classes. That is a hard structural limit and not a stylistic choice. It is not a claim that there are six feelings, or six kinds of feeling, or that six is enough; it is a claim about how many *distinct pitch classes* an index of twelve can generate under a square law. Failure Mode F5 says what happens to readers who conflate those.

7.5 The twelve-state spectrum, computed

The full spectrum. Every value computed; every value matches SPINE §1 exactly.

n	state class (Ch 5)	f = k.n² (Hz)	j	folded (220–440)	odd part	pitch class	cents above 220
1	undifferentiated activation	0.4834	9	247.514	1	1	204.008
2	valenced activation	1.9337	7	247.514	1	1	204.008
3	directed feeling	4.3508	6	278.453	3	3	407.918
4	situated feeling	7.7348	5	247.514	1	1	204.008
5	attributed feeling	12.0856	5	386.741	5	5	976.635
6	temporal feeling	17.4033	4	278.453	3	3	407.918
7	reflexive feeling	23.6879	4	379.006	7	7	941.660
8	social feeling	30.9392	3	247.514	1	1	204.008
9	normative feeling	39.1575	3	313.260	9	9	611.828
10	narrative feeling	48.3426	3	386.741	5	5	976.635

n	state class (Ch 5)	f = k.n² (Hz)	j	folded (220–440)	odd part	pitch class	cents above 220
11	reciprocal feeling	58.4945	2	233.978	11	11	106.644
12	committed feeling	69.6133	2	278.453	3	3	407.918

```
distinct classes  = {1, 3, 5, 7, 9, 11}   count 6 = ceil(12/2)    [verified]
 max f = k * 144    = 69.61330555033166 Hz
 ln N  = ln 12      = 2.4849066497880004
2/ln2            = 2.8853900817779268    (the octave conversion, CANON3 §1)
```

The maximum matters beyond this chapter. `f = k N^2 = 69.6133 Hz` is simultaneously the pitch of the most differentiated state on the ladder and, by CANON3 §1, the **ceiling on `f_min`** — the highest minimum sustainable frequency that a twelve-category system can ever meet. Chapter 4 owns that reading; it is noted here only because the same number does both jobs and a reader will meet it twice.

7.6 The 50/49 regime — stated once, here

This book renders entirely in the 50/49 regime, and this is the only place the fact is derived.

The folded class lattice for `N = 12`, sorted by folded pitch, with the interval to the next class above (the last row wraps to the octave):

class	folded (Hz)	interval to next	cents	Hz gap
11	233.978	-> 1	97.3641	13.5359
1	247.514	-> 3	203.9100	30.9392
3	278.453	-> 9	203.9100	34.8067
9	313.260	-> 7	329.8318	65.7459
7	379.006	-> 5	**34.9756**	**7.7348**
5	386.741	-> 11 (octave)	330.0085	81.2155

The tightest pair is class 7 against class 5. Their frequency ratio is exact:

```
f(5)/f(7) folded  =  (2 * 5^2) / 7^2  =  50/49
1200 * log2(50/49) =  34.975614791419815 cents
```

CANON3 §5 records the folded-lattice cliff at `N = 17`. Computed, sweeping `N` and taking the minimum adjacent separation over the whole folded class lattice:

N	min separation (cents)	between classes	ratio
8	34.9756	7 and 5	50/49
10	34.9756	7 and 5	50/49
12	**34.9756**	**7 and 5**	**50/49**
14	34.9756	7 and 5	50/49

N	min separation (cents)	between classes	ratio
16	34.9756	7 and 5	50/49
17	6.0008	3 and 17	289/288
18	6.0008	3 and 17	289/288
24	6.0008	3 and 17	289/288
25	6.0008	3 and 17	289/288
40	6.0008	3 and 17	289/288
41	1.0296	41 and 29	1682/1681

```
1200 * log2(289/288)   = 6.0008172700399145 cents
1200 * log2(1682/1681) = 1.0295772227721520 cents
```

The cliff at 17 is `17^2 = 289` colliding with `2 * 12^2 = 288`, i.e. class 17 arriving a sixth of a comma away from class 3. The second cliff at 41 is `41^2 = 1681` against `2^4 * 105 = 1680`, reached through class 29 (`29^2 = 841`, `1682/1681`). Both are exact rational coincidences, not approximations.

`N = 12` sits below the first cliff. Every folded distinction this book asks a listener or an instrument to make is at least 34.9756 cents wide — roughly a third of a semitone. That is a comfortable margin, and it is the reason the twelve-state ladder is renderable at all. **No other chapter re-derives this. Chapters 9, 16 and 26 cite it.**

One rendering consequence, computed here and used in §7.18. The tightest folded gap in absolute frequency is

```
386.740586 − 379.005775 = 7.734812 Hz
```

and the Gabor limit `dt * df >= 1/(4 pi)` (CANON3 §5) therefore imposes

```
dt >= 1 / (4 * pi * 7.734812) = 0.0102882 s = 10.2882 ms
```

as the *formal* lower bound on the duration of a tone that carries that distinction. At the widest gap in the lattice (class 5 up to class 11 at the octave, 81.2155 Hz) the same bound is 0.9798 ms. CANON3 §5 also records that the rate factor between the binding pairs is exactly 8, because they sit at different octave heights (49/32 against 9/8); that figure is cited, not re-derived. **The Gabor bound is a bound on the signal, not a statement about hearing.** Human discrimination needs far longer tones than 10 ms and the book never confuses the two.

7.7 Honesty clauses, restated in place

Restated because this is a chapter that publishes a table of frequencies next to a table of feelings, and that adjacency is exactly where drift happens.

- **These are ESSENCE frequencies, not physical resonances.** No claim is made that any person, tissue, cell, molecule, or nervous system physically resonates at 247.514 Hz, at 39.1575 Hz, or at any other value in this chapter. The frequency is the resonance of the essence — a number produced by a square law applied to a count. Nothing vibrates.
- **The twelve states are not diagnoses.** The ladder is a differentiation index. A reading is not an assessment, a score, a screen, or a stage. **No clinical, diagnostic or therapeutic claim is made anywhere in this chapter**, and no reader should treat a pitch class as a category of person.

- **k is postulated, not measured.** It cancels from every interval in the system.
- **The square law is a postulate about essence structure.** Its formal agreement with the eigenvalue spacing of a one-dimensional infinite square well (E_n proportional to n^2) is a genuine mathematical correspondence and may be noted as such — but coincidence of form is not evidence of mechanism, and this sentence exists so that the two never travel apart.
- **The frequency inherits the defensibility of the count** (SPINE §1).
- **All worked cases in this chapter are CONSTRUCTED and labelled as such.** None is an observation.

The Engineering

7.8 Two kinds of mood change

Here is the chapter's result, and everything from here to the end of Part II leans on it.

A mood change is a move from state `m` to state n on the ladder. Its interval, in cents, is

```
interval(m -> n) = 1200 * log2( f(n)/f(m) ) = 1200 * log2( n^2/m^2 ) = 2400 * log2( n/m )
```

Every such interval is a **ratio of squares**, which is Result One of CANON_SONIC §2, and k cancels out of it entirely.

THEOREM 7.3 (the intensification/transformation dichotomy). A transition `m -> n` on the ladder falls into exactly one of two classes:

```
(I)  INTENSIFICATION   m and n share an odd part
                       the interval is an exact WHOLE number of octaves
                       and that number is EVEN (Corollary 7.1a)
                       the pitch changes; the pitch CLASS does not

(T)  TRANSFORMATION    m and n have different odd parts
                       the interval is NEVER a whole number of octaves
                       the pitch changes AND the pitch class changes
```

Proof. The interval is a whole number of octaves iff `f(n)/f(m) = (n/m)^2` is a power of two, iff (by the argument of Theorem 7.1) `m` and n share an odd part. Corollary 7.1a supplies the evenness. []

Verified exhaustively over all 66 unordered pairs of the twelve-state ladder:

```
whole-octave transitions found:
  (1,2,+2)  (1,4,+4)  (1,8,+6)  (2,4,+2)  (2,8,+4)
  (3,6,+2)  (3,12,+4) (4,8,+2)  (5,10,+2) (6,12,+2)
count: 10       all octave counts even: True
same-odd-part pairs, counted independently: 10       [identical set]
```

The two counts agree because they are the same set. That agreement is the theorem, checked rather than asserted.

The distinction is therefore not imposed on the psychology. It is read off the index. A move that quadruples the frequency and leaves the class alone is arithmetically a different animal from a move that shifts the class, and the arithmetic knows which is which before anyone has decided what the states mean.

7.9 The orbit structure of the twelve-state ladder

Group the twelve states by odd part. Each group is an **orbit** under doubling — the set of ladder positions that sound the same pitch class at different registers.

```
class  1  ->  states { 1, 2, 4, 8 }      four degrees
class  3  ->  states { 3, 6, 12 }        three degrees
class  5  ->  states { 5, 10 }           two degrees
class  7  ->  states { 7 }               one degree
class  9  ->  states { 9 }               one degree
class 11  ->  states { 11 }              one degree

six orbits, twelve states, mean orbit size 2.0
```

Read against Chapter 5's names, the orbits are the substance of this chapter:

Class 1 — {1, 2, 4, 8}. Undifferentiated activation, valenced activation, situated feeling, social feeling. Four rungs of the ladder, one pitch class, spanning six octaves of raw pitch (0.4834 Hz to 30.9392 Hz, exactly 7200 cents). All four fold to **247.514 Hz**.

Class 3 — {3, 6, 12}. Directed feeling, temporal feeling, committed feeling. Three rungs, one class, spanning four octaves (4.3508 Hz to 69.6133 Hz, exactly 4800 cents). All three fold to **278.453 Hz**.

Class 5 — {5, 10}. Attributed feeling and narrative feeling. Two rungs, two octaves apart, both folding to **386.741 Hz**. This collision is the one SPINE §3.1 exhibits in the carried case, and §7.20 works it.

Classes 7, 9, 11 — singletons. Reflexive feeling, normative feeling, reciprocal feeling. Each alone in its class, because 14, 18 and 22 all exceed `N = 12`.

Two structural facts follow directly, and both are computed rather than asserted:

```
states possessing a double INSIDE the ladder (2n <= 12):  1,2,3,4,5,6   (six)
 states possessing none     (2n >  12):                 7,8,9,10,11,12  (six)
```

Exactly half the ladder can be intensified without leaving it. The upper half cannot — not because the upper states are somehow final, but because `N = 12` truncates their orbits. That is an artefact of the index length and it is honest to name it as one. Chapter 8 will find the same truncation waiting when it asks what a durable version of state 11 would be.

And the proportions:

```
class-preserving unordered pairs : 10 / 66 = 15.1515 %
class-changing   unordered pairs : 56 / 66 = 84.8485 %
```

Transformation is the generic move; intensification is the rare and highly structured one. Of the ten class-preserving pairs, only six are single doublings — `(1,2) (2,4) (3,6) (4,8) (5,10) (6,12)` — the rest being compositions of those.

7.10 The same feeling at three degrees of differentiation

Take the class-3 orbit and walk it. **Constructed trajectory M1, labelled as constructed.**

```
  state  3  directed feeling    f =  4.3508 Hz  fold j=6  -> 278.453 Hz  class 3
  state  6  temporal feeling    f = 17.4033 Hz  fold j=4  -> 278.453 Hz  class 3
  state 12  committed feeling   f = 69.6133 Hz  fold j=2  -> 278.453 Hz  class 3

   step 3 ->  6 : +2400.0000 cents = +2 octaves exactly
   step 6 -> 12 : +2400.0000 cents = +2 octaves exactly
  total 3 -> 12: +4800.0000 cents = +4 octaves exactly, ratio f(12)/f(3) = 144/9 = 16 = 2^4
```

Chapter 5's ladder says what has been added at each step: state 3 has an intentional object; state 6 adds a projected duration; state 12 adds a bound future action. Three genuinely different specifications of a state — and **one pitch class throughout.**

The reading this licenses, stated carefully:

> **The pitch has moved four octaves and the class has not moved at all. On the index, the state has become far more differentiated. On the lattice, it is the same note played higher.**

That is what "intensification" means in this book, and it is a technical term with an arithmetic definition, not an intuition. It says: *the specification acquired components, and the components it acquired were of a kind that doubled the count.* It does **not** say the experience is unchanged; a feeling with a projected duration is not the same experience as one without. It says the two sit at the same position in the folded lattice, and therefore that everything the lattice can express about their relationship is expressed by an octave number.

The class-1 orbit gives the longest such walk available at `N = 12`. **Constructed trajectory M3, labelled as constructed.**

```
      state  1   undifferentiated  activation   f  =   0.4834  Hz   j=9   -
> 247.514 Hz  class 1
  state 2  valenced activation       f =  1.9337 Hz  j=7  -> 247.514 Hz  class 1
  state 4  situated feeling          f =  7.7348 Hz  j=5  -> 247.514 Hz  class 1
  state 8  social feeling           f = 30.9392 Hz  j=3  -> 247.514 Hz  class 1

   total span 1 -> 8 : 7200 cents = 6 octaves exactly
```

Bare arousal and a fully socially-modelled feeling — the widest differentiation gap the ladder offers within one orbit — **sound the identical pitch class.** Six octaves of raw pitch, zero change of class.

7.11 A change of class

Now the other kind. **Constructed trajectory M2, labelled as constructed.**

```
   state 3  directed feeling    f =  4.3508 Hz  -> 278.453 Hz  class 3
   state 5  attributed feeling  f = 12.0856 Hz  -> 386.741 Hz  class 5

   interval 3 -> 5 : 2400 * log2(5/3) = 1768.7174 cents = 1.473931 octaves
   frequency ratio : 25/9  -- not a power of two
```

One rung apart on the count. Class 3 to class 5. **1768.7174 cents is not a whole number of octaves and by Theorem 7.3 it never could be**, because 25/9 is odd over odd in lowest terms and no odd number other than 1 is a power of two.

Compare directly:

move	Δ index	cents	octaves	class before -> after	kind
3 -> 6	+3	2400.0000	+2 exactly	3 -> 3	intensification
6 -> 12	+6	2400.0000	+2 exactly	3 -> 3	intensification
3 -> 12	+9	4800.0000	+4 exactly	3 -> 3	intensification
3 -> 5	+2	1768.7174	1.473931	3 -> 5	transformation
5 -> 7	+2	1165.0244	0.970854	5 -> 7	transformation
4 -> 5	+1	772.6274	0.643856	1 -> 5	transformation
11 -> 12	+1	301.2741	0.251062	11 -> 3	transformation
9 -> 11	+2	694.8159	0.579013	9 -> 11	transformation
1 -> 8	+7	7200.0000	+6 exactly	1 -> 1	intensification

Notice what the table refuses to do. It does not order the moves by size of index change and call that the answer. **The +9 move from 3 to 12 is an intensification. The +1 move from 11 to 12 is a transformation.** Distance on the ladder is not what separates the two kinds; the odd part is. A reader who arrived expecting “bigger jump = more different” has to give that up, and giving it up is the point.

7.12 What the dichotomy does and does not claim

The distinction is load-bearing enough that its limits need stating before anyone builds on it.

It claims: that transitions on the twelve-state ladder partition exactly into whole-even-octave moves and everything else; that the partition is computable from the indices alone; that the first kind preserves pitch class and the second does not; and that consequently the lattice has a formal representation of “more of the same thing” that is distinct from its representation of “a different thing”.

It does not claim: that an intensification is smaller, easier, less consequential, or less distressing than a transformation. The class-1 orbit spans six octaves. Moving from bare arousal to a fully social feeling is not a minor event in anybody’s afternoon, and the formalism says nothing about how it feels to cross it.

It does not claim that the odd part means anything on its own. Class 7 is not “the reflexive class” in any sense beyond the fact that 7 is the only index in `1..12` with odd part 7. Change `N` and the singleton would acquire company at index 14.

It does not claim to be a taxonomy of emotion, a stage model, or a diagnostic scheme. Failure Mode F7 addresses this directly and the prohibition is absolute.

And it does not survive a bad index. The dichotomy is a theorem about integers. It becomes a statement about a person only through Chapter 5’s count, and it inherits every weakness of that count. A decomposition that is defensible only to its author yields a dichotomy that is defensible only to its author.

7.13 Why intensification is scarce, and what that costs

There is a structural point hiding in the 10/66 figure that is worth pulling out, because it constrains what Part III can promise.

For an index of length N, the number of same-class unordered pairs is

```
sum over odd m <= N of  C(s_m, 2)      where s_m = floor(log2(N/m)) + 1
```

For `N = 12` the orbit sizes are `4, 3, 2, 1, 1, 1`, giving `6 + 3 + 1 + 0 + 0 + 0 = 10`. The orbit sizes fall off like `log2(N/m)`, so **the supply of intensification moves grows only logarithmically while the total supply of moves grows quadratically.** Lengthen the ladder and transformation dominates ever more completely.

This is not an inconvenience to be engineered around. It is the reason the lattice is informative at all. Chapter 10 will make the matching argument from the other side — that the most consonant pairs in the essence lattice are precisely the pitch-class collisions, so **consonance and redundancy are the same phenomenon**, and a self composed entirely of one orbit would be a single note played four times. That chapter owns the argument. This one supplies its arithmetic.

The Specification

Two procedures, a set of reference tables, and a reporting format. Everything here is executable with a calculator and a definition of the states.

7.14 Procedure MOOD-1 — reading a single transient state

```
INPUT   a described state, and Chapter 5's twelve-state ladder
OUTPUT  ( n, f_raw, j, f_folded, pitch_class ), plus the warrant for n

step 1  DECOMPOSE.  List the distinguishable components required to specify
        the state, using Chapter 5's definition and Chapter 6's procedure.
         Record the list, not just its length. The list IS the warrant.

step 2  COUNT.  n := the number of components. Check 1 <= n <= 12.
        If the decomposition yields n > 12, the ladder does not cover the
         state; say so and stop. Do not compress to fit.

step 3  APPLY THE LAW.  f_raw = k * n^2,  k = 0.483425732988414 Hz.

step 4  FOLD.  Find the unique integer j with 220 <= f_raw * 2^j < 440.
        f_folded = f_raw * 2^j.   Publish BOTH the band and j.

step 5  CLASS.  Strip factors of two from n until odd. That odd number is
        the pitch class. It is also, by Theorem 7.1, the index whose raw
        frequency f_folded folds from.

step 6  BADGE.  Attach: essence frequency not physical resonance;
```

```
          not a diagnosis; frequency inherits the defensibility of the count.
```

Step 1 is the only step that can go wrong in an interesting way, and it is the only step this chapter does not own. Step 2's ceiling is a real constraint: **the ladder is twelve long and a thirteenth component has nowhere to go.** Compressing a thirteen-component description into twelve to make the table work is index fabrication, and Failure Mode F1 treats it as such.

7.15 Procedure MOOD-2 — reading a mood CHANGE

```
    INPUT   two readings from MOOD-1, at times t0 and t1: indices m and n
    OUTPUT  ( interval_cents, octaves, kind, class_before, class_after )

    step 1  INTERVAL.   interval_cents = 2400 * log2(n/m)
                        octaves        = 2 * log2(n/m)
          Sign is meaningful: positive = up the ladder = more differentiated.

    step 2  CLASSIFY.   odd_part(m) == odd_part(n)  ?
                         yes -> INTENSIFICATION.  octaves is an even integer.
                          Verify it: if it is not, one of the indices is wrong.
                             no  -> TRANSFORMATION.   octaves is irrational-
in-the-relevant-sense:
                                       it is never an integer (Theorem 7.3).

   step 3  REDUCE (optional).  For the class-to-class interval independent of
             register, fold both to the band and take
           1200 * log2( folded(n) / folded(m) ).  Table 7.17 tabulates all 36.

    step 4  REPORT.  index change, interval, kind, both classes, and the two
           component lists from step 1 of MOOD-1. Without the component lists
             the reading is unauditable.
```

Step 2's verification is a genuine error check and costs nothing. If the odd parts match, the octave count *must* come out an even integer. If it does not, the arithmetic is wrong or an index was mistyped. This is one of the few places in the book where the formalism catches its own mistakes.

7.16 Reference table — the full transition matrix

Interval in cents from row state `m` to column state `n`, `2400 * log2(n/m)`. Whole-even-octave entries — the intensifications — are the ones divisible by 2400.

```
       n=1     2     3     4     5     6     7     8     9    10    11    12
 m=1    0.0 2400.0 3803.9 4800.0 5572.6 6203.9 6737.7 7200.0 7607.8 7972.6 8302.6 8603.9
  2 -2400.0   0.0 1403.9 2400.0 3172.6 3803.9 4337.7 4800.0 5207.8 5572.6 5902.6 6203.9
  3 -3803.9-1403.9   0.0  996.1 1768.7 2400.0 2933.7 3396.1 3803.9 4168.7 4498.7 4800.0
  4 -4800.0-2400.0 -996.1   0.0  772.6 1403.9 1937.7 2400.0 2807.8 3172.6 3502.6 3803.9
  5 -5572.6-3172.6-1768.7 -772.6   0.0  631.3 1165.0 1627.4 2035.2 2400.0 2730.0 3031.3
  6 -6203.9-3803.9-2400.0-1403.9 -631.3   0.0  533.7  996.1 1403.9 1768.7 2098.7 2400.0
  7 -6737.7-4337.7-2933.7-1937.7-1165.0 -533.7   0.0  462.3  870.2 1235.0 1565.0 1866.3
```

```
  8 −7200.0−4800.0−3396.1−2400.0−1627.4 −996.1 −462.3   0.0  407.8  772.6 1102.6 1403.9
  9 −7607.8−5207.8−3803.9−2807.8−2035.2−1403.9 −870.2 −407.8   0.0  364.8  694.8  996.1
 10 −7972.6−5572.6−4168.7−3172.6−2400.0−1768.7−1235.0 −772.6 −364.8   0.0  330.0  631.3
    11  −8302.6−5902.6−4498.7−3502.6−2730.0−2098.7−1565.0−1102.6  −694.8  −
330.0    0.0  301.3
 12 −8603.9−6203.9−4800.0−3803.9−3031.3−2400.0−1866.3−1403.9 −996.1 −631.3 −
301.3    0.0
```

The matrix is antisymmetric, as it must be, since reversing a transition inverts the ratio. Reading down any row, the intervals compress: the same one-rung step is worth 2400 cents at the bottom of the ladder (1->2) and 301.3 cents at the top (11->12). **This is the square law's signature. Differentiation is cheap in pitch at low index and expensive in pitch at high index —** or, put the other way, the same increment of pitch buys progressively more components as you climb. Chapter 21 will price this properly when it treats transmutation as re-indexing.

7.17 Reference table — reduced class intervals

Interval in cents between folded pitch classes, register removed. Rows and columns are the six classes.

```
            1         3         5         7         9        11
 1     0.0000  203.9100  772.6274  737.6518  407.8200  −97.3641
 3  −203.9100    0.0000  568.7174  533.7418  203.9100 −301.2741
 5  −772.6274 −568.7174    0.0000  −34.9756 −364.8074 −869.9915
 7  −737.6518 −533.7418   34.9756    0.0000 −329.8318 −835.0159
 9  −407.8200 −203.9100  364.8074  329.8318    0.0000 −505.1841
11   97.3641  301.2741  869.9915  835.0159  505.1841    0.0000
```

Every entry lies in `(−1200, +1200)` because register has been folded out. The 5–7 entry is the 34.9756 cents of §7.6 and is the smallest magnitude in the table, as the regime analysis requires.

A note on reading this table that matters more than it looks. The reduced interval is a statement about the *folded lattice*, not about the states. Classes 5 and 7 are 34.9756 cents apart; states 5 and 7 are 1165.0244 cents apart. Both are true. They answer different questions, and the report format of §7.19 requires that whichever one is quoted, it is labelled.

7.18 Rendering

If a reading is to be heard rather than read:

```
    band        220–440 Hz, stated with every value, with its j
  tuning     none. The values are exact ratios of squares; do NOT temper them
             to twelve-tone equal temperament. 34.9756 cents is not 0 cents.
    duration    Gabor bound dt * df >= 1/(4 pi).  For the tightest N=12 pair
                (df = 7.734812 Hz) this gives dt >= 10.2882 ms.
             This is a bound on the SIGNAL. It is not a perceptual threshold,
                and practical rendering uses tones far longer.
    rate       CANON3 §5: the rate factor between binding pairs is exactly 8
                (49/32 against 9/8). Cited, not re-derived here.
    capacity    CANON3 §5: single-transposition audible capacity is N <= 25.
```

```
            N = 12 is comfortably inside it.
disclosure  ABSOLUTE. Nothing masked, nothing below threshold, nothing
          concealed from the listener. No subliminals of any kind, ever.
            A message hidden from its recipient cannot be consented to.
            Open-label works; disclosure costs nothing that matters.
```

The no-subliminals constraint is not a stylistic preference and is not negotiable at any point in this book. It is repeated here rather than cited because this is the first chapter that produces renderable audio material, and the first chapter to produce it is where the constraint has to be loudest.

7.19 Reporting format

A published mood reading carries, minimally:

```
STATE READING
  components        [the list, in full]
  n                 [count]
  f_raw             [Hz, 4 dp]     k = 0.483425732988414 Hz
  band / j          220–440 Hz / j
  f_folded          [Hz, 3 dp]
  pitch class       [odd part of n]
  badges            essence frequency, not physical resonance
                    not a diagnosis; no clinical claim
                    frequency inherits the defensibility of the count

CHANGE READING (add)
  m -> n            [indices]
  interval          [cents, 4 dp]  and  [octaves, 4 dp or "exactly +k"]
  kind              INTENSIFICATION | TRANSFORMATION
  class             [before] -> [after]
  basis             [both component lists]
```

The `basis` line is the one that gets dropped and the one that must not be. **A frequency without its component list is a number that cannot be checked, and an unfalsifiable number in a book about measurement is worse than an absent one.**

The Failure Modes

Ten ways to get this chapter wrong. Several of them are attractive.

F1 — Editing the ladder to fit the reading. The index is fixed by SPINE §1 and owned by Chapter 5. Renaming a state, splitting one, merging two, adding a thirteenth, or reordering the rungs changes every frequency, every class, every orbit, and the entire dichotomy of §7.8. *Detection:* any state name in a reading that is not verbatim from Chapter 5's ladder. *Remedy:* use the ladder, or write a different book. If a case genuinely needs more than twelve rungs, that is a finding about the case and it is reported as one — never fixed by compression.

F2 — Reading class identity as sameness of feeling. States 1 and 8 share class 1. They fold to the same 247.514 Hz. **They are not the same feeling and this chapter never says they are.** They occupy the same position in the folded lattice, which is a statement about a lattice. *Detection:* the words "is really just" appearing near a class collision. *Remedy:* quote the raw frequencies and the octave count alongside the class, always. Six octaves is not nothing.

F3 — Treating the folded pitch as a magnitude. 386.741 Hz (classes 5 and 10) is not "more" than 247.514 Hz (class 1) in any sense the formalism supports. Folding destroys magnitude by design; that is what it is for. The magnitude is `n`, and `n` is a count. *Detection:* folded values being summed, averaged, ranked, or subtracted as though they were quantities. *Remedy:* rank by `n` or by `f_raw`; never by `f_folded`.

F4 — Importing certainty the count does not have. The single most likely failure in practical use. `f = 39.1575 Hz` looks exact to six figures. It is exactly as exact as the claim that the state required nine distinguishable components and not eight or ten. **A contested decomposition yields a contested frequency, and printing more decimal places does not settle the contest.** *Detection:* readings published without their component lists. *Remedy:* §7.19's `basis` line, mandatory.

F5 — Mistaking the class ceiling for a claim about feelings. `ceil(N/2) = 6` says a twelve-long index generates six pitch classes. It does **not** say there are six moods, six emotion families, six anything psychological. It is a fact about odd numbers below 12. *Detection:* "the six fundamental" appearing in any sentence. *Remedy:* say "six pitch classes" and never drop the noun.

F6 — Physical-resonance drift. The most persistent failure mode in the whole series, because the numbers are in the audible band and the audible band is where physical things happen. Nothing resonates. Not a neuron, not a tissue, not a room, not a person. **The output is the resonance of the essence** — a square law applied to a count — and the number's presence in a hearing range is a consequence of the transposition in §7.2, which is a choice. *Detection:* any sentence combining a frequency from this chapter with a biological noun and a verb of oscillation. *Remedy:* the badge, in the sentence, every time.

F7 — Diagnosis drift. The twelve states are not diagnoses. A pitch class is not a type of person. An index is not a severity. A trajectory is not a prognosis. This chapter makes no clinical, diagnostic or therapeutic claim, offers no assessment, and supports no decision about anybody's care. *Detection:* a reading being used to sort people, or a class being spoken of as something a person *is*. *Remedy:* stop. The prohibition is standing and admits no exception.

F8 — Band-shopping. Choosing the rendering band to make two states collide, or to pull them apart. It does not work, and the fact that it does not work is a strength of the method. Folding is multiplication by `2^j`; pitch class is invariant under it (Theorem 7.1). **A class collision cannot be tuned away and a class distinction cannot be tuned in.** *Detection:* a band chosen after the states, rather than before. *Remedy:* fix the band at 220–440 Hz and publish `j`.

F9 — Cents as effect size. 1768.7174 cents from state 3 to state 5 is an interval on the essence lattice. It is not a measure of how much a person changed, how hard the change was, how long it took, or whether it was good. Those questions belong to other parameters, and one of them — persistence — is Chapter 8's entire subject. *Detection:* cents being compared across cases as though they were outcomes. *Remedy:* report cents with `kind` and both component lists, and let the reader see what produced them.

F10 — Re-deriving the regime, or quietly changing `N`. The 50/49 result belongs to `N = 12`.

Anyone who lengthens the index past 16 has left the regime — at `N = 17` the minimum separation collapses from 34.9756 to 6.0008 cents, a factor of 5.8 — and everything §7.18 says about rendering has to be recomputed. *Detection:* an `N` other than 12 anywhere in Parts II or III without an explicit note. *Remedy:* `N = 12` throughout this book; the regime statement lives in §7.6 and is cited, not repeated.

Worked Example

All cases below are CONSTRUCTED and labelled as such (SPINE §3). None is an observation of any person. No clinical or diagnostic claim is made or implied.

7.20 The Subject — the carried case, read for classes

SPINE §3.1 fixes the carried case: one situation held under three framings. Chapter 9 owns the chord and its width; Chapter 11 owns the full worksheet. **This chapter takes exactly one thing from the case — the pitch-class structure of the states it spans — because that is what this chapter owns.**

The three framings, as fixed:

```
threat = {3:0.8, 5:0.4, 7:0.2}
story  = {3:0.5, 5:0.7, 10:0.4}
mirror = {5:0.3, 7:0.8, 10:0.5}
```

The union of the supports is `{3, 5, 7, 10}` — four states held across the three framings. Applying MOOD-1 to each:

state	Ch 5 name	f = k.n² (Hz)	j	folded	pitch class
3	directed feeling	4.3508	6	278.453	3
5	attributed feeling	12.0856	5	386.741	5
7	reflexive feeling	23.6879	4	379.006	7
10	narrative feeling	48.3426	3	386.741	5

```
states spanned : 4
pitch classes  : {3, 5, 7}  --  THREE distinct
```

Four states are held and only three classes sound. States 5 and 10 share odd part 5. Their interval is `2400 * log2(10/5) = 2400.0000` cents — two octaves exactly — and they fold to the same 386.741 Hz. **One of the four states adds pitch but no new class.**

This is not an artefact of the case. It is Theorem 7.1 doing its work on a set that happens to contain a doubling pair, and by §7.9's proportions it is not even unlikely: any four states drawn from `1..12` have a fair chance of containing one of the six doubling pairs.

The reading it supports, and its limit:

> On the folded lattice, `attributed feeling` and `narrative feeling` are the same note two octaves apart. The Subject's framings differ in how *far* they carry that note,

not in whether they sound it. **Whether that identity is meaningful about the Subject depends entirely on whether Chapter 5's decomposition of "attributed" and "narrative" is right — and if it is contested, so is this.**

SPINE_NUMBERS records the same result from the other direction — the chord width as pitch, 36.6686 Hz, folded to 293.349 — and Chapter 9 works it there. This chapter's contribution is the count: four states, three classes, one collision, and the arithmetic reason for it.

7.21 Constructed trajectory M1 — the same feeling, three degrees

Case M1, constructed and labelled. A single mood tracked across a sequence in which the description acquires components. This is *not* a claim that moods develop in this order.

t	state	Ch 5 name	components added	f (Hz)	j	folded	class
t_0	3	directed feeling	arousal, valence, object	4.3508	6	278.453	3
t_1	6	temporal feeling	+ situation, cause, duration	17.4033	4	278.453	3
t_2	12	committed feeling	+ self, other, standard, story, reciprocity, bound action	69.6133	2	278.453	3

MOOD-2 applied to each step:

```
t0 -> t1   3 ->  6   interval = +2400.0000 cents = +2 octaves EXACTLY
                     class 3 -> 3      kind: INTENSIFICATION
                   check: odd parts equal, octaves even integer     PASS

t1 -> t2   6 -> 12   interval = +2400.0000 cents = +2 octaves EXACTLY
                     class 3 -> 3      kind: INTENSIFICATION
                   check: odd parts equal, octaves even integer     PASS

t0 -> t2   3 -> 12   interval = +4800.0000 cents = +4 octaves EXACTLY
                     class 3 -> 3      kind: INTENSIFICATION
                     ratio f(12)/f(3) = 144/9 = 16 = 2^4
```

Report: the pitch has moved four octaves; the folded value has not moved at all; the class has not moved at all. Nine components were added and the lattice registers every one of them as register, none of them as class.

That is the clean statement of what intensification is, and it is worth pausing on how unusual it is for a formalism to have such a thing. Most measurement schemes have one axis: more or less. This one has two, and the second axis — class — is *invariant* along an entire family of changes that the first axis registers as enormous.

7.22 Constructed trajectory M2 — a change of class

Case M2, constructed and labelled. The same starting state, a different second reading.

t	state	Ch 5 name	f (Hz)	j	folded	class
t_0	3	directed feeling	4.3508	6	278.453	3
t_1	5	attributed feeling	12.0856	5	386.741	5

```
3 -> 5   interval = +1768.7174 cents = 1.473931 octaves
      ratio 25/9 -- not a power of two, so NOT a whole octave (Theorem 7.3)
         class 3 -> 5        kind: TRANSFORMATION
         reduced class interval (register folded out): +568.7174 cents
```

Two components added — situation, then cause — against M1's nine over the same span of the ladder. **M2 is the smaller move on the count and the larger move on the lattice.** M1 moved four octaves and stayed in class 3. M2 moved one and a half octaves and left class 3 permanently.

Put the two readings next to each other:

	M1 (3->12)	M2 (3->5)
index change	+9	+2
interval (cents)	4800.0000	1768.7174
octaves	+4 exactly	1.473931
whole octaves?	yes	no, and never
class	3 -> 3	3 -> 5
folded	278.453 -> 278.453	278.453 -> 386.741
kind	INTENSIFICATION	TRANSFORMATION

The larger index change is the class-preserving one. The smaller index change is the transformative one. Any intuition that ranks mood changes by how far they move on the ladder gets this exactly backwards, and the arithmetic is not open to negotiation about it.

7.23 Constructed trajectory M3 — the six-octave orbit

Case M3, constructed and labelled. The longest single-class walk N = 12 permits.

state	Ch 5 name	f (Hz)	j	folded	class	cents from state 1
1	undifferentiated activation	0.4834	9	247.514	1	0.0
2	valenced activation	1.9337	7	247.514	1	2400.0
4	situated feeling	7.7348	5	247.514	1	4800.0
8	social feeling	30.9392	3	247.514	1	7200.0

state	Ch 5 name	f (Hz)	j	folded	class	cents from state 1

```
every step: +2400.0000 cents = +2 octaves exactly
total span: +7200.0000 cents = +6 octaves exactly
folded value: 247.514 Hz at every rung, without exception
kind of every transition in this orbit: INTENSIFICATION
```

Four rungs. Seven added components between the first and the last. Six octaves of raw pitch. **One folded value, unchanged to three decimal places, because it is the same value and not an approximation of one.**

And the honest counterweight, stated in the same breath: bare arousal and a socially-modelled feeling are not remotely the same experience, and this chapter has no instrument that says otherwise. What it says is that the *folded lattice* cannot tell them apart, and that this is a property of the lattice worth knowing — because Chapter 10 is going to argue that exactly this inability is where the redundancy in a self lives.

7.24 A complete worked reading, in report format

Combining everything, for one constructed transition.

```
STATE READING  --  t0                          [CONSTRUCTED CASE, labelled]
  components     arousal; valence; intentional object
  n              3
  f_raw          4.3508 Hz          k = 0.483425732988414 Hz
  band / j       220–440 Hz / j = 6      (4.3508 * 64 = 278.4532)
  f_folded       278.453 Hz
  pitch class    3      (odd part of 3)
  badges         essence frequency, NOT a physical resonance
                 NOT a diagnosis; no clinical claim
                 frequency inherits the defensibility of the count

STATE READING  --  t1                          [CONSTRUCTED CASE, labelled]
  components     arousal; valence; intentional object; situation; cause;
                 projected duration
  n              6
  f_raw          17.4033 Hz
  band / j       220–440 Hz / j = 4      (17.4033 * 16 = 278.4532)
  f_folded       278.453 Hz
  pitch class    3      (odd part of 6)
  badges         as above

CHANGE READING
  m -> n         3 -> 6
  interval       +2400.0000 cents  =  +2 octaves exactly
 verification   odd_part(3) = odd_part(6) = 3, octave count even      PASS
  kind           INTENSIFICATION
  class          3 -> 3   (unchanged)
```

```
folded          278.453 -> 278.453   (unchanged)
basis           both component lists above, in full
limit           the reading is exactly as defensible as the claim that
              the t1 state required six components and not five or seven
```

That is the complete instrument for a transient state. It has one moving part that is not arithmetic — the count — and the report format is built so that the moving part is visible to anyone who wants to argue with it.

7.25 What is left over, and what Chapter 8 takes

Two things have been deliberately left on the table.

The first is the chord. A person does not hold one state at a time, and the carried case in §7.20 made that obvious the moment four states appeared where the procedure expected one. **Chapter 9 owns simultaneity** — the set of held states as a chord, its intervals as ratios of squares, and the internal surplus `F_int` as the chord's width. It uses this chapter's classes and does not redefine them.

The second is time, and it is the larger omission. Everything in this chapter is a snapshot or a pair of snapshots. Nothing in `f = k n^2` says how long a state lasts, how fast it fades, or whether it comes back. **The formalism has been entirely silent about persistence, and that silence is not an oversight — it is a fact about the law.**

There is no `delta` in `f = k n^2`. There is no time in it at all. Which means the law that assigns a frequency to a mood assigns exactly the same frequency to a standing disposition at the same index, with no term available to tell them apart.

Chapter 8 takes that up, and finds that the frequency does not distinguish a mood from a trait. Only the decay does.

Chapter 7 owns octave folding and the odd-part class rule (SPINE §4). Chapters 9, 16 and 26 cite this chapter for both, and for the 50/49 regime statement in §7.6. The twelve-state ladder is Chapter 5's and is used here unaltered.

Chapter 8 . The Frequency of a Trait

Chapter 7 ended on an omission it could not repair from inside itself. The essence law assigns a frequency to a state, and the assignment is complete, exact, and entirely silent about time. There is no duration in `f = k n^2`. There is no rate of fade, no return interval, no persistence term, no `t` of any kind. The law tells you what a state sounds like and refuses to tell you how long it lasts.

This is usually where a formalism quietly acquires a second parameter. A frequency for moods and a different frequency for traits; a modifier; a coefficient of stability folded into `n`; something. The temptation is strong because the intuition behind it is strong: a passing irritation and a lifelong disposition toward irritability *feel* like different kinds of thing, and one expects a serious formalism to say so.

It does not say so. This chapter's central result is a negative one, and it is worth stating before any of the machinery that supports it:

> **A trait and a mood at the same index have the same frequency. Not a similar one — the same one, to every decimal place, in the raw band and in the folded band, at the same pitch class. The essence law cannot distinguish them and no honest reading of it pretends otherwise. What separates them is not the frequency law at all. It is the storage law, and specifically it is `delta`.**

That is a real result and a slightly surprising one, and the correct response to it is not to patch the frequency law. It is to notice that the series already contains the parameter that does the job, that it has been sitting in Paper E since Book Two, and that it belongs to a different equation entirely.

A trait is a state with low retention loss. That definition is this chapter's to own (SPINE §4), and Chapters 22 and 28 cite it. What follows is the definition made precise, the arithmetic of persistence worked out in full, and an accounting of exactly which questions the frequency reading answers and which it cannot touch.

The Law

8.1 The frequency law, unchanged

Nothing about the essence law changes when the state being read is a standing disposition rather than a passing one. Restated in full because a chapter that claims two things sound identical had better show the law that makes them so:

POSTULATE (the essence law; Book Two Ch 29, CANON_SONIC §1).

```
f(n) = k * n^2

k = (9/8)^2 / phi^2 = (81/64)/(phi + 1) = 0.483425732988414 Hz
```

computed: `phi = 1.618033988749895`, `phi^2 = phi + 1 = 2.618033988749895`, `(9/8)^2 = 1.265625`, `k = 0.48342573298841435 Hz`.

The index n is Chapter 5's twelve-state differentiation ladder, fixed by SPINE §1:

```
n := the number of distinguishable components required to specify the state.
```

This chapter does not redefine a state, rename one, or add one. Octave folding into the standing 220–440 Hz band, and the odd-part class rule that governs it, belong to Chapter 7 and are cited here without re-derivation. So does the regime statement: `N = 12` sits below the `N = 17` cliff and the whole book renders at a minimum class separation of 34.9756 cents.

Now the trait spectrum. Twelve standing dispositions, one at each rung, each read by Chapter 7's Procedure MOOD-1 with no modification whatsoever:

n	state class (Ch 5)	$f = k.n^2$ (Hz)	j	folded (220–440)	pitch class
1	undifferentiated activation	0.4834	9	247.514	1
2	valenced activation	1.9337	7	247.514	1
3	directed feeling	4.3508	6	278.453	3
4	situated feeling	7.7348	5	247.514	1
5	attributed feeling	12.0856	5	386.741	5
6	temporal feeling	17.4033	4	278.453	3
7	reflexive feeling	23.6879	4	379.006	7
8	social feeling	30.9392	3	247.514	1
9	normative feeling	39.1575	3	313.260	9
10	narrative feeling	48.3426	3	386.741	5
11	reciprocal feeling	58.4945	2	233.978	11
12	committed feeling	69.6133	2	278.453	3

This table is byte-for-byte Chapter 7's mood spectrum. Every column agrees: the same raw frequencies, the same transposition exponents, the same folded values, the same six pitch classes `{1, 3, 5, 7, 9, 11}`. Nothing was changed to make it agree; there was no mechanism available to change it.

THEOREM 8.1 (frequency does not distinguish disposition from episode). The essence law is a function of n alone. Persistence is not an argument of it. Therefore two states of equal index have equal essence frequency, equal folded pitch, and equal pitch class, irrespective of how long either lasts. []

The proof is one line because the content is one line: **you cannot read persistence off a pitch, because the pitch was never told about it.**

8.2 The storage law, and where `delta` actually lives

The parameter that does distinguish them comes from the other half of the book — the half Chapter 3 joined to the first. Paper E's contracting loop, restated in full:

```
Q      =  (1 − delta) ∗ Q  +  ( eta ∗ F(u∗) − c∗ )
 t+1                     t

Q     =  ( eta ∗ F(u∗) − c∗ ) / delta                Paper E, Theorem 3.5
 inf
```

with the standard readings:

```
eta    conversion  −− how much of an interaction's surplus becomes stored order
 F(u∗)    surplus     −− the interaction surplus at the operating point
 c∗      cost       −− the per−cycle cost of maintaining the arrangement
delta   LOSS      −− the per−cycle fraction of stored order that leaks away
 Q_inf    the fixed point −− where the loop arrives if the parameters hold
                      CANON3 §4 calls this DESTINY, and means it literally
```

`delta` is the retention loss. It is a per-cycle rate in `(0, 1]`. It is the only parameter in the system that speaks about time, and it is the only one absent from the essence law.

DEFINITION 8.2 (a trait). A **trait** is a state held at low `delta`. A **mood** is the same state held at high `delta`. The index — and therefore the frequency, the folded pitch, and the pitch class — is common to both. The distinction is entirely and exclusively a claim about retention loss.

Section 8.6 makes "low" precise as a declared convention rather than pretending it is a natural boundary.

8.3 The half-life of stored order

Set the input to zero and let the loop run. From `Q_{t+1} = (1−delta) Q_t`:

```
Q_t = (1 − delta)^t ∗ Q_0
```

Solve for the `t` at which half remains:

```
(1 − delta)^t = 1/2
t ∗ ln(1 − delta) = −ln 2

t_half  =  ln 2 / ( −ln(1 − delta) )              THE STORAGE HALF−LIFE
```

Computed across the range of `delta` this book uses, with three companion quantities: the time to lose ninety per cent, the time to reach ninety-five per cent of the fixed point, and the naive linear estimate `1/delta` that people reach for instead.

delta	-ln(1-δ)	t_half (cycles)	t to 90% lost	t to 95% of Q_inf	1/δ
0.005	0.005013	138.2826	459.3648	597.6473	200.0000
0.010	0.010050	68.9676	229.1053	298.0729	100.0000
0.020	0.020203	34.3096	113.9741	148.2837	50.0000
0.030	0.030459	22.7566	75.5957	98.3523	33.3333
0.050	0.051293	13.5134	44.8906	58.4040	20.0000
0.070	0.072571	9.5513	31.7289	41.2802	14.2857
0.090	0.094311	**7.3496**	24.4149	31.7645	11.1111
0.100	0.105361	6.5788	21.8543	28.4332	10.0000
0.150	0.162519	4.2650	14.1681	18.4331	6.6667
0.200	0.223144	3.1063	10.3189	13.4251	5.0000
0.250	0.287682	2.4094	8.0039	10.4133	4.0000
0.300	0.356675	1.9434	6.4557	8.3991	3.3333
0.400	0.510826	1.3569	4.5076	5.8645	2.5000
0.500	0.693147	1.0000	3.3219	4.3219	2.0000
0.700	1.203973	0.5757	1.9125	2.4882	1.4286
0.900	2.302585	0.3010	1.0000	1.3010	1.1111

`delta = 0.09` is bolded because SPINE §3 uses it in both carried cases, and its half-life — **7.3496 cycles** — is the number to carry around when reading those.

Two features of the table earn their keep.

The `1/delta` column is not `t_half` and the gap is large. At `delta = 0.30` the naive estimate is 3.3333 and the truth is 1.9434, an error of seventy-two per cent. Only as `delta` gets small do they converge on the ratio `ln 2 = 0.693147`. At `delta = 0.001`, `ln2/delta = 693.1472` against the exact `692.8005` — agreement to four figures. **`t_half` approximates `ln 2 / delta` for small delta and nothing at all for large delta**, and Failure Mode F6 is about people who use the linear form anyway.

Halving `delta` does not exactly double the half-life, but it nearly does, and it does so from above. Computed:

```
t_half(0.150)/t_half(0.300) = 4.2650 / 1.9434  = 2.1946
t_half(0.075)/t_half(0.150) = 8.8909 / 4.2650  = 2.0846
t_half(0.0375)/t_half(0.075)= 18.1351 / 8.8909 = 2.0397
```

converging on 2 as `delta` falls. Retention is very close to a doubling business in the low-delta regime where traits live, which makes the intuition safe exactly where traits are and unsafe exactly where moods are.

8.4 Retention, tabulated

`(1 − delta)^t`, the fraction of stored order surviving `t` cycles with no input:

delta	t=1	t=2	t=3	t=5	t=10	t=20	t=50	t=100
0.01	0.990000	0.980100	0.970299	0.950990	0.904382	0.817907	0.605006	0.366032
0.03	0.970000	0.940900	0.912673	0.858734	0.737424	0.543794	0.218065	0.047553
0.05	0.950000	0.902500	0.857375	0.773781	0.598737	0.358486	0.076945	0.005921
0.09	0.910000	0.828100	0.753571	0.624032	0.389416	0.151645	0.008955	0.000080
0.15	0.850000	0.722500	0.614125	0.443705	0.196874	0.038760	0.000296	0.000000
0.30	0.700000	0.490000	0.343000	0.168070	0.028248	0.000798	0.000000	0.000000

At `delta = 0.30` nothing survives twenty cycles. At `delta = 0.01`, thirty-seven per cent survives a hundred. **Between those two rows lies the entire difference between a mood and a trait, and there is not a single frequency in the table.**

8.5 `delta` is a per-cycle rate, and the cycle must be declared

This is a specification detail that behaves like a law, so it is stated here rather than in the procedures.

`delta` has no meaning without a stated cycle. Rescaling is exact:

```
over r cycles:    delta_r = 1 − (1 − delta_1)^r
```

Computed:

δ per day	per week (r=7)	per month (r=30)	per year (r=365)
0.02	0.131874	0.454516	0.999373
0.05	0.301663	0.785361	1.000000
0.09	0.483239	0.940947	1.000000
0.30	0.917646	0.999977	1.000000

The half-life is invariant in *time*; only its units change. Verified:

```
delta/day = 0.02  ->  t_half = 34.3096 days = 4.9014 weeks
delta/week= 0.131874 -> t_half = 4.9014 weeks           [identical]

delta/day = 0.09  ->  t_half =  7.3496 days = 1.0499 weeks
delta/week= 0.483239 -> t_half = 1.0499 weeks           [identical]
```

A `delta` quoted without its cycle is not a number. “delta = 0.09” describes a disposition with a seven-cycle half-life, and whether that is a week or a decade depends on a unit nobody stated. Failure Mode F1 treats this as the most common practical error in the whole chapter.

8.6 The trait cut is a declared convention

DEFINITION 8.6 (the trait window). Fix an observation window `W`, in cycles, over which a description is meant to hold. A state qualifies as a **trait relative to `W`** when its half-life meets or exceeds `W`:

```
t_half >= W    <=>    delta <= 1 − 2^(−1/W)
```

Computed thresholds:

W (cycles)	δ must be <=
5	0.129449
10	0.066967
20	0.034064
30	0.022840
50	0.013767
100	0.006908
365	0.001897

This is a convention, declared, and it is not a discovery about persons. There is no `delta` at which something switches from being an episode to being a disposition; there is a continuous rate and a cut that somebody chose. What Definition 8.6 buys is not truth but auditability: two readings that declare `W` can be compared, and two that do not, cannot. Failure Mode F4 is about readers who forget the word "convention".

This book's working labels, used only for legibility and never as categories of person:

```
MOOD    delta ~ 0.30    t_half   1.9434 cycles
HABIT   delta ~ 0.09    t_half   7.3496 cycles
TRAIT   delta ~ 0.02    t_half  34.3096 cycles
CORE    delta ~ 0.005   t_half 138.2826 cycles
```

8.7 The division of labour — what frequency decides and what `delta` decides

Here is the structural result that makes the chapter's negative finding useful rather than merely awkward.

THEOREM 8.7 (sign independence). For any `delta` in `(0, 1]`,

```
sign( Q_inf )  =  sign( eta * F(u*) - c* )
```

and `delta` never changes it. Moreover `Q_inf` is exactly inversely proportional to `delta`:

```
Q_inf(delta_2) / Q_inf(delta_1) = delta_1 / delta_2    exactly, for any numerator
```

Proof. `Q_inf = (eta F - c)/delta` with `delta > 0`. Division by a positive number preserves sign, and the ratio of two such expressions with a common numerator is `delta_1/delta_2`. []

Trivial to prove, and it settles a lot. Its companions:

```
f_min  =  k * e^( 2c / eta )                    CONTAINS NO delta
octave deficit = 2.885*(c/eta) - log2(f/k)      CONTAINS NO delta
```

Verified on all three of SPINE §3's parameter sets:

```
eta=0.30, c=0.62  ->  f_min = 30.1588 Hz    (delta absent from the expression)
 eta=0.44, c=0.40   ->  f_min =  2.9782 Hz
 eta=0.34, c=0.825  ->  f_min = 61.9349 Hz
ceiling: c/eta <= ln 12 = 2.4849066  =>  f_min <= k * 144 = 69.6133 Hz  (CANON3 §1)
```

So the two halves of the instrument divide cleanly, and the division is exact:

```
THE FREQUENCY READING (Ch 4's octave deficit) DECIDES THE SIGN.
  Does the relation accumulate or decay?  f vs f_min.  delta is irrelevant.

delta DECIDES THE MAGNITUDE AND THE CLOCK.
  How far does it go, and how fast does it get there?  delta and nothing else.
```

A trait's frequency tells you which way it is going. Its `delta` tells you how far and how fast. Neither substitutes for the other, and a reading that reports one without the other is half a reading.

8.8 The honesty clauses, restated in place

- **These are ESSENCE frequencies, not physical resonances.** No person, tissue, cell or molecule is claimed to resonate at 39.1575 Hz, at 313.260 Hz, or at any other value here. A trait does not hum.
- **The twelve states are not diagnoses, and a trait reading is not a personality assessment.** No clinical, diagnostic or therapeutic claim is made anywhere in this chapter. A low-`delta` state is not a type, a disorder, a trait score, or a prediction about a person. This warning is sharper here than in Chapter 7, because "trait" is a word with a large and respectable psychometric literature attached to it, and this chapter is not contributing to that literature. It is defining a term inside a formalism.
- **k is a chosen constant, not a measured one**, and it cancels from every interval.
- **The square law is a postulate about essence structure.**
- **The frequency inherits the defensibility of the count** (SPINE §1). A contested decomposition yields a contested frequency, and a `delta` estimated from too few cycles yields a contested half-life on top of it. **Trait readings carry two independent sources of softness, not one.**
- **All cases below are CONSTRUCTED and labelled as such.**

The Engineering

8.9 Durability and revisability are the same number

The storage law's convergence identity, from Book Two (Chapter 22 owns the refinement treatment; this is a citation and a consequence):

```
| Q_t − Q_inf |  =  (1 − delta)^t ∗ | Q_0 − Q_inf |
```

Read it once for decay, with `Q_inf` below `Q_0`: the state fades toward the fixed point at rate `(1−delta)` per cycle. Read it again for approach, with `Q_inf` above `Q_0`: the state builds toward the fixed point at rate `(1−delta)` per cycle.

It is the same rate. There is one time constant, and it governs both directions.

delta	t_half of decay	t_half of approach	identical?
0.02	34.3096	34.3096	yes
0.05	13.5134	13.5134	yes

delta	t_half of decay	t_half of approach	identical?
0.09	7.3496	7.3496	yes
0.30	1.9434	1.9434	yes

COROLLARY 8.9 (no free durability). A state cannot be made durable without being made slow to revise, and cannot be made quick to revise without being made quick to lose. The two properties are the same parameter read in two directions.

This is worth sitting with, because it dissolves a wish that a lot of practical work is built around. The wish is for a disposition that persists when you want it and yields when you want it to — high retention on demand, low retention on demand. The storage law does not have a term for "on demand". It has `delta`, and `delta` is symmetric. Anything that lowers the leak also lowers the rate of change of the thing that is leaking.

Tabulated as the arrival problem: starting from `Q_0 = 0` with a fixed positive numerator `a = 0.5476136` (which is SPINE §3.2's post-intervention dyad, §8.16), how far the loop has come at each `delta`:

delta	Q_inf	t_half	t to 95%	Q at t=1	t=5	t=10	t=25	t=50	t=100
0.30	1.8254	1.9434	8.3991	0.5476	1.5186	1.7738	1.8251	1.8254	1.8254
0.09	6.0846	7.3496	31.7645	0.5476	2.2876	3.7152	5.5088	6.0301	6.0841
0.02	27.3807	34.3096	148.2837	0.5476	2.6307	5.0087	10.8574	17.4095	23.7495

Every row starts at the same place after one cycle. `Q_1 = a` in all three, because one cycle of input is one cycle of input regardless of the leak. The rows separate afterwards, and by `t = 100` the low-`delta` row holds thirteen times what the high-`delta` row holds — while still not having arrived, because it will not arrive for another fifty cycles.

A trait is not something that happens quickly. It is, by construction, the thing that takes longest to finish arriving.

8.10 The price of holding a level

The other side of the same coin, and the one that makes low `delta` attractive.

To *hold* stored order at a level `Q*` — steady state, no growth — the per-cycle input must exactly replace the leak:

```
Q* = (1 − delta) Q* + a        =>       a = delta * Q*
```

Computed, the per-cycle input required to hold a level:

delta	Q*=1	Q*=5	Q*=10	Q*=27.38
0.005	0.0050	0.0250	0.0500	0.1369
0.020	0.0200	0.1000	0.2000	0.5476
0.050	0.0500	0.2500	0.5000	1.3690

delta	Q*=1	Q*=5	Q*=10	Q*=27.38
0.090	0.0900	0.4500	0.9000	2.4643
0.300	0.3000	1.5000	3.0000	8.2142

Maintenance cost is linear in `delta` and linear in the level held. Holding `Q* = 10` at trait loss (`delta = 0.02`) costs 0.2000 per cycle. Holding the same level at mood loss (`delta = 0.30`) costs 3.0000 — fifteen times as much, every cycle, forever.

Put Corollary 8.9 next to this table and the trade is fully priced:

> **Low `delta` buys cheap maintenance and expensive revision. High `delta` buys cheap revision and expensive maintenance. There is no setting that is cheap at both, because it is one setting.**

8.11 The durable liability

Now the case the whole thing exists to catch, and it is R15 in its trait form.

Theorem 8.7 says `delta` cannot flip a sign. It scales. So if the numerator is *negative* — if `eta F < c`, if the arrangement is below `f_min`, if the octave deficit is positive — then lowering `delta` scales a negative number.

SPINE §3.2's dyad as found (Case 2a), which is deliberately marginal and deliberately below threshold:

```
u = 0.068735    F = ln(1 + 11u) = 0.563085 nats    eta = 0.30    c = 0.62
numerator = eta*F − c = 0.30 * 0.563085 − 0.62 = −0.4510739     NEGATIVE
f_rel = k(1+11u)^2 = 1.4908 Hz      f_min = 30.1588 Hz
octave deficit = +4.3384        ->  DECAYS
```

Sweep `delta` against a fixed negative numerator:

delta	Q_inf	t_half	reading
0.30	-1.5036	1.9434	decays, shallow, fast
0.09	**-5.0119**	7.3496	decays — SPINE's as-found reading
0.05	**-9.0215**	13.5134	decays deeper, slower
0.02	-22.5537	34.3096	decays far deeper, far slower
0.01	-45.1074	68.9676	decays deepest, slowest

The two bolded values are exactly SPINE's, and the ratio between them is exactly what Theorem 8.7 requires:

```
Q_inf(0.05) / Q_inf(0.09) = −9.0215 / −5.0119 = 1.800000...  = 0.09/0.05
```

No approximation. `delta` is a pure scale.

The reading, and it is the hardest sentence in Part II to hear correctly:

> **A trait is not a good thing. A trait is a persistent thing. Low `delta` deepens whichever fixed point is already there, and if the numerator is negative, low `delta`**

> **is the mechanism by which a problem becomes a permanent feature rather than an episode.**

The intuitive repair for a struggling arrangement is to add trust. Trust lowers `delta`. SPINE §3.2 prices it: on the as-found dyad, moving `delta` from 0.09 to 0.05 takes `Q_inf` from -5.0119 to -9.0215. **The intervention that everyone reaches for first makes the reading eighty per cent worse**, and it does so for the entirely mechanical reason that it multiplied a negative number by 1.8.

Chapter 15 owns the resulting rule and Chapter 18 applies it. Stated here in its trait register because this is the chapter where `delta` is defined:

> **THE ORDER OF OPERATIONS (R15). Fix the numerator, then lower the loss. Reverse the order and the same intervention accelerates the failure.**

8.12 A trait whose numerator is positive

The mirror case, so that the chapter is not read as an argument against durability.

SPINE §3.2's dyad after the intervention (Case 2b), where B has taken on two states A does not hold and the pair have adopted a decision log:

```
    u = 0.692400    eta = 0.44    c = 0.40
    F = ln(1 + 11u) = 2.153667 nats
    numerator = 0.44 * 2.153667 - 0.40 = +0.5476136        POSITIVE
     f_rel = 35.8907 Hz      f_min = 2.9782 Hz      deficit = -3.5911    -
>  ACCUMULATES
```

(SPINE_NUMBERS prints `f = 35.8882 Hz`, `Q_inf = +6.0844` at `delta = 0.09` and `+10.9520` at `delta = 0.05`. Recomputing from the printed `u = 0.692400` — itself a rounded value — gives 35.8907, +6.0846 and +10.9523: agreement to four significant figures, the residual being the rounding of u in the printed table. The result this chapter depends on is the ratio, and the ratio is exact at either precision.)

```
    Q_inf(0.05) / Q_inf(0.09) = 1.800000...  = 0.09/0.05     [exact]
    SPINE printed:  6.0844 * 1.8 = 10.9519  vs printed 10.9520
```

Same `delta` move. Same multiplier. Opposite meaning:

	numerator	δ = 0.09	δ = 0.05	effect of lowering δ
Case 2a (as found)	-0.4510739	-5.0119	-9.0215	**worse by 4.0096**
Case 2b (after)	+0.5476136	+6.0846	+10.9523	**better by 4.8677**

The identical intervention, on the identical parameter, in the identical direction, helps in one case and hurts in the other — and the frequency reading is what tells you which case you are in before you act. That is what the octave deficit is *for*, and it is why Chapter 4's diagnostic comes before Chapter 8's parameter in the book's order.

8.13 The `N = 12` truncation, revisited for traits

One structural consequence carries over from Chapter 7 §7.9 and deserves naming here.

Chapter 7 computed that exactly half the ladder possesses a double inside it:

```
states with 2n <= 12 : 1, 2, 3, 4, 5, 6
states with 2n >  12 : 7, 8, 9, 10, 11, 12
```

An intensification — a class-preserving move — requires the double to exist on the ladder. So for a standing disposition read at index 7, 9, 11 or above, **there is no more-differentiated version of the same pitch class available within `N = 12`.** Any move from there is a transformation, a change of class, by Chapter 7's Theorem 7.3.

This is an artefact of index length and is reported as one. It is not a claim that reflexive, normative or reciprocal dispositions cannot deepen. It is a claim that *this ladder, at this length, has no rung to represent the deepening as an octave.* Lengthening the index would supply one — and would also, past `N = 16`, drop the whole book out of the 50/49 regime (Chapter 7 §7.6), which is why this book does not lengthen it.

CANON3 §1's fourth lever — add genuinely distinct categories, raise `N` — has a cost, and this is part of it. Chapter 29 prices the lever properly.

8.14 What a trait reading actually contains

Assembling the pieces, a complete reading of a standing disposition has four numbers and two of them are soft:

```
n       the index       -- soft: inherits the defensibility of the count (Ch 5)
 f          the frequency      -- HARD given n: exact arithmetic, k n^2
 class      the pitch class    -- HARD given n: odd part (Ch 7, Thm 7.1)
delta    the retention loss -- soft: a rate, requiring repeated observation
                                  and a DECLARED cycle
 t_half     the half-life      -- HARD given delta: ln2 / -ln(1-delta)
```

The two soft numbers are the two that carry all the psychological content, and the two hard ones are pure arithmetic on top of them. That is the honest shape of the instrument, and it is better to say so than to let the six-figure frequencies imply a precision that lives nowhere in the procedure.

The Specification

8.15 Procedure TRAIT-1 — reading a standing disposition

```
INPUT   a described standing disposition; Chapter 5's twelve-state ladder;
          a declared cycle unit; a declared observation window W
 OUTPUT  ( n, f_raw, j, f_folded, class, delta, t_half, trait-relative-
to-W )

step 1  DECOMPOSE and COUNT.  Exactly Chapter 7 Procedure MOOD-1 steps 1-2.
          Nothing about the count changes because the state is standing.
```

```
            Record the component list; it is the warrant.

    step 2  FREQUENCY.  f_raw = k n^2, fold to 220–440 Hz, take the odd part.
           Exactly MOOD–1 steps 3–5. No modification for persistence exists.

    step 3  DECLARE THE CYCLE.  State the unit of t. Without it, delta is not
            a number (§8.5).

    step 4  ESTIMATE delta.  See TRAIT–2 below. Report the estimate WITH the
            number of cycles it was estimated over.

    step 5  HALF–LIFE.  t_half = ln2 / ( –ln(1 – delta) ).
           Do NOT use 1/delta. Do NOT use ln2/delta unless delta < 0.01 and
            the approximation is labelled.

     step 6  CLASSIFY AGAINST W.  trait iff t_half >= W, i.e. delta <= 1 –
2^(–1/W).
            State W. The cut is a convention (Definition 8.6).

   step 7  BADGE.  essence frequency, not physical resonance; not a diagnosis
            and not a personality assessment; frequency inherits the count's
            defensibility; delta inherits the observation's.
```

8.16 Procedure TRAIT-2 — estimating `delta`, honestly

This book does not supply a validated instrument for `delta` and says so plainly. Chapters 11, 13 and 24 give the elicitation procedures the book does have; Appendix C carries the worksheets. What follows is the shape of the estimate and the conditions under which it is worth anything.

```
    step 1  DECLARE THE CYCLE.  The unit of t. Everything downstream is in it.

    step 2  ESTABLISH A NO–INPUT INTERVAL.  delta is the leak with the input
           removed. An interval during which the arrangement is being actively
           maintained measures ( delta, eta, c ) jointly and delta not at all.

    step 3  OBSERVE OVER >= 3 * t_half_expected CYCLES.  Below that, the decay
            curve is indistinguishable from a line and delta is unidentified.
             At delta = 0.02 this means ~103 cycles. THIS IS THE BINDING
             CONSTRAINT AND IT IS EXPENSIVE. Say so in the report.

    step 4  FIT.  delta_hat = 1 – ( Q_t / Q_0 )^(1/t).

    step 5  REPORT WITH ITS SUPPORT.  delta_hat, the cycle unit, t, and the
            number of observations. A delta from a single episode is not an
           estimate of a rate; it is one point on a curve (Failure Mode F10).
```

Step 3 is the reason trait readings are hard and mood readings are easy, and it is a real asymmetry in the method rather than a gap in the exposition. The lower a `delta` is, the longer you must watch to measure it, and the definition of a trait is a low `delta`. Traits are, by construction, the states

whose defining parameter is the most expensive to observe.

8.17 Reference table — the trait spectrum at a uniform `delta`

For a disposition set held at a common trait-grade loss, `delta = 0.02`, half-life 34.3096 cycles:

n	f (Hz)	j	folded	class	t_half	trait rel. W=30?
1	0.4834	9	247.514	1	34.3096	yes
2	1.9337	7	247.514	1	34.3096	yes
3	4.3508	6	278.453	3	34.3096	yes
4	7.7348	5	247.514	1	34.3096	yes
5	12.0856	5	386.741	5	34.3096	yes
6	17.4033	4	278.453	3	34.3096	yes
7	23.6879	4	379.006	7	34.3096	yes
8	30.9392	3	247.514	1	34.3096	yes
9	39.1575	3	313.260	9	34.3096	yes
10	48.3426	3	386.741	5	34.3096	yes
11	58.4945	2	233.978	11	34.3096	yes
12	69.6133	2	278.453	3	34.3096	yes

The `t_half` column is constant because `t_half` does not depend on `n`, and the frequency columns are Chapter 7's because `f` does not depend on `delta`. The two laws are orthogonal and the table makes it visible.

8.18 Reporting format

```
TRAIT READING
  components        [the list, in full]
  n                 [count]
  f_raw             [Hz, 4 dp]      k = 0.483425732988414 Hz
  band / j          220–440 Hz / j
  f_folded          [Hz, 3 dp]
  pitch class       [odd part of n]
  -------- persistence, and it is a SEPARATE law --------
  cycle unit        [declared]
  delta             [rate per cycle]
 support          [cycles observed, observations, no-input interval y/n]
  t_half            ln2 / -ln(1-delta)   [cycles, 4 dp]
  window W          [declared]
  classification    TRAIT | not-a-trait, relative to W
  -------- if a fixed point is being reported --------
  eta, F, c         [with source]
  numerator         eta*F - c, and ITS SIGN
  Q_inf             (eta*F - c)/delta
 octave deficit   2.885*(c/eta) - log2(f/k)    [Ch 4; contains no delta]
  badges            essence frequency, NOT a physical resonance
```

```
NOT a diagnosis, NOT a personality assessment
frequency inherits the defensibility of the count
delta inherits the defensibility of the observation
```

The horizontal rules in the format are load-bearing. **The frequency block and the persistence block come from different equations and fail independently**, and a report that runs them together invites exactly the error Theorem 8.1 exists to prevent.

The Failure Modes

F1 — `delta` without a declared cycle. The most common practical error. "delta = 0.09" is not a number until the unit of `t` is fixed; the same figure describes a half-life of 7.3496 days or 7.3496 years. *Detection:* any `delta` in a report with no cycle unit beside it. *Remedy:* §8.5's rescaling identity, and the cycle unit in the report format. It is not optional.

F2 — Reading "trait" as "good". Definition 8.2 says durable and nothing else. Theorem 8.7 says `delta` scales the fixed point and cannot flip it. **A low-`delta` state with a negative numerator is a durable liability, and it is more durable precisely because `delta` is low** (§8.11). *Detection:* any sentence in which lowering `delta` is presented as an unconditional improvement. *Remedy:* read the sign of the numerator first, every time. That is R15.

F3 — Reading persistence off the frequency. Theorem 8.1 forbids it. There is no pitch, no class, no interval, and no folded value that distinguishes a mood from a trait, because the essence law has no persistence argument. *Detection:* "high-frequency states are more stable", or any variant. *Remedy:* the two-block report format of §8.18.

F4 — Treating the trait cut as a natural kind. `t_half >= W` is a convention with a declared `W`. Nothing switches at the boundary; a state with half-life 29.9 cycles at `W = 30` differs from one at 30.1 by nothing anybody could observe. *Detection:* trait/mood used as a binary without `W` stated. *Remedy:* report `t_half`, report `W`, and let the classification be a derived convenience.

F5 — Lowering `delta` as a universal remedy. The trust intervention. It multiplies; it does not add. On the as-found dyad it makes the reading eighty per cent worse (§8.11). *Detection:* a proposed intervention that acts on `delta` in an arrangement whose octave deficit is positive. *Remedy:* Chapter 4's diagnostic before Chapter 8's parameter. Chapters 15 and 18 own the rule.

F6 — Using `1/delta` or `ln2/delta` for the half-life. At `delta = 0.30` the linear estimate overstates the half-life by seventy-two per cent (3.3333 against 1.9434). The approximation `ln2/delta` is good to four figures only below about `delta = 0.001`. *Detection:* a half-life that is a clean reciprocal. *Remedy:* the exact form, `ln2 / -ln(1-delta)`, which costs one logarithm.

F7 — Personality typing. The sharpest prohibition in this chapter. **The twelve states are not diagnoses. A trait reading is not a personality assessment, a profile, a type, a score, or a prediction about anyone.** The word "trait" is borrowed here as a term inside a formalism and carries none of the psychometric apparatus that word has elsewhere. *Detection:* a reading used to sort, select, screen, place or characterise a person; a pitch class spoken of as something someone *is*. *Remedy:* stop. The prohibition is standing (SPINE §5) and admits no exception, no matter how carefully hedged.

F8 — Physical-resonance drift. Unchanged from Chapter 7's F6 and no weaker for standing states. Nothing resonates. A trait does not hum at 313.260 Hz; 313.260 Hz is a folded square of a count. *Detection:* a frequency from this chapter beside a biological noun and a verb of oscillation. *Remedy:* the badge, in the sentence.

F9 — Index drift under persistence. A subtle one specific to traits. A standing disposition is read once, its n recorded, and the index is then carried for the whole observation window as though it were fixed. But the count is a count of *components required to specify the state*, and that can change while the disposition persists. *Detection:* a long trait reading with a single index and a single component list. *Remedy:* re-run TRAIT-1 step 1 at intervals, and report an index change as an index change (Chapter 7's MOOD-2 classifies it) rather than absorbing it into `delta`.

F10 — `delta` from a single episode. One decay observation gives one point, and one point does not identify a rate. TRAIT-2 step 3's requirement — observation over at least three expected half-lives — is the binding constraint of the whole procedure and it is expensive. *Detection:* a `delta` reported without its support line. *Remedy:* report the support, or report that `delta` is unmeasured and give the reading as frequency-only. **A frequency-only reading is honest. A fabricated `delta` is not.**

Worked Example

All cases below are CONSTRUCTED and labelled as such (SPINE §3). None is an observation of any person. No clinical, diagnostic or therapeutic claim is made or implied, and nothing here is a personality assessment.

8.19 The identical pair — the chapter's result, on one page

Case T0, constructed and labelled. Two readings at the same index, one transient and one standing. The point is what is *not* different.

```
state index n = 9     Ch 5: normative feeling (+ standard applied)

f_raw     = k * 9^2 = 0.483425732988414 * 81 = 39.1575 Hz
band / j  = 220–440 Hz / j = 3        (39.1575 * 8 = 313.2599)
f_folded  = 313.260 Hz
odd part  = 9        pitch class 9

---- READING A : as a MOOD ----          ---- READING B : as a TRAIT ----
  n          9                               n          9
  f_raw      39.1575 Hz                      f_raw      39.1575 Hz
  f_folded   313.260 Hz                      f_folded   313.260 Hz
  class      9                               class      9
  delta      0.30                            delta      0.02
  t_half     1.9434 cycles                   t_half     34.3096 cycles
  retention at t=30   0.00002254           retention at t=30   0.54548432

columns that differ:  delta, t_half, retention
```

```
      columns that agree:     n, f_raw, j, f_folded, pitch class   --
ALL of them, exactly

    ratio of half-lives:  34.3096 / 1.9434 = 17.6548
```

Rendered as tone, Reading A and Reading B are the same tone. Not close, not approximately — identical to every decimal the arithmetic produces, at the same pitch class, folded by the same exponent. **Nothing in the sound carries the seventeen-fold difference in persistence, because the law that produced the sound was never given the persistence.**

That is the whole result, and it is worth having produced in this bare a form, because it is the kind of statement a formalism will otherwise be quietly assumed to make.

8.20 Case 2-T — the carried dyad's member A, read for persistence

Case 2-T, a constructed and explicitly-labelled extension of SPINE §3.2. Member A of the carried dyad holds three states with intensities `A = {4: 0.7, 6: 0.5, 9: 0.4}`. SPINE fixes the states and the intensities; **the `delta` values below are a constructed extension and are not in SPINE.** Cycle unit: declared as one week. Window `W = 20` weeks.

n	Ch 5 name	intensity	f (Hz)	j	folded	class	delta (per week)	t_half (weeks)	rel. W=20
4	situated feeling	0.70	7.7348	5	247.514	1	0.02	34.3096	**TRAIT**
6	temporal feeling	0.50	17.4033	4	278.453	3	0.30	1.9434	not a trait
9	normative feeling	0.40	39.1575	3	313.260	9	0.06	11.2023	not a trait

Read the columns in two groups, as §8.18 requires.

The frequency block. Three states, three distinct pitch classes `{1, 3, 9}` — no collision here, unlike the Subject's `{3,5,7}` in Chapter 7 §7.20, because none of 4, 6, 9 is a doubling of another. Interval 4->6 is `2400*log2(6/4) = 1403.9100` cents, a transformation (class 1 -> class 3). Interval 6->9 is `2400*log2(9/6) = 1403.9100` cents, also a transformation (class 3 -> class 9). The two intervals are equal because `6/4 = 9/6 = 3/2`; that is the square law's ratio structure and it is Chapter 7's, cited.

The persistence block. Three states, three very different clocks. The index-4 state has a half-life of 34.3096 weeks and qualifies as a trait at `W = 20`. The index-6 state has a half-life of 1.9434 weeks and does not come close. The index-9 state, at 11.2023 weeks, sits between and fails the declared cut — **and would pass at `W = 10`, which is exactly why `W` must be declared and exactly why Failure Mode F4 exists.**

Now put the blocks together and notice what the reading says that neither block says alone:

> A's most differentiated state (index 9, the highest pitch of the three at 39.1575 Hz) is *not* A's most persistent. A's most persistent state is the least differentiated of the three

(index 4, 7.7348 Hz). **Differentiation and persistence are independent axes and this constructed case has them running opposite.**

There is no law preventing that, because there is no law connecting them. Which is the point.

8.21 The `delta` intervention, priced in both directions

Cases 2a and 2b, from SPINE §3.2, constructed and labelled there. The dyad before and after one intervention, with `delta` moved from 0.09 to 0.05 in each.

The frequency readings first, since Theorem 8.7 says they decide the sign:

```
2a AS FOUND     u = 0.068735   eta = 0.30   c = 0.62
                F = 0.563085 nats
                f_rel  =  1.4908 Hz        f_min = 30.1588 Hz
                octave deficit = +4.3384   ->  SHORT. DECAYS.
                numerator = eta*F - c = -0.4510739     NEGATIVE

2b AFTER        u = 0.692400   eta = 0.44   c = 0.40
                F = 2.153667 nats
                f_rel  = 35.8907 Hz        f_min =  2.9782 Hz
                octave deficit = -3.5911   ->  ACCUMULATES.
                numerator = eta*F - c = +0.5476136     POSITIVE
```

Then the `delta` move, identical in both:

	numerator	Q_inf at δ=0.09	Q_inf at δ=0.05	change	ratio
2a as found	-0.4510739	-5.0119	-9.0215	-4.0096	1.800000
2b after	+0.5476136	+6.0846	+10.9523	+4.8677	1.800000

```
  both ratios are exactly 0.09/0.05 = 1.8, as Theorem 8.7 requires
  SPINE printed for 2a: -5.0119 -> -9.0215        [reproduced exactly]
SPINE printed for 2b: +6.0844 -> +10.9520     [reproduced to 4 s.f.; see §8.12]
check on the printed values:  -5.0119 * 1.8 = -9.0214 ;  6.0844 * 1.8 = 10.9519
```

One parameter, one direction, one multiplier — and two opposite verdicts. The multiplier is blind. It does not know whether it is deepening an accumulation or a decay, and nothing in `delta` can tell it.

What tells it is the frequency reading. `f_rel` against `f_min`; the octave deficit; the sign of `eta F - c`. **The instrument that decides whether to lower `delta` is the instrument from Chapter 4, and it contains no `delta` at all** (§8.7). The two halves of the book do genuinely different jobs, and this case is where the division becomes operational rather than merely formal.

8.22 A complete trait reading, in report format

Case 2-T-4, constructed and labelled. Member A's index-4 state, read in full.

```
TRAIT READING                                  [CONSTRUCTED CASE, labelled]
```

```
  components      arousal; valence; intentional object; situation
  n               4
  f_raw           7.7348 Hz        k = 0.483425732988414 Hz
  band / j        220–440 Hz / j = 5        (7.7348 * 32 = 247.5140)
  f_folded        247.514 Hz
  pitch class     1        (odd part of 4)
note          classes with states 1, 2 and 8 (Ch 7 §7.9, orbit of class 1)

---- persistence : a SEPARATE law ----
  cycle unit      one week          [declared]
  delta           0.02 per week
  support         CONSTRUCTED. In a real reading this line carries the
                cycles observed, the number of observations, and whether
                  a no-input interval was established (TRAIT-2 step 3).
                At delta = 0.02 that requires ~103 weeks of observation.
                  A reading without this line is frequency-only.
  t_half         ln2 / -ln(1-0.02) = 0.693147 / 0.020203 = 34.3096 weeks
  window W        20 weeks          [declared convention, Def 8.6]
  threshold       delta <= 1 - 2^(-1/20) = 0.034064
  classification  TRAIT relative to W = 20     (0.02 <= 0.034064)

---- maintenance ----
  to hold Q* = 10   per-cycle input required = delta * Q* = 0.2000 / week
  the same level at delta = 0.30 would cost 3.0000 / week   (15x)

---- symmetry ----
  t_half of decay    = 34.3096 weeks
t_half of approach = 34.3096 weeks    (Corollary 8.9 -- one time constant)
  consequence: this disposition is cheap to hold and slow to revise.
               Those are the same fact.

---- badges ----
  essence frequency, NOT a physical resonance
  NOT a diagnosis, NOT a personality assessment, no clinical claim
  frequency inherits the defensibility of the count (Ch 5)
  delta inherits the defensibility of the observation (TRAIT-2)
  constructed case, labelled
```

8.23 What has been established, and what Chapter 9 takes

Three things are settled and cited from here forward.

First, and it is the surprising one: the essence law does not distinguish a mood from a trait. Same index, same frequency, same folded pitch, same pitch class, same rendered tone. Theorem 8.1, demonstrated on one page in §8.19. **Only the decay distinguishes them** — `delta`, the half-life `ln2 / -ln(1-delta)`, and the retention curve `(1-delta)^t`.

Second, the division of labour is exact. The frequency reading decides the sign of the fixed point and contains no `delta`; `delta` decides the magnitude and the clock and cannot change

the sign. Theorem 8.7. That is why Chapter 4's diagnostic must run before any intervention on `delta`, and it is the arithmetic behind R15.

Third, durability is not a virtue and it is not free. A trait is a persistent thing, not a good thing; low `delta` deepens whatever fixed point is already present; and by Corollary 8.9 the same parameter that makes a disposition cheap to hold makes it slow to revise. **There is no setting that is cheap at both, because it is one setting.**

What has been assumed throughout, and is now the next omission to close: that a person holds one state at a time. Chapter 7 broke that assumption the moment its carried case produced four states where the procedure expected one. Chapter 8 has quietly kept it, reading each of member A's three states as though the other two were not there.

They are there, simultaneously, and their simultaneity has a structure. Chapter 9 owns the chord: the set of states a person holds at once, its intervals as ratios of squares, the internal surplus `F_int` as the chord's width — and the one-hot degeneracy trap that makes a naive chord meaningless. It uses this chapter's `delta` for nothing at all, and Chapter 7's pitch classes for everything.

Chapter 8 owns the definition of a trait as a low-`delta` state (SPINE §4); Chapters 22 and 28 cite it. The twelve-state ladder is Chapter 5's. Octave folding, the odd-part class rule and the 50/49 regime are Chapter 7's. `f_min`, the octave deficit and the four levers are Chapter 4's. The R15 order-of-operations rule is Chapter 15's and is used here in its trait register only.

Chapter 9 . Internal Intervals — the Self as a Chord

Chapter 7 gave a mood a frequency. Chapter 8 gave a trait one. Both chapters described a person as though a person were at any moment in one state, so that the reading was a single note and the only question was which note.

No one is ever in one state. A person walking into a difficult conversation is holding, at the same instant, a reading of the other party's intent, a story about how the situation arose, and a running commentary on their own performance inside it. These are not three moods in sequence. They are three *framings* held together, each with its own primitives, its own salient variables, and its own inference rules, and each of them assigns intensity to several of the twelve states at once.

The set of indices a person holds simultaneously is a **chord**. This chapter establishes what that chord is, how its intervals are computed, what its width means, and — at length, because it is the single most consequential error available in this part of the book — the exact way the calculation degenerates into meaninglessness if the framings are encoded carelessly.

The result the chapter is built to reach is this:

> **THE CHORD'S WIDTH IS THE INTERNAL SURPLUS.** The set of framings has a mean non-parallelism `u_int`; the internal surplus is `F_int = ln(1 + (N−1) u_int)`; and the essence bridge converts that surplus into a pitch, `f = k e^(2 F_int)`. The width of the chord and the amount of order the person has available are the same quantity in two units.

And the result the chapter is built to *protect*:

> **ONE-HOT FRAMINGS ARE AUTOMATICALLY ORTHOGONAL.** A subject represented as holding exactly one state per framing yields `u = 1` on every pair, for every subject, regardless of content — the maximum possible reading, produced by the encoding and not by the person. **Framings must be represented with overlapping support or the internal surplus is degenerate and measures nothing.**

Both are computed in full below, and the degenerate calculation is shown before the correct one, because a reader who has not seen the trap spring will not believe it is there.

All cases in this chapter are CONSTRUCTED and labelled as such. Nothing here is an observation of any person.

The Law

9.1 What a chord is in this system

Chapter 5 fixed the index. n is the number of distinguishable components required to specify the state — a count, with a true zero and equal units, and therefore ratio-scaled by construction. Chapter 6 built the twelve-state ladder on that count and defended it. Chapter 7 mapped it to pitch by the essence law:

```
f(n) = k * n^2          k = (9/8)^2 / phi^2 = 0.483425732988414 Hz
```

k is a chosen reference constant, not a measured one [POSTULATE]. Its form — a Pythagorean ditone divided by the golden section — is a design decision by the author, badged here as it must be badged wherever the law appears. **And the output is an essence frequency, not a physical resonance. No person, tissue, or molecule is claimed to resonate at any value in this chapter or anywhere in this book.**

The twelve-state ladder, recomputed:

```
      n    state class                        f = k n^2 (Hz)    folded 220–
440   j    pitch class
    1   undifferentiated activation       0.4834        247.514     +9       1
    2   valenced activation               1.9337        247.514     +7       1
    3   directed feeling                  4.3508        278.453     +6       3
    4   situated feeling                  7.7348        247.514     +5       1
    5   attributed feeling               12.0856        386.741     +5       5
    6   temporal feeling                 17.4033        278.453     +4       3
    7   reflexive feeling                23.6879        379.006     +4       7
    8   social feeling                   30.9392        247.514     +3       1
    9   normative feeling                39.1575        313.260     +3       9
   10   narrative feeling                48.3426        386.741     +3       5
   11   reciprocal feeling               58.4945        233.978     +2      11
   12   committed feeling                69.6133        278.453     +2       3

     Six distinct pitch classes {1,3,5,7,9,11} = ceil(12/2), per the odd–
part rule (Ch 7).
    ln N = 2.484907.  Max f = k*144 = 69.6133 Hz.
```

A **chord** is the set of indices carrying nonzero intensity across the framings a person holds at one moment. It is a set of indices, not a single index, and it inherits everything the square law says about relationships between indices.

9.2 The intervals are ratios of squares, and the constant cancels

The first structural fact about a chord follows immediately from the essence law and was proved in Book Two Chapter 29 as its Result One:

```
THE INTERVAL LAW

    f(m) / f(n)  =  (k m^2) / (k n^2)  =  m^2 / n^2
```

k cancels. Whatever one thinks of the constant — and it is a postulate, badged as one — it sets the absolute reference pitch and nothing else. Every relationship inside a chord is a ratio of squared integers, and is therefore independent of the constant entirely. A reader who rejects k must still accept every interval in this chapter, because no interval contains it.

This is the reason the chapter can be about *intervals* rather than about *pitches*. The pitches depend on a design decision. The intervals do not.

Consequences worth stating once, since they govern everything that follows:

```
index ratio    essence interval    what it is
    2:1             4/1           two octaves exactly
    3:2             9/4           an octave plus a whole tone
    4:3            16/9           a minor seventh
    5:4            25/16          a near minor sixth
    3:1             9/1           three octaves plus a whole tone
```

Because `(2n)^2 = 4 n^2` is exactly two octaves above `n^2`, doubling an index moves the pitch by two octaves and leaves the pitch class untouched. Two essences share a pitch class if and only if their indices have the same odd part. That is Result Two of the essence law, and Chapter 7 owns it; this chapter uses it and does not re-derive it.

9.3 The width of the chord is the internal surplus

Now the joining that makes this a chapter rather than a table.

Book Two Chapter 39 established the **internal surplus** by substitution of domain:

```
THE INTERNAL SURPLUS

    F_int = ln( 1 + (N − 1) u_int )                Paper A, Theorem 2.1

    u     = 1 − (x . y)^2 = sin^2 theta   in [0,1]
  N_eff = 1 + (N − 1) u_int                    the audited framing count
    F_int = ln N_eff
```

where x and y are two **framings held by one observer**, encoded as unit states over the twelve categories, and `u_int` is the mean non-parallelism across the observer's own framing pairs.

This is a derivation, not an analogy, and the claim is checkable. Paper A's axioms S1–S4 quantify over states in an inner-product space. S1: `F` depends on the pair only through `u`. S2: `f(0) = 0`. S3: `g(u) = e^{f(u)}` is affine in `u` — the design axiom, chosen, not derived. S4: `f(1) = ln N`. Not one of those four axioms contains a term referring to persons, agents, groups, or the ownership of states. They refer to states and to the geometry between them. The theorem that follows is therefore silent on who holds the states, and applying it inside one observer changes the encoder and nothing else.

Book One's Node 03, HORIZON, put the same statement in the older register: *the many windows of the one tower*. The tower does not see more because it has more glass; it sees more because the windows face different ways. That sentence is `F_int = ln(1 + (N−1)u)` with u doing the work of "face different ways." Book Two showed it is the same statement and not a resemblance; this chapter is what you get when you then take the pitch of it.

Two postulates travel with the derivation and are restated here rather than assumed:

> **THE FRAMING ENCODING [POSTULATE].** A framing — a way of construing a situation, with its own primitives, its own salient variables, and its own inference rules — admits representation as a unit vector over the twelve states, such that two framings that make the same predictions on the same inputs map to parallel states and two framings whose predictions are uncorrelated map to orthogonal ones.
>
> **THE INTERNAL ROSTER REDUCTION [POSTULATE].** For an observer holding some nominal number of framings, `N` is the number of state categories the system spans — here twelve — and `u_int` is a representative non-parallelism across the observer's framing pairs, normally the mean pairwise u over all pairs. `N_eff = 1 + (N−1) u_int` is then read as the audited framing count.

Now apply the essence bridge (Book Two Ch 30, restated in Chapter 2 of this book):

```
THE CHORD WIDTH

  f_width  =  k ∗ e^( 2 F_int )  =  k ∗ N_eff^2  =  k ∗ ( 1 + (N−1) u_int )^2

   octaves above the ground essence  =  2 F_int / ln 2  =  2.885 ∗ F_int
```

The chord's width, expressed as a pitch, IS the internal surplus expressed as a pitch. They are one number in two units. A person whose framings are parallel has `u_int = 0`, `N_eff = 1`, `F_int = 0`, and a chord width of exactly `k` — the ground essence, no width at all, a single note. A person whose framings are orthogonal has `u_int = 1`, `N_eff = 12`, `F_int = ln 12 = 2.484907`, and a chord width of `k ∗ 144 = 69.6133 Hz`, which is 7.17 octaves above the ground.

One nat of internal surplus is exactly 2.885 octaves. The internal surplus *is* a pitch interval.

A precision note, since the book will use the rounded constant repeatedly: `2 / ln 2 = 2.885390`. The book writes `2.885` throughout and that rounding costs about a thousandth of an octave at the ceiling — `2.885 ∗ ln 12 = 7.1690` against the exact `7.1699`. Where four decimal places matter, use `2/ln 2`; where the reading is a decision between "sustains" and "decays," the rounded form is sufficient and the book says so here once.

9.4 THE ONE-HOT TRAP — the degeneracy that makes the whole construction worthless

Everything above is correct and everything above can be made to produce a meaningless number by a single encoding decision that looks, at first sight, like the *cleanest* possible one.

Suppose one represents a framing by the state it most characteristically involves. Threat-reading is about directed feeling, so encode it as state 3. Story-telling is about attributed feeling, so encode it as state 5. Self-monitoring is about reflexive feeling, so encode it as state 7. Each framing becomes a **one-hot vector**: a single 1 in one position, zeros elsewhere.

This is tidy. It is also fatal.

> **THE ORTHOGONALITY DEGENERACY [THEOREM, and it is one line].** Let x and y be unit vectors with **disjoint support** — that is, no index carries nonzero intensity in both. Then `x . y = 0`, so `u = 1 − 0^2 = 1`, and `F = ln(1 + (N−1)) = ln N`, the

maximum. **Distinct one-hot vectors always have disjoint support. Therefore any set of distinct one-hot framings scores the maximum internal surplus, for every subject, regardless of which states they name.**

Proof. `x . y = sum_i x_i y_i`. Disjoint support means every term of the sum has a zero factor. The sum is zero. Substituting into `u = 1 − (x.y)^2` gives `u = 1`, and into `F = ln(1+(N−1)u)` gives `F = ln N`. Nothing in either step consulted the identity of the states. []

Computed, so the reader sees it rather than takes it:

```
ONE-HOT FRAMINGS -- FOUR DIFFERENT SUBJECTS, IDENTICAL READING

framings at states {3, 5, 7}    u = 1.000000 on all three pairs
                    mean u = 1.000000  F = 2.484907  N_eff = 12.0000  f = 69.6133 Hz
framings at states {1, 2, 3}    u = 1.000000 on all three pairs
                    mean u = 1.000000  F = 2.484907  N_eff = 12.0000  f = 69.6133 Hz
framings at states {4, 9, 12}   u = 1.000000 on all three pairs
                    mean u = 1.000000  F = 2.484907  N_eff = 12.0000  f = 69.6133 Hz
framings at states {1, 11, 12}  u = 1.000000 on all three pairs
                    mean u = 1.000000  F = 2.484907  N_eff = 12.0000  f = 69.6133 Hz
```

Four subjects with nothing in common produce the same reading, and it is the maximum reading the system can produce. A subject whose three framings are “undifferentiated activation, valenced activation, directed feeling” — three descriptions that are, in content, a nested sequence in which each contains the last — scores exactly what a subject spanning states 4, 9 and 12 scores. The encoding has erased the content and reported its own shape.

Worse: the degenerate reading is not merely wrong, it is *flattering*. It says the subject holds the full twelve-fold vocabulary. An instrument whose failure mode is to award everyone full marks will be adopted enthusiastically and will never be questioned by the people it flatters. That is why this trap gets a section of the law and not a footnote in the failure modes.

And the trap is not really about one-hot vectors. One-hot is the extreme case of the real condition, which is *disjoint support*. Consider three framings each spanning three states, none shared:

```
DISJOINT SUPPORT WITHOUT ONE-HOT -- the same degeneracy, better disguised

A = {1:0.6, 2:0.5, 4:0.3}
B = {3:0.7, 6:0.4, 9:0.5}
C = {5:0.8, 7:0.2, 11:0.6}

u(A,B) = 1.000000   u(A,C) = 1.000000   u(B,C) = 1.000000
mean u = 1.000000   F = 2.484907   N_eff = 12.0000   f = 69.6133 Hz
```

Each framing now looks rich — three states, graded intensities, nothing one-hot about it — and the reading is still the maximum, still identical for every such subject, still produced entirely by the encoding. A reader who fixed the one-hot problem by “using more states per framing” and stopped there has not fixed anything.

9.5 The overlap requirement, stated as a validity condition

The repair is not cosmetic and it is not optional.

> **THE OVERLAP CONDITION [BINDING].** A set of framings may be used to compute an internal surplus only if the framings share support — that is, only if there exist states carrying nonzero intensity in more than one framing, and the shared states carry enough weight that the inner products are not artefacts of a stray small term. **Where the condition fails, the computation is reported as degenerate and no `F_int` is published.**

The condition has a plain-language reading, and it is the reading that makes it obviously right: two ways of construing *the same situation* must be about the same situation. If a person's threat-reading and their story-telling have literally no ingredient in common, they are not two framings of one thing. They are two things. The non-parallelism of two framings is only interesting because they are aimed at one target; measure the angle between vectors pointed at different targets and you have measured nothing but the fact that they point at different targets.

A framing set with disjoint support is a category error rendered as a number.

There is a second, quieter reason the condition matters, and it will drive Chapter 10. Overlap is what makes u *variable*. On disjoint supports u is pinned at 1 and carries no information. On overlapping supports u ranges over the interval and its value is a fact about the person. **An instrument that cannot vary cannot measure. Overlap is what restores the variance that measurement requires.**

9.6 States and pitch classes are different counts

The last piece of law in this chapter is a distinction that a chord makes unavoidable and a single note conceals.

A chord spans a set of *states*. It also occupies a set of *pitch classes*. These are not the same count, because the odd-part rule collapses indices that differ by a power of two:

```
THE CLASS COLLAPSE

    n = 2^a * m  with m odd     =>     pitch class depends only on m

 Over indices 1..12 the odd parts are  1,1,3,1,5,3,7,1,9,5,11,3
giving classes {1,3,5,7,9,11} -- six classes for twelve states, = ceil(12/2).
```

Adding a state to a chord always adds a pitch. It does not always add a pitch class. A chord containing state 5 that acquires state 10 gains a note two octaves higher and gains no new class: `10 = 2 * 5`, same odd part, same class. The chord gets taller. It does not get harmonically wider.

This matters because the two counts answer different questions. The state count is what the essence index measures — how many distinguishable components the person's framings collectively require. The class count is what the folded chord can express — how many distinct positions in the harmonic vocabulary the chord occupies. A chord of four states in three classes is a real object with a real property: one of its members is a doubling of another.

Whether that doubling is a redundancy or a legitimate distinction is the subject of Chapter 10, and this chapter does not settle it. **It only insists that the two counts be reported separately and never conflated.** Reporting "three classes" as though it were the roster size understates the chord. Reporting "four states" as though it were the harmonic vocabulary overstates it.

The Engineering

9.7 Building framings that overlap without faking it

The overlap condition creates an immediate practical risk in the other direction: if overlap is required, an operator under pressure to produce a valid reading will manufacture it. That is a real failure and it gets its own entry below. What follows is how overlap arises honestly.

Chapter 6 owns the elicitation procedure and this chapter cites it. In outline, and only in outline, because the details belong there: the subject is asked to describe the situation once per framing; each description is decomposed by asking, of each of the twelve state classes, whether the description requires that component to be specified; the required components are rated for intensity on a common scale; the resulting twelve-vector is normalised to unit length. The scale status of those intensities is settled by Chapter 6 and it is not the ordinal status a reader might expect. Chapter 6 shows that ordinal-preserving transformations of the intensities move u from 0.362115 to 0.525275 on the carried Subject's closest pair — nearly nine tenths of an octave of disagreement from transformations that preserve every ordinal fact in the data. The intensities must therefore be ratio-scaled up to a common free multiplier, and Chapter 6 defends that by building the elicitation as a magnitude-estimation procedure. What R1 demotes is `D/P`, and any frequency derived THROUGH `D/P` inherits the demotion; the state intensities are defended separately and on their own evidence. Say which of the two applies, in place, every time.

Overlap arises honestly because **the twelve states are components of specification, not topics.** A threat-reading of a conversation and a story about the same conversation both require a directed feeling — both are *about* something. They will both carry state 3. They may weight it very differently; that difference is the signal. What produces overlap is that the framings are framings of one situation, and one situation has components that any construal of it must specify.

Overlap becomes fake when the operator adds a state to a framing because the arithmetic needs it rather than because the description requires it. The guard is procedural and simple: **the state membership of each framing vector must be fixed and recorded before any inner product is computed.** An operator who has seen `u = 1.000000` and then revisits the vectors is no longer measuring.

9.8 The chord as a set of raw pitches, and the fold

Once the framings are built, the chord is read off as the union of their supports. Take the carried Subject of this book, whose full construction is given in the Worked Example below and whose states are `{3, 5, 7, 10}`:

```
 RAW CHORD                                  FOLDED TO 220–440 Hz
state  3   f =  4.3508 Hz              278.453 Hz   (j = +6)   class 3
state  5   f = 12.0856 Hz              386.741 Hz   (j = +5)   class 5
```

```
state  7   f = 23.6879 Hz                  379.006 Hz   (j = +4)   class 7
state 10   f = 48.3426 Hz                  386.741 Hz   (j = +3)   class 5
```

The band and the transposition `j` are stated for every value, as the house method requires. Octave transposition preserves pitch class exactly and is not a change to the reading; it is a change to the units in which the reading can be heard.

The fold makes the class collapse audible. **States 5 and 10 fold to the identical value, 386.741 Hz. In the folded chord they are one note. In the raw chord they are two notes exactly two octaves apart.** Which representation is correct depends entirely on what one is asking, and both are published because neither is a measurement.

9.9 The interval table

The intervals within a chord are computed directly from the index ratios, with k absent. For the carried Subject's chord `{3, 5, 7, 10}`:

```
INTERVALS OF THE CHORD {3, 5, 7, 10}

index pair   essence ratio    raw cents   folded cents
   3 : 5        25 / 9         1768.72       568.72
   3 : 7        49 / 9         2933.74       533.74
   3 : 10      100 / 9         4168.72       568.72
   5 : 7        49 / 25        1165.02      1165.02
   5 : 10        4 / 1         2400.00         0.00
   7 : 10      100 / 49        1234.98        34.98
```

Three features of this table are worth pausing on, and each will be used later.

First, `5:10` is exactly two octaves and folds to exactly 0.00 cents. It is the class collision, visible as an interval. The two states occupy one position in the folded vocabulary.

Second, `3:5` and `3:10` fold to the same 568.72 cents, for the same reason: folding is a quotient by octaves, and 10 is 5 doubled. The chord's relation to state 3 is the same relation twice.

Third, `7:10` folds to 34.98 cents — a separation of about a third of a semitone. That is the 50/49 interval, `1200 * log2(50/49) = 34.9756 cents`, and Chapter 7 established that N = 12 sits below the N = 17 minimum-gap cliff so that the whole book renders in the 50/49 regime. The carried Subject's own chord contains the tightest pair the system permits. That is not a coincidence arranged for effect; it is what happens when a chord contains both 7 and 10, and it is a fact about the arithmetic of the ladder rather than about the person.

9.10 What raises the width and what does not

An operator who has a chord and wants a wider one has a small number of real moves and a large number of moves that look real. The arithmetic separates them.

The width is a function of `u_int` alone. Given `N = 12`, `f_width = k (1 + 11 u_int)^2`, and `u_int` is the mean over pairs of `1 − cos^2`. So the only thing that raises the width is a reduction in the mean squared alignment between framings.

What that means in practice:

- **Adding a framing that is genuinely angled from the existing ones raises the mean.** It adds new pairs, and if the new pairs have higher u than the existing mean, the mean rises.
- **Adding a framing that restates an existing one lowers the mean.** It adds pairs with u near zero and drags the average down. Chapter 10 computes exactly how far.
- **Increasing the intensity of a state uniformly across a framing changes nothing.** Normalisation removes it. A framing at `{3:0.8, 5:0.4, 7:0.2}` and one at `{3:1.6, 5:0.8, 7:0.4}` are the *same unit vector*, `u = 0.000000` between them, `F = 0`, width k exactly. Intensity that scales the whole framing is not information. Only the shape survives normalisation.
- **Adding a state to an existing framing changes the shape and therefore can move u in either direction.** There is no general rule; it must be computed.

The third of these is worth stating as a law, because it is the exact answer to a question readers ask at this point:

> **SCALE INVARIANCE OF THE FRAMING VECTOR.** Multiplying every intensity in a framing by a positive constant leaves the unit vector unchanged and therefore leaves every u, every `F_int`, and the chord width unchanged. **Feeling the same thing more strongly does not widen the chord. Feeling a differently-shaped thing does.**

This is not a claim about how intense experiences matter; it is a statement about what this particular instrument is sensitive to. **The instrument reads composition, not amplitude.** Anything one wishes to say about amplitude must be said with a different instrument, and Chapter 8's treatment of `delta` is where the book puts persistence, which is the nearest available proxy.

9.11 Drop-one analysis, and an honest surprise

A natural audit of a chord is to remove each framing in turn and see what the width does. Applied to the carried Subject:

```
DROP-ONE AUDIT (Subject, three framings, F_int = 2.164390)

drop threat  -> remaining pair (mirror, story)  u = 0.809410  F = 2.292890  f = 47.4142 H
drop story   -> remaining pair (mirror, threat) u = 0.904762  F = 2.393557  f = 57.9892 H
drop mirror  -> remaining pair (story, threat)  u = 0.388360  F = 1.662402  f = 13.4361 H
```

Two of the three drops raise the width. Removing the threat framing takes F from 2.164390 to 2.292890. Removing the story framing takes it to 2.393557. Only removing the mirror framing lowers it, and it lowers it a great deal, to 1.662402.

This is not a paradox and it is not a bug, but it is a warning that must be published with any drop-one audit. **The mean pairwise u is an average, and removing a member removes its pairs. A framing that sits at a moderate angle to two framings that are at a wide angle to each other will drag the average down by existing.** The threat framing is angled 0.388360 from story and 0.904762 from mirror; its presence adds one high pair and one low pair, and the low one wins.

The deeper statement, inherited from Book Two Chapter 7 and Chapter 39 and binding here:

> **MEAN PAIRWISE u DOES NOT DETERMINE THE JOINT GEOMETRY.** The triangle inequality fails for geometric triples (Paper A §5), so a set of pairwise angles is consistent with more than one arrangement, and no scalar summary of pairwise values

recovers the arrangement. `N_eff` computed from a mean is an *audited count under the roster reduction postulate*, not a measured dimension.

What follows practically is that a drop-one audit tells you about the average, not about the person's dependence on a framing. The mirror framing is not "the important one" because dropping it costs the most; it is the one whose removal leaves behind the closest remaining pair. Those are different claims and the arithmetic supports only the second.

9.12 The minimum roster, and what a chord is not

Two framings are enough to compute a width. One is not: with a single framing there are no pairs, `u_int` is undefined, and the correct output is not `F_int = 0` but *no reading at all*. **A single framing has no internal geometry, and reporting zero surplus for it confuses "measured to be parallel" with "not measured."** The distinction is the same one that separates a null result from a missing one, and the specification below requires the operator to report which they have.

With two framings the reading is a single u and the mean is trivially that value. It is a valid reading and it is a thin one, because a single pairwise angle can be produced by many arrangements and there is no second pair to constrain it. Three framings is the smallest roster at which the drop-one audit of 9.11 says anything at all, and it is the roster the carried Subject uses for exactly that reason.

It is worth marking, before the specification, what a chord is *not*, because the object invites three misreadings and all three are easy to make.

A chord is not a personality. It is computed from framings of one situation, elicited on one occasion. Chapter 8 treats standing dispositions, and the difference between a trait's frequency and a mood's is a question about `delta` — about how long the reading persists — not a question about the chord. A chord read on Tuesday about a colleague's message is a Tuesday reading about a colleague's message. Whether it generalises is an empirical question that no single reading can answer, and the correct way to find out is to read the same subject on other situations and compare, which Chapter 11's worksheet provides for.

A chord is not a preference or a value. Nothing in the construction records what the subject wants, endorses, or would choose. u is a geometric quantity over an encoding of construals. Two framings can be wildly non-parallel and both be ones the subject disowns.

A chord is not evidence about the situation. The Subject's threat framing scores `u = 0.904762` against their mirror framing whether or not the colleague meant any harm. **The instrument measures the structure of the construals, and it is silent — completely, by construction — on whether any of them is correct.** A wide chord over a misread situation is a wide chord over a misread situation. This is worth saying because the alternative reading, in which internal breadth implies accuracy, is both attractive and unsupported by anything in the formalism.

9.13 The ceiling, restated at internal scope

`F_int <= ln N` always (Paper D, Cor 2.2), with equality only at orthogonality. At `N = 12`:

```
F_int      <=  ln 12   = 2.484907 nats
f_width    <=  k * 144 = 69.6133 Hz
octaves    <=  2.885 * 2.484907 = 7.1690   (exactly 7.1699 with 2/ln 2)
```

There is no arrangement of twelve categories that produces more than 7.17 octaves of internal width. A reading that claims more has an arithmetic error or has silently changed N.

And the ceiling is why the one-hot trap is so damaging rather than merely wrong. **The degenerate encoding does not produce a random number; it produces the ceiling.** It reports every subject as maximally differentiated, which is the one reading that cannot be improved upon and therefore the one reading that ends the inquiry.

The Specification

What follows is the complete procedure for computing a chord and its width. Chapter 11 embeds this specification in the full personal worksheet and adds the sustainability reading; **this chapter owns the chord and the degeneracy trap, and Chapter 11 and Chapter 16 cite it rather than restating it.**

SPEC-9.1 — Inputs

```
    REQUIRED
      1. A state ladder with N categories and a defensible ordinal index.
         This book uses N = 12 and the ladder of Chapter 6.
      2. Two or more FRAMINGS of ONE situation, elicited per Chapter 6.
      3. For each framing, an intensity on each of the twelve states
       (zero permitted; the elicitation determines which states are required).

    RECORDED WITH THE INPUTS
      4. The situation, stated once, in the subject's words.
      5. The date and the elicitation conditions.
           6. The scale status of the intensities, per Ch 6: ratio-
scaled up to a common
      free multiplier, to the extent the magnitude-estimation procedure delivers it.
         (R1's demotion applies to D/P and to anything derived through it.)
     7. The raw L2 norm of every vector, per Ch 6's MAGNITUDE REPORTING RULE:
         it is reported alongside the normalised vector and never discarded.
```

SPEC-9.2 — Procedure

```
    STEP 1  BUILD.  For each framing, write the twelve-vector of intensities.
              Fix and record the support (which states are nonzero) BEFORE step 3.

    STEP 2  NORMALISE.  x <- x / ||x||_2  for each framing.
                    Check: ||x|| = 1.000000 for every framing.

    STEP 3  OVERLAP GATE.  For every pair, compute the shared support.
                    If ANY pair has empty shared support, STOP.
                  Report DEGENERATE. Publish no F_int. Return to Chapter 6.
```

```
    STEP 4  PAIRWISE.  For every pair:   cos = x . y      u = 1 - cos^2
                    Publish all pairwise u. Do not publish only the mean.

     STEP 5  AGGREGATE.  u_int = mean of the pairwise u over all P = m(m-
1)/2 pairs,
                    where m is the number of framings.

    STEP 6  SURPLUS.  N_eff = 1 + (N-1) u_int        F_int = ln N_eff

    STEP 7  WIDTH.  f_width = k * N_eff^2 = k * e^(2 F_int)
                    octaves above ground = 2 F_int / ln 2

    STEP 8  CHORD.  States spanned = union of the supports.
                    Raw pitch of each = k n^2.
              Folded pitch = k n^2 * 2^j into the stated band; publish the band and j.
                 Pitch classes = the distinct odd parts of the spanned states.

   STEP 9 INTERVALS.  For each pair of spanned states (i, j), the essence ratio is
                    j^2 / i^2 in lowest terms; cents = 1200 log2(j^2/i^2);
                    folded cents = cents mod 1200.
```

SPEC-9.3 — Required outputs

A published chord reading contains all of the following, and a reading missing any of them is incomplete:

```
    (a) the framing vectors, unnormalised and normalised, with supports marked
    (b) every pairwise u, not the mean alone
    (c) u_int, N_eff, F_int, f_width, and the octave figure
    (d) the spanned STATE set and its size
    (e) the spanned PITCH CLASS set and its size, reported SEPARATELY from (d)
    (f) the interval table, raw and folded cents
    (g) the band and transposition j for every folded value
    (h) the overlap-gate result
    (i) the scale-status statement (R1)
    (j) the non-resonance statement
    (k) the non-diagnostic statement
```

Items (j) and (k) are not decoration. They are the two sentences that most reliably fall off a reading as it is copied from one document into another, and they are the two the book cannot afford to lose. They read, in the house form:

> *These are essence frequencies, not physical resonances. No claim is made that any person, tissue or molecule resonates at any value in this reading.*
>
> *The twelve states are not diagnoses and this reading is not an assessment. It describes the structure of a set of framings under a stated encoding. It says nothing about health, illness, function, or fitness, and nothing in it constitutes clinical advice.*

SPEC-9.4 — The gates, as pass/fail conditions

```
     GATE 1   UNIT NORM         every framing has ||x|| = 1 to within 1e-
9         FAIL -> recompute
   GATE 2  OVERLAP       every pair shares at least one state           FAIL -
> DEGENERATE
    GATE 3  WEIGHT           shared support carries >= 10% of each vector's
                       squared norm (a stated convention, not a law)    FAIL -
> flag WEAK
   GATE 4  DISTINCTNESS   no two framings have u < 0.01                 FAIL -
> flag RESTATEMENT
    GATE 5  CEILING        F_int <= ln N                                FAIL -
> arithmetic error
   GATE 6  PRE-REGISTRATION supports fixed before inner products computed   FAIL -
> reading void
   GATE 7  ROSTER       at least two framings, so at least one pair     FAIL -
> NO READING
```

Gate 3's threshold is a convention chosen by the author and badged as one. There is no derivation that puts it at 10% rather than 5% or 20%. It exists because a pair sharing one state at intensity 0.02 is arithmetically overlapping and substantively disjoint, and the gate should catch that. A different operator may choose a different threshold; what is not optional is choosing one, stating it, and applying it before seeing the result.

Gate 4 is the mirror of Gate 2 and catches the opposite failure: framings so nearly parallel that they are one framing described twice. It flags rather than fails, because a genuine near-restatement is a finding — Chapter 10 is about exactly that finding — and should be reported, not suppressed.

The Failure Modes

FM-9.1 — Disjoint support, of which one-hot is the extreme case

The failure. Framings encoded so that no state appears in more than one, producing `u = 1` on every pair, `F_int = ln N`, and the ceiling reading for every subject.

Why it happens. One-hot encoding is the default in most adjacent technical practice, it is easy to elicit ("which state best describes this framing?"), and it produces clean-looking vectors. The disguised version — several states per framing, still disjoint — happens when an operator partitions the twelve states among the framings as though assigning territory.

Detection. Gate 2. Also detectable by inspection: a reading of exactly `F_int = 2.484907` at `N = 12` should be treated as a defect until proven otherwise.

Repair. Return to elicitation. Ask, of each framing, which components of the situation it requires to be specified — not which component it is *about*. Framings of one situation share components.

What it is not. It is not repaired by adding states. It is repaired by adding *shared* states.

FM-9.2 — Manufactured overlap

The failure. Having learned Gate 2, the operator adds a small intensity on a shared state to each framing so the gate passes.

Why it happens. Because the gate is a gate, and gates create incentives to pass them.

Detection. Gate 3 catches the crude version — overlap carrying negligible weight. Gate 6 catches the honest-operator version, where the support was revised after seeing the arithmetic. Neither gate catches a determined bad actor, and the book says so plainly. **No procedural gate defends against an operator who wants a particular answer. What the gates do is stop an honest operator from producing a degenerate answer by accident, and make a dishonest one leave a trace.**

Repair. Pre-registration of supports, per Gate 6, and publication of the unnormalised vectors so a reader can see where the weight sits.

FM-9.3 — Reading the mean as the geometry

The failure. Treating `N_eff = 8.7093` as though it said the subject's framings span 8.71 dimensions, or treating a drop-one audit as a dependency ranking.

Why it happens. `N_eff` is called an effective count and counts feel like dimensions.

Detection. Any claim about *joint* structure derived from *pairwise* values.

Repair. State the roster reduction postulate wherever `N_eff` is published. `N_eff` is an audited count under a stated postulate, and because the triangle inequality fails for geometric triples, a set of pairwise angles does not determine an arrangement.

FM-9.4 — The contested decomposition

The failure. Two competent operators decompose the same framing differently, get different supports, and get different frequencies — and the discrepancy is reported as measurement noise.

Why it happens. Because deciding what counts as one component is a judgment, and Chapter 6 defends the index against exactly this objection rather than pretending it does not exist.

Detection. Independent double-coding of the same framings, which should be standard practice for any reading that will be published.

Repair. There is no repair that makes the judgment go away. **The honest limit, stated wherever the index is used: the frequency inherits exactly the defensibility of the count. A contested decomposition yields a contested frequency.** Publish both decompositions and both readings. Do not average them; averaging two decompositions produces a third that neither operator endorses.

FM-9.5 — The chord read as a diagnosis

The failure. A narrow chord is read as pathology, a wide one as health, and the reading is used to make a claim about a person's condition.

Why it happens. Because `F_int = 0` has a description in Book Two Chapter 39 that sounds clinical — a person with forty ways of describing their situation, all of them the same description in different words — and a sentence that sounds clinical will be used clinically.

Detection. Any sentence in a reading containing a condition name, a severity, a recommendation, or a comparison to a norm.

Repair. **The twelve states are not diagnoses. The reading is not an assessment. This book makes no clinical, diagnostic, or therapeutic claim of any kind, and a chord width is a property of an encoding of a set of framings, not a property of a person's health.** A narrow chord means the framings elicited on that day, under that decomposition, were close to parallel. It means that and nothing further. Chapter 10 will take this considerably further and considerably more carefully, because Chapter 10 is where the temptation is strongest.

FM-9.6 — Physical resonance drift

The failure. "The subject's chord is 36.67 Hz" becomes "the subject resonates at 36.67 Hz" becomes a recommendation to expose someone to a 36.67 Hz tone.

Why it happens. Frequencies are physical quantities in every other context a reader has met them in.

Detection. Any sentence in which the frequency does something.

Repair. The essence frequencies are essence frequencies. The value is the surplus in different units — `f = k e^(2F)` is a change of variable and nothing more. **Nothing in this book asserts that any physical system is driven at, responds to, or resonates with any of these values.**

FM-9.7 — Conflating states with pitch classes

The failure. Reporting the Subject as holding three states, or as occupying four pitch classes.

Why it happens. The fold makes 5 and 10 the same audible note, so a reader working from the folded chord counts three.

Detection. Cross-check: the class count is the number of distinct odd parts, and it can never exceed `ceil(N/2)` or the state count.

Repair. Report both, labelled, per SPEC-9.3 (d) and (e).

FM-9.8 — Amplitude mistaken for width

The failure. A subject reports everything more intensely; the operator expects a wider chord and does not get one, and adjusts something until they do.

Why it happens. Because intensity feels like it should matter and the instrument is blind to it by construction.

Detection. Compare unnormalised to normalised vectors. If the change is a uniform scaling, the reading cannot move.

Repair. Accept the invariance. It is a property of the construction, stated in 9.10, and the correct response is to say what the instrument does not see rather than to make it appear to see it.

FM-9.9 — Silent change of N

The failure. An operator adds categories to the ladder mid-analysis, or computes u on a twelve-state encoding and F with `N = 20`.

Why it happens. N appears in two places — the dimensionality of the encoding and the multiplier in `F = ln(1 + (N−1)u)` — and it is easy to change one.

Detection. Gate 5. A reading with `F_int > 2.484907` at `N = 12` has changed N somewhere.

Repair. State N once at the head of the reading and derive both uses from it. And note that raising N genuinely is one of the four levers of Chapter 4 — adding genuinely distinct categories raises the ceiling. What is forbidden is raising it silently or raising it with categories that are not distinct, which Book Two Chapter 11 treats at length as pseudo-diversity.

Worked Example

THE SUBJECT — three framings of one situation

CONSTRUCTED CASE. This is not an observation of any person. It is the carried case of this book, defined in SPINE §3.1, and every chapter of Part II that needs an example uses it and no other.

The situation, stated once: a person has received a message from a colleague that could be read several ways, and must decide how to respond.

Three framings are elicited by the magnitude-estimation procedure of Chapter 6. The intensities are ratio-scaled up to a common free multiplier, to the extent that procedure delivers it — Chapter 6 owns that defence and its honest limit. R1's demotion applies to D/P and to any frequency derived through it; nothing below is derived through D/P. The raw L2 norms are reported with the vectors, per Chapter 6's magnitude reporting rule.

```
    FRAMING VECTORS, UNNORMALISED

    threat  = {3: 0.8, 5: 0.4, 7: 0.2}
         "they are coming at me"  -- a directed feeling, with an attributed cause
              and a reflexive component

    story   = {3: 0.5, 5: 0.7, 10: 0.4}
                   "this is the latest instalment of an old pattern"   --
directed, heavily
              attributed, and placed in a narrative

    mirror  = {5: 0.3, 7: 0.8, 10: 0.5}
         "what does my reaction say about me"  -- heavily reflexive, attributed,
              and narrated
```

Step 1–2: normalise

```
||threat|| = 0.916515     ||story|| = 0.948683     ||mirror|| = 0.989949

threat  = {3: 0.872872, 5: 0.436436, 7: 0.218218}
story   = {3: 0.527046, 5: 0.737865, 10: 0.421637}
mirror  = {5: 0.303046, 7: 0.808122, 10: 0.505076}
```

Each has unit norm to within floating-point tolerance. Gate 1 passes.

Step 3: the overlap gate

```
shared support (threat, story)  = {3, 5}       PASS
shared support (threat, mirror) = {5, 7}       PASS
shared support (story,  mirror) = {5, 10}      PASS
```

Gate 2 passes. **Every pair shares two states.** No pair is disjoint, and the shared states carry substantial weight in both members of each pair, so Gate 3 passes as well.

Note that state 5 — attributed feeling, the component that says the situation has a cause — is carried by all three framings. That is what makes them three framings of one situation rather than three situations. It is the overlap condition satisfied not by construction but by content.

The degenerate calculation, shown first

Before the correct reading, here is what the same Subject produces under the one-hot encoding, so that the difference is visible rather than asserted.

Encode each framing by its heaviest state: threat is heaviest at 3, story at 5, mirror at 7.

```
ONE-HOT ENCODING OF THE SUBJECT

threat  = {3: 1}      story = {5: 1}      mirror = {7: 1}

cos(mirror, story)  = 0.000000    u = 1.000000
cos(mirror, threat) = 0.000000    u = 1.000000
cos(story,  threat) = 0.000000    u = 1.000000

u_int = 1.000000    N_eff = 12.0000    F_int = 2.484907 nats
f_width = k * 144 = 69.6133 Hz    (folded 278.453 Hz, j = +2)
octaves above ground = 7.1699
```

The Subject scores the ceiling. So does every other subject encoded this way. The reading is `F_int = ln 12` because three distinct one-hot vectors are mutually orthogonal, and that is a fact about one-hot vectors. Nothing in the number came from the Subject.

The damage compounds downstream. Chapter 4's octave-deficit diagnostic, applied to this degenerate width at an illustrative cost ratio of `c/eta = 1.5714`:

```
DEGENERATE READING, CARRIED THROUGH THE DIAGNOSTIC

present octaves = log2(69.6133 / 0.483425732988414) = 7.1699
```

```
required        = 2.885 * 1.5714 = 4.5336
octave deficit  = 4.5336 - 7.1699 = -2.6364    "SUSTAINS"
```

The subject is pronounced sustainable by 2.64 octaves, and so is everyone else. A diagnostic that returns the same verdict for every input is not a diagnostic. This is the whole reason the trap gets a section of the law: it does not fail loudly, it fails by succeeding uniformly.

Step 4: the correct pairwise calculation

With overlapping support, computed from the normalised vectors above:

```
PAIRWISE READING

pair           cos theta    u        F = ln(1+11u)  N_eff    f = k(1+11u)^2
mirror / story    0.436566  0.809410     2.292890    9.9035    47.4142 Hz
mirror / threat   0.308607  0.904762     2.393557   10.9524    57.9892 Hz
story  / threat   0.782074  0.388360     1.662402    5.2720    13.4361 Hz

folded (220-440):  47.4142 -> 379.314 Hz (j=+3)
                   57.9892 -> 231.957 Hz (j=+2)
                   13.4361 -> 429.956 Hz (j=+5)

octaves above ground: 6.6159, 6.9063, 4.7967
```

All three pairwise u values are different, and they are different because the framings are different. That sentence is trivial and it is the entire point: under the degenerate encoding it was false.

The structure the numbers report is legible. Threat and story are the closest pair at `u = 0.388360` — both construe the situation as something arriving from outside with a cause, and they share the two heaviest states of the threat framing. Mirror and threat are the widest pair at `u = 0.904762` — one looks outward at an agent, the other looks inward at the self, and their overlap is a single state at modest weight in one of them. Mirror and story sit between at `0.809410`.

Every pairwise value is published, per SPEC-9.3 (b). The mean alone would have concealed that the Subject's three framings are not evenly spread — that two of them are relatively close and the third stands apart.

Step 5–7: aggregate, surplus, width

```
u_int   = (0.809410 + 0.904762 + 0.388360) / 3 = 0.700844

N_eff   = 1 + 11 * 0.700844 = 8.7093
F_int   = ln 8.7093         = 2.164390 nats

f_width = k * 8.7093^2      = 36.6686 Hz
folded  = 293.349 Hz  (band 220-440, j = +3)

octaves above ground = 2 * 2.164390 / ln 2 = 6.2451
```

The chord's width is 36.6686 Hz, which is 6.2451 octaves above the ground essence, and it is the same statement as `F_int = 2.164390 nats` of internal surplus.

Against the ceiling:

```
F_int / ln 12       = 2.164390 / 2.484907 = 87.10%
N_eff / N           = 8.7093 / 12         = 72.58%
```

The Subject holds three framings and audits to an effective 8.71 of the twelve available categories. That is the roster reduction postulate speaking, and it should be read as what it is: a stated construction, not a measured dimension. The framings are three; `N_eff` is not a count of framings. `N_eff = 1 + (N−1)u` is the effective count of *categories*, and the fact that three framings can audit to 8.71 categories is exactly Paper A's content — that non-parallel states span more than they number.

Step 8: the chord

```
STATES SPANNED  =  {3, 5, 7, 10}    -- FOUR states

state  3  directed feeling    raw  4.3508 Hz   folded 278.453 Hz (j=+6)  class 3
state  5  attributed feeling   raw 12.0856 Hz   folded 386.741 Hz (j=+5)  class 5
state  7  reflexive feeling    raw 23.6879 Hz   folded 379.006 Hz (j=+4)  class 7
state 10  narrative feeling    raw 48.3426 Hz   folded 386.741 Hz (j=+3)  class 5

PITCH CLASSES  =  {3, 5, 7}         -- THREE classes
```

Four states, three pitch classes. States 5 and 10 have the same odd part — `10 = 2 * 5` — so they fold to the identical value, 386.741 Hz, and occupy one position in the six-position harmonic vocabulary that `N = 12` permits.

The fourth state adds a pitch and does not add a class. In the raw chord it is a distinct frequency two octaves above state 5. In the folded chord it is the same note. Both statements are true; they answer different questions, and SPEC-9.3 requires both to be published.

What it means substantively — and this is a statement about the encoding, not about the person: **the Subject's narrative framing and their attribution of cause require components that this ladder places at the same odd part.** Whether that constitutes redundancy in the Subject's construal or merely redundancy in the ladder's vocabulary is precisely the question Chapter 10 opens, and this chapter deliberately does not answer it.

Step 9: the intervals

```
INTERVAL TABLE, SUBJECT'S CHORD {3, 5, 7, 10}

pair    essence ratio   raw cents   folded cents   note
3:5        25 / 9        1768.72       568.72
3:7        49 / 9        2933.74       533.74
3:10     100 / 9       4168.72      568.72     same folded interval as 3:5
5:7        49 / 25       1165.02      1165.02      just under an octave
5:10     4 / 1     2400.00      0.00    CLASS COLLISION, exactly 2 octaves
7:10      100 / 49       1234.98        34.98      the 50/49 minimum gap
```

Every ratio in this table is a ratio of squared integers, and k appears nowhere. The intervals survive any dispute about the reference constant.

Two entries deserve their own line.

`5:10` at exactly 0.00 folded cents is the collision, visible. It is also, as Chapter 10 will establish from Book Two Chapter 33, the most *consonant* interval available in the essence lattice — harmonic distance `HD_ess = 2.0000`, the minimum over all sixty-six index pairs at `N = 12`, attained by exactly the six pairs `{1:2, 2:4, 3:6, 4:8, 5:10, 6:12}` and by nothing else. **The Subject's chord contains the most consonant interval the system has, and it is also the chord's only redundancy. Chapter 10 is about the fact that those are the same sentence.**

`7:10` at 34.98 folded cents is the tightest pair the system permits. `1200 log2(50/49) = 34.9756 cents`. Chapter 7 established that `N = 12` sits below the `N = 17` cliff and that the whole book therefore renders in the 50/49 regime; the Subject's chord happens to contain the pair that realises it. In a rendered version of this chord, states 7 and 10 would be about a third of a semitone apart, and the Gabor limit `dt * df >= 1/(4 pi)` governs whether that separation is resolvable in a given duration. That is a rendering constraint, treated in Chapter 26, and it does not affect the reading.

The complete reading

```
=========================================================================
 CHORD READING -- THE SUBJECT (CONSTRUCTED CASE, SPINE 3.1)
=========================================================================
ladder            N = 12, Chapter 6 ordinal on n; intensities ratio-scaled
              up to a free multiplier per Ch 6 (R1 governs D/P, not used here)
 situation             one ambiguous message from a colleague
 framings              3: threat, story, mirror

 GATES                 1 unit norm      PASS
                      2 overlap        PASS  (every pair shares 2 states)
                       3 weight         PASS
                      4 distinctness   PASS  (min pairwise u = 0.388360)
                       5 ceiling        PASS  (2.164390 <= 2.484907)
                       6 pre-registered PASS
                       7 roster         PASS  (3 framings, 3 pairs)

 PAIRWISE u            mirror/story  0.809410
                       mirror/threat 0.904762
                       story/threat  0.388360

 u_int                 0.700844
N_eff               8.7093       (of N = 12; 72.58% of the roster audited)
 F_int                 2.164390 nats (87.10% of the ln 12 ceiling)

CHORD WIDTH          36.6686 Hz    folded 293.349 Hz, band 220-440, j = +3
                       6.2451 octaves above the ground essence k
```

```
  STATES SPANNED        {3, 5, 7, 10}   -- four
  PITCH CLASSES         {3, 5, 7}       -- three
                      states 5 and 10 share class 5: the fourth state adds
                        pitch but no new class

  RAW CHORD             4.3508 / 12.0856 / 23.6879 / 48.3426 Hz
  FOLDED CHORD          278.453 / 386.741 / 379.006 / 386.741 Hz

  DEGENERATE CHECK      one-hot encoding of the same three framings gives
                  u = 1.000000 on all pairs, F_int = 2.484907, f = 69.6133 Hz
                        -- the ceiling, and identical for every subject.
                        The reading above is the non-degenerate one.

  These are essence frequencies, not physical resonances.
  The twelve states are not diagnoses and this is not an assessment.
================================================================================
```

What the reading says, and what it does not

It says: under the Chapter 6 decomposition, on this situation, on this occasion, the Subject's three framings are substantially non-parallel; the mean non-parallelism is 0.700844; the internal surplus is 2.164390 nats, which is 87% of the maximum this twelve-category ladder can express; the chord spans four states in three pitch classes; and one of the four states is a doubling of another.

It does not say that the Subject is well or unwell, that a wider chord would be better, that a narrower one would be worse, that the Subject's framings are accurate, that the colleague's message means anything in particular, or that any frequency in the reading corresponds to a physical oscillation anywhere.

And it does not say that the Subject is "8.71 people," or has 8.71 perspectives, or contains 8.71 of anything. `N_eff` is the effective count under the roster reduction postulate, computed from a mean of pairwise angles, and because the triangle inequality fails for geometric triples that mean does not determine the joint arrangement. It is an audited count with a stated audit, and the audit is the postulate.

The one line to carry forward

A person is a chord, its intervals are ratios of squares, and its width is the order they have available. Chapter 10 asks what happens when the chord is consonant — and finds, uncomfortably, that the most consonant chord the lattice permits is a single note played four times.

Chapter 9 owns the chord and the one-hot degeneracy trap. Chapters 11 and 16 cite this chapter's specification rather than restating it. The elicitation of intensities is owned by Chapter 6; octave folding and the odd-part class rule are owned by Chapter 7; the octave-deficit diagnostic is owned by Chapter 4.

Chapter 10 . Dissonance and the Divided Self

Chapter 9 established that a person holding several framings at once is a chord, that its intervals are ratios of squares, and that its width is the internal surplus `F_int`. It left one question deliberately open. The carried Subject's chord contains states 5 and 10, which fold to the identical pitch. Is that a redundancy in the Subject, or only a redundancy in the vocabulary the ladder makes available?

This chapter answers it, and the answer is uncomfortable enough that the chapter has to be built around a firewall rather than around a result.

Book Two Chapter 33 proved that the essence lattice **doubles** Tenney harmonic distance: every essence pair is exactly twice as harmonically remote as the corresponding integer-ratio pair. From that theorem a second result follows immediately, and Chapter 33 stated it:

> **CONSONANCE IS REDUNDANCY.** The pairs that sound most related are precisely the pairs that occupy the same pitch class — the pairs the lattice cannot distinguish. **In the essence lattice, harmonic agreement and informational redundancy are the same phenomenon.**

Carry that into the internal scope of Chapter 9 and the consequence is arithmetic and immediate:

```
A PERFECTLY CONSONANT SELF IS A PERFECTLY REDUNDANT ONE

    all framings parallel    =>   u_int = 0
                             =>   N_eff = 1 + 11 * 0 = 1
                             =>   F_int = ln 1 = 0 nats
                  =>  f_width = k * 1^2 = k = 0.4834 Hz, the ground essence
```

Zero surplus. No width. A single note played several times.

That sentence is true, it is derived rather than asserted, and it is also the sentence in this book most likely to be misused. So before the chapter proves anything, it fixes what it is and is not claiming, and it will fix it again in the middle and again at the end.

> **THE SEPARATION, STATED FIRST AND RESTATED THROUGHOUT.**
>
> **This chapter is about information content. It is not about pain.**

`F_int` measures how much order is available in a set of framings under a stated encoding. It does not measure distress, and it does not measure well-being. **Internal difference that generates surplus is NOT distress, and distress is not a source of surplus.** They are different quantities, they are not computed from each other, and the formalism contains no term that would connect them.

Nothing in this chapter recommends suffering, values suffering, or claims that suffering is informative. Nothing in it is clinical advice, and the twelve states are not diagnoses. A person in pain is not thereby better instrumented, and a person at ease is not thereby impoverished.

The chapter says exactly one thing about redundancy, and it is a statement about an encoding: **framings that make the same assignments carry the same information once.** That is all it says, and §10.6 shows formally why nothing further can be derived from it.

All cases in this chapter are CONSTRUCTED and labelled as such, and every extension of the carried Subject is labelled as an extension.

The Law

10.1 What "dissonance" means here, and the four things it does not mean

The word is doing technical work and it will be misheard unless the work is specified.

```
DISSONANCE, IN THIS BOOK

   A computed property of a PAIR OF INDICES.
 Given indices i and j, reduce j/i to p/q in lowest terms; the essence interval is
   p^2/q^2; its dissonance is a chosen distance measure on that ratio.

 It is a fact about arithmetic. It contains no term for a person, an experience,
   a preference, or a state of health.
```

Four things it is not, listed because each has been read into the word by someone reading a draft of this chapter:

It is not psychological conflict. Two framings can be maximally non-parallel and coexist without friction. Reading a colleague's message as a threat and also as an episode in a long story are not at war; they are two things a person is doing at once, and a person can do both comfortably.

It is not ambivalence. Ambivalence is a state about a decision. Dissonance here is a relation between indices, computed before any decision is in view.

It is not distress. This is the load-bearing one and it is the subject of §10.6.

It is not cognitive dissonance in the social-psychological sense. That is a different construct with its own literature, its own operationalisations, and no relation to the arithmetic here beyond a shared word. This book does not import it, does not cite it as support, and does not claim

its findings. Where a term collides with an established construct, the collision is named and the constructs are kept apart.

10.2 The two consonance measures, both chosen

Book Two Chapter 33 used two indices and this chapter uses the same two, with the same badge.

```
    TENNEY HARMONIC DISTANCE       HD(p/q) = log2(p * q)
     A defined measure of the complexity of a frequency ratio. Larger = more remote.

   EULER'S GRADUS SUAVITATIS    G(n) = 1 + sum over prime factors of e_i * (prime_i -
1)
     Euler, Tentamen novae theoriae musicae, 1739. G(p/q) = G(p*q). Larger = less smooth.
```

> **BOTH ARE CHOSEN MEASURES WITH A DEFINED ARITHMETIC, NOT MEASURED FACTS ABOUT HUMAN HEARING [POSTULATE].** Neither is an experiment. They are defined functions on rationals, adopted because they are explicit and checkable. Where they disagree, the book publishes both and does not adjudicate.

10.3 The doubling theorem, restated

> **THE DOUBLING THEOREM** (Book Two, Ch 33). *For any index pair reducing to p/q, the essence interval p^2/q^2 has exactly twice the Tenney harmonic distance of the ratio p/q, and Euler gradus 2G − 1.*

Proof. `HD(p^2/q^2) = log2(p^2 q^2) = 2 log2(pq) = 2 HD(p/q)`. For the gradus: squaring doubles every exponent in the prime factorisation of `pq`, so the sum `sum e_i (prime_i − 1)` doubles, and `G(p^2 q^2) = 1 + 2 sum e_i (prime_i − 1) = 2(1 + sum e_i (prime_i − 1)) − 1 = 2 G(pq) − 1`. □

Verified exhaustively over all sixty-six pairs `1 <= i < j <= 12`; both identities exact in every case.

Consonance is scarce in the essence lattice, and it is scarce by a factor of exactly two. The perfect fifth `3:2` — the most consonant interval after the octave in ordinary just intonation — becomes `9:4` here, an octave plus a whole tone, at double the harmonic distance. The major third `5:4` becomes `25:16`.

There is a consequence of the doubling that matters specifically at internal scope and that Book Two, working at relational scope, had no occasion to draw.

A chord built on the essence ladder cannot contain a mild interval. In ordinary just intonation a set of four pitches will typically contain several relations of low harmonic distance — fifths, thirds, fourths — and the ear has somewhere to rest. In the essence lattice every one of those relations has been pushed to twice its distance, so the only low-distance relations that survive are the ones that were already trivial: the octave-equivalent pairs, which are the collisions. The lattice offers exactly two kinds of interval — the redundant kind and the remote kind — and almost nothing in between.

Look at the ranking below and the shape is immediate: six pairs at `HD_ess = 2.0000`, then a gap to 3.1699, and then a long tail running to 14.0888 with fifty-six of the sixty-six pairs above 3.1699. The distribution is not a gradient. It is a small cluster of redundancies and a large spread

of remoteness. That is what “consonance is scarce by a factor of exactly two” looks like when the scarcity is enumerated rather than described.

For a chapter about the internal case, the reading is this: **there is no such thing as a chord of framings that is comfortably related without being partly redundant.** The middle ground the metaphor promises — framings that harmonise without repeating — is not a region of this lattice. It is not that it is hard to reach. It is not there. Whether that is a discovery about selves or an artefact of the square law is exactly the kind of question this book is obliged to leave open, and it leaves it open: **the doubling theorem is a theorem about ratios, and the square law under it is a postulate.**

10.4 The identity: the most consonant pairs are exactly the collisions

Rank all sixty-six index pairs at `N = 12` by essence harmonic distance. The head of the list:

HD_ess	G_ess	pair	essence ratio	raw cents	folded cents	same class?
2.0000	3	1:2	4/1	2400.00	0.0000	YES
2.0000	3	2:4	4/1	2400.00	0.0000	YES
2.0000	3	3:6	4/1	2400.00	0.0000	YES
2.0000	3	4:8	4/1	2400.00	0.0000	YES
2.0000	3	5:10	4/1	2400.00	0.0000	YES
2.0000	3	6:12	4/1	2400.00	0.0000	YES
3.1699	5	1:3	9/1	3803.91	203.9100	no
3.1699	5	2:6	9/1	3803.91	203.9100	no
3.1699	5	3:9	9/1	3803.91	203.9100	no
3.1699	5	4:12	9/1	3803.91	203.9100	no
4.0000	5	1:4	16/1	4800.00	0.0000	YES
4.0000	5	2:8	16/1	4800.00	0.0000	YES
4.0000	5	3:12	16/1	4800.00	0.0000	YES
4.6439	9	1:5	25/1	5572.63	772.6274	no
...						
14.0888	29	11:12	144/121	301.27	301.2741	no

Three exact statements follow, and the chapter is careful to give the precise ones rather than the convenient one.

> **(A) THE MINIMUM IS EXACTLY THE 2:1 COLLISIONS.** The minimum essence harmonic distance over all sixty-six nontrivial pairs is `HD_ess = 2.0000`, and it is attained by exactly six pairs: `{1:2, 2:4, 3:6, 4:8, 5:10, 6:12}`. Every one of them is a pitch-class collision, and no non-collision attains it. Verified exhaustively.
>
> **(B) ZERO FOLDED SEPARATION IS EXACTLY COLLISION.** Across all sixty-six pairs, the folded interval is exactly 0.0000 cents **if and only if** the two indices share an odd part. Verified exhaustively: true for all ten collision pairs, false for all fifty-six others. This is a biconditional, not a tendency.
>
> **(C) THE HONEST CAVEAT — the ordering is not total.** It is **not** true that every collision is more consonant than every non-collision. `1:3` sits at `HD_ess = 3.1699`, more consonant than the collisions `1:4` (4.0000), `2:8` (4.0000), `3:12` (4.0000) and `1:8` (6.0000). The collision set is dramatically more consonant on average — mean `HD_ess`

3.0000 over the ten collisions against 8.6617 over the fifty-six non-collisions — but the ranges overlap and the book says so.

Statement (C) is the kind of thing a book drops when it wants a clean slogan. It is kept because Chapter 33's own rule requires it: **where two defensible indices disagree, or where a claim is true at the extremum but not across the order, publish the precise version.** The slogan "consonance is redundancy" is exactly true at the minimum and true on average; it is not a total order, and a reader who builds on the total-order version will eventually be wrong about `1:4`.

One more exact fact from the same table, carried forward because the Subject will need it:

```
The smallest NONZERO folded separation in the whole N = 12 lattice is 34.9756 cents,
 = 1200 * log2(50/49), and it is attained by EXACTLY ONE pair: 7:10.
```

Chapter 7 established that `N = 12` sits below the `N = 17` minimum-gap cliff and that the book therefore renders in the 50/49 regime. The pair that realises the regime is unique, and it is `7:10`.

10.4b The extremal chords, computed

The identity in (A) and (B) has a consequence that can be exhibited rather than argued: among all four-state chords available at `N = 12` — there are `C(12,4) = 495` of them — there is a most consonant one and a most dissonant one, and they are what the identity predicts.

```
EXTREMAL FOUR-STATE CHORDS AT N = 12   (all 495 enumerated)

MOST CONSONANT   {1, 2, 4, 8}
    mean HD_ess = 3.3333   min = 2.0000   max = 6.0000
    collisions  = 6 of 6 pairs           pitch classes = 1

MOST DISSONANT   {7, 9, 10, 11}
    mean HD_ess = 12.7586  min = 11.9546  max = 13.5627
    collisions  = 0 of 6 pairs           pitch classes = 4

THE SUBJECT      {3, 5, 7, 10}
    mean HD_ess = 8.4882   min = 2.0000   max = 12.2586
    collisions  = 1 of 6 pairs           pitch classes = 3
```

The most consonant four-state chord in the entire system is a chord of one pitch class. States 1, 2, 4 and 8 all have odd part 1; every one of their six pairs is a collision; folded, all four arrive at 247.514 Hz — one note, four times. That is the identity made visible: maximal consonance is maximal redundancy, and here they are the same chord.

The other pole is equally instructive. `{7, 9, 10, 11}` has no collisions at all, four distinct classes, and a mean harmonic distance nearly four times the consonant extreme. It is also, and this is worth noticing, an entirely ordinary set of states: reflexive, normative, narrative, and reciprocal feeling. **Nothing about holding those four components is difficult, unusual, or distressing. The chord is maximally dissonant in the lattice's arithmetic and completely unremarkable as a description of a person attending to a social situation.** If ever a case were needed against reading harmonic dissonance as psychological difficulty, this is it, and it is enumerated rather than asserted.

Two further counts from the same enumeration, useful for calibration:

```
    of the 495 four-state chords at N = 12:
        159 have ZERO collisions                (32.1%)
        the maximum possible collision count in a four-state chord is 6,
        attained only by {1, 2, 4, 8}
    the single-class families are: {1,2,4,8}, {3,6,12}, {5,10}, and singletons
```

Only three pitch classes at `N = 12` contain more than one index — class 1 with four members, class 3 with three, class 5 with two. Classes 7, 9 and 11 are singletons, so any pair drawn from `{7, 9, 11}` and any state outside their own doubling is automatically collision-free. The redundancy structure of the whole lattice is that small and that explicit, and it is entirely a fact about which integers under twelve are powers of two times which odd numbers.

10.5 The consequence at internal scope

Now put (A) and (B) together with Chapter 9's construction.

Two framings that assign intensity in the same proportions across the same states are, after normalisation, the **same unit vector**. Their inner product is 1, their u is 0, and they contribute nothing to the mean. Extend to a whole roster:

```
    THE REDUNDANT SELF

        every pair parallel   =>   u_int = 0
                              =>   N_eff = 1
                              =>   F_int = ln 1 = 0
                              =>   f_width = k = 0.483426 Hz
                              =>   octaves above ground = 0.0000
```

A person with forty ways of describing their situation, all of them the same description in different words, has an internal surplus of exactly zero. Book Two Chapter 39 put it as a single window with forty panes. They will experience this as thoroughness, because forty descriptions feel like forty descriptions, and the arithmetic is the only thing in the room that knows otherwise.

And the harmonic version of the same statement: the chord such a person produces sits entirely inside one pitch class if the framings share support, or spans several pitches with zero width if they do not. The most consonant possible internal chord is the one with the least in it.

There is a branch of this that Book Two's R8 makes at relational scope, and it deserves its internal statement here because at internal scope it behaves differently.

> **R8 AT INTERNAL SCOPE, AND ITS LIMIT.** Because `u = 1 − cos^2 theta`, perfect opposition (`cos = −1`) gives `u = 0` exactly as perfect agreement does — the contrarian and the sycophant sound the identical note. Chapter 14 owns that result at relational scope. Internally, the opposition branch is UNREACHABLE under this book's encoding, because intensities are non-negative by construction, so `cos theta >= 0` for every pair of framing vectors and `u = 0` can arise only from parallelism. Verified: over 100,000 random non-negative unit pairs the minimum observed cosine was 0.260867, and the bound `cos >= 0` is exact, not empirical.

That is a real limitation of the internal encoding and it is stated as one. A framing that is the exact negation of another cannot be represented here, because the twelve states record which

components a construal requires, not which direction it points. A reader who wants an internal analogue of the contrarian must build a signed encoding, and this book does not have one.

10.6 THE FIREWALL — why nothing about distress follows, formally

This is the section the chapter exists for, and it makes a formal argument rather than a reassurance, because reassurances are ignored and arguments can be checked.

The claim to be blocked is: *low `F_int` indicates a problem, high `F_int` indicates health, and therefore internal difference — including painful internal difference — should be sought.*

Every step of that is unsupported, and the formalism can be made to demonstrate it.

First: `F_int` is a function of `u_int` alone, and `u_int` is a mean of pairwise angles. It contains no term for valence, for intensity, for duration, for the subject's evaluation of any framing, or for anything else that a word like "distress" would have to refer to. A quantity cannot report on a variable it does not contain.

Second, and stronger: `F_int` is invariant under relabelling of the states, so it does not even determine the chord. Take the carried Subject and relabel state 10 as state 9, leaving every intensity untouched. Inner products are unchanged, because relabelling is a permutation of coordinates:

```
THE RELABELLING TEST  (EXTENSION 3.1-e of the carried Subject, LABELLED)

SUBJECT, states {3,5,7,10}
    u = 0.388360, 0.904762, 0.809410     u_int = 0.700844
    F_int = 2.164390    N_eff = 8.7093    f = 36.6686 Hz
    pitch classes {3,5,7} -- THREE      collisions: (5,10)
  chord HD_ess: 7.8138 8.7846 9.8138 10.2586 2.0000 12.2586  mean 8.4882  min 2.0000

RELABELLED 10 -> 9, states {3,5,7,9}
    u = 0.388360, 0.904762, 0.809410     u_int = 0.700844
    F_int = 2.164390    N_eff = 8.7093    f = 36.6686 Hz
    pitch classes {3,5,7,9} -- FOUR     collisions: NONE
  chord HD_ess: 7.8138 8.7846 3.1699 10.2586 10.9837 11.9546  mean 8.8275  min 3.1699

difference in F_int: 0.000000000000
```

Two chords with identical internal surplus to twelve decimal places: one contains a pitch-class collision, the other contains none. A third relabelling, to states `{2,8,9,11}`, again gives `F_int = 2.164390` exactly, with classes `{1,9,11}` and the collision `(2,8)`.

> **THE FIBER ARGUMENT.** `F_int` is many-to-one on chords. Chords that differ in their state set, their pitch classes, their collision structure, and their entire harmonic content map to the same `F_int`. A quantity that does not determine the chord cannot determine anything the chord does not determine, and the chord determines nothing about a person's suffering. The inference from `F_int` to distress fails two levels above where anyone would look for it.

Third: the four quadrants are all occupiable, and the formalism forbids ruling any of them out.

- *High `F_int`, no distress.* A person who reads a situation three genuinely different ways and is perfectly comfortable doing so. Nothing in the construction prevents this. It is probably the common case.
- *High `F_int`, severe distress.* The same structure in someone having a terrible week. The arithmetic is identical.
- *Low `F_int`, no distress.* A person with a settled, coherent single reading of a situation, at ease. The formalism calls this zero surplus. It does not call it a problem.
- *Low `F_int`, severe distress.* A person locked into one construal they cannot escape.

All four are consistent with the formalism, because the formalism has nothing to say about the second coordinate. A measure that is compatible with every value of a variable carries no information about that variable. That is the firewall, and it is a theorem about the construction rather than a promise about its use.

Fourth, the ethical statement, which does not follow from the arithmetic and is made on its own authority: nothing in this book recommends that anyone acquire, retain, or cultivate suffering. **Suffering is not a resource, is not an input to any equation here, and appears nowhere in the formalism.** Where a reader finds a sentence in this book that seems to praise internal division, that sentence is about the *width of an encoding of construals* and about nothing else, and it should be read that way or discarded.

Fifth, the clinical statement, restated because it is a standing prohibition of this book: **the twelve states are not diagnoses. A chord reading is not an assessment. No claim, diagnostic, therapeutic, or prognostic, is made anywhere in this chapter, and nothing here is advice about any person's care.** A reader who wants to know whether something is wrong should ask someone qualified to answer that, and this instrument is not qualified to answer it and does not attempt to.

The Engineering

10.7 The consonance audit of a chord

Given a chord — a set of spanned states from Chapter 9's SPEC-9.2 — three quantities summarise its harmonic content, and all three are computed from the index set alone.

```
CONSONANCE AUDIT

    for every pair (i, j) in the chord, with j/i reduced to p/q:
        HD_ess(i,j)  = 2 * log2(p * q)
        G_ess(i,j)   = 2 * G(p*q) - 1
        folded cents = ( 1200 * log2(j^2/i^2) ) mod 1200

    MIN HD_ess       the chord's most consonant interval
    MEAN HD_ess      the chord's overall remoteness
    COLLISION COUNT  the number of pairs with folded cents = 0
                     equivalently, pairs sharing an odd part
```

The collision count is the redundancy count, by result (B), exactly. It is not a proxy and not an

estimate: folded separation is zero if and only if the odd parts agree, so counting collisions and counting exact pitch-class redundancies are the same operation.

10.8 What a collision costs and what it does not

Here is where the chapter must be careful in the second direction, because having established that consonance is redundancy it would be easy to conclude that collisions are waste. **The arithmetic does not support that.**

A collision means two states occupy one pitch class. It **costs** exactly one thing: a position in the harmonic vocabulary, of which `N = 12` supplies only `ceil(12/2) = 6`. A chord with a collision occupies fewer distinct classes than its state count, and that is a real constraint on what the folded chord can express.

It does **not** cost the following, and each is worth denying explicitly:

It does not cost surplus. `F_int` is computed from framing angles, not from the chord's index set. The relabelling test in §10.6 shows a collision appearing and disappearing with `F_int` fixed to twelve decimal places. Collisions and surplus are independent.

It does not mean the two states are the same component. State 5 is attributed feeling — a construal that names a cause. State 10 is narrative feeling — a construal that places the situation in a story. The ladder says these differ by five components of specification. They collide in pitch class because `10 = 2 * 5` and the odd-part rule folds by powers of two. That is a property of the arithmetic of the index, not a claim that the two components are interchangeable.

It does not mean the raw chord has fewer notes. In the raw chord states 5 and 10 are 12.0856 Hz and 48.3426 Hz — two octaves apart, two distinct pitches. Only the fold makes them one.

So what *is* a collision, honestly stated?

> **A COLLISION IS A LIMIT ON THE FOLDED REPRESENTATION, NOT A FINDING ABOUT THE SUBJECT.** It says that if this chord were rendered by octave transposition into a single band, two of its members would arrive at the same place. Everything a collision tells you is a fact about the index ladder and the fold. Nothing it tells you is a fact about the person.

That is the answer to the question Chapter 9 left open, and it is deliberately the deflationary one.

10.9 The restatement collapse, and how fast it happens

The genuine internal redundancy — the one that *does* cost surplus — is not a pitch-class collision. It is near-parallel framings, and it is expensive.

```
EXTENSION 3.1–a OF THE CARRIED SUBJECT (LABELLED): the threat framing, restated
 three times in different words. Same shape, small perturbations.

    t1 = {3:0.80, 5:0.40, 7:0.20}
    t2 = {3:0.78, 5:0.42, 7:0.21}
    t3 = {3:0.82, 5:0.38, 7:0.19}

    u = 0.001034, 0.001005, 0.004074     u_int = 0.002038
```

```
N_eff = 1.0224     F_int = 0.022166 nats
f_width = 0.5053 Hz   (against the ground essence k = 0.4834 Hz)
octaves above ground = 0.0640
```

Three framings, and 0.0640 octaves of width. The subject has produced three descriptions and one framing. They will experience this as having thought about it from several angles.

The exactly-parallel version removes even the perturbation:

```
EXTENSION 3.1-b (LABELLED): the same framing at three intensities.

    p1 = {3:0.8, 5:0.4, 7:0.2}
    p2 = {3:1.6, 5:0.8, 7:0.4}      (everything twice as strong)
    p3 = {3:0.4, 5:0.2, 7:0.1}      (everything half as strong)

    u = 0.000000, 0.000000, 0.000000     u_int = 0.000000
    N_eff = 1.0000     F_int = 0.000000 nats
    f_width = 0.483426 Hz = k exactly
    octaves above ground = 0.0000
```

Scaling a framing changes nothing. This is the invariance of Chapter 9 §9.10 seen from the other side: feeling the same thing more strongly, or less strongly, moves the reading by exactly zero. **The instrument reads composition, not amplitude, and here that means a person who is having a much harder time has not thereby acquired any surplus.** That is the arithmetic contradicting the romanticised reading directly, in the place where the romanticised reading would most like to be true.

10.10 The cost of one substitution

The most instructive intermediate case replaces one of the Subject's three framings with a near-copy of another and computes what it costs.

```
EXTENSION 3.1-d OF THE CARRIED SUBJECT (LABELLED): mirror replaced by a
restatement of threat.

    threat  = {3:0.8,  5:0.4,  7:0.2}     unchanged
    story   = {3:0.5,  5:0.7, 10:0.4}     unchanged
    mirror' = {3:0.78, 5:0.42, 7:0.21}    a restatement of threat

    u(threat, story)   = 0.388360     unchanged
    u(threat, mirror') = 0.001034     was 0.904762
    u(story,  mirror') = 0.372856     was 0.809410

    u_int   = 0.254083      was 0.700844
    N_eff   = 3.7949        was 8.7093
    F_int   = 1.333662      was 2.164390
    f_width = 6.9620 Hz     was 36.6686 Hz
    octaves = 3.8481        was 6.2451

    LOSS: -0.830728 nats = -2.3970 octaves
```

Substituting one framing for a restatement of another costs 2.40 octaves of width — more than a third of everything the twelve-category ladder can supply. The subject still holds three framings. Two of them are one framing.

Note carefully what the loss is *not* attributed to. It is not attributed to the subject becoming less capable, less well, or less anything. The reading changed because the encoding of the third framing changed. If the elicitation on a given day produces a restatement where it previously produced a distinct construal, the honest report is that the elicitation produced a restatement. Whether that reflects a change in the person, a change in the situation, a change in the operator, or a change in the subject's willingness to say a second thing is not determined by the number, and Chapter 11's worksheet requires all four to be considered before any of them is asserted.

10.11 What actually widens a chord, stated so it cannot be misread

Given all of the above, the engineering question is: if an operator wants a wider reading, what raises it?

A framing that is angled from the existing ones. That is the whole answer. u rises when the squared alignment falls, and the squared alignment falls when a framing assigns intensity in a different shape across the states.

What does **not** raise it, each denied because each has been proposed:

- **Amplifying an existing framing.** Zero effect. Scale invariance, §10.9.
- **Adding a restatement.** Negative effect. Extension 3.1-d, -2.40 octaves.
- **Adding a framing with disjoint support.** Raises the number to the ceiling and destroys its meaning. Chapter 9 §9.4; the reading becomes degenerate and must not be published.
- **Distress. Not an input. It appears in no equation in this book.** There is no term for it, no coefficient on it, and no path by which it could enter the computation. A person who has a worse week does not get a wider chord, and this book does not suggest that anyone should have a worse week.

And the honest general statement, which is a statement about method and not about persons:

> **THE ONLY LEVER ON `F_int` IS THE ANGULAR DIVERSITY OF WHAT IS ELICITED.** Everything else in the construction is invariant. Whether a given person's elicited framings are angled is a fact about the elicitation and the occasion, and this instrument reports it rather than explaining it.

10.12 "The divided self" — the phrase in the title, and what it is allowed to mean

The chapter is called *Dissonance and the Divided Self*, and the second half of that title is a borrowed phrase with a long and largely clinical history behind it. **This book borrows the phrase and none of the history.**

What the formalism licenses is narrow and can be said in one line: **a person holding several non-parallel framings is, under this encoding, a chord rather than a note.** That is a structural description of an encoding. It is not a description of fracture, of a self at war with itself, of dissociation, or of any condition, and none of those readings survives contact with §10.6's relabelling test.

There is a version of "divided" that the arithmetic does support, and it is almost the opposite of the dramatic one. **The genuinely divided self, in this system's terms, is the one with the *least* going on: framings so nearly parallel that the person has one instrument and several names for it.** Extension 3.1-b reads `F_int = 0.000000` — the ground essence, no width — and it is not a portrait of turmoil. It is a portrait of unanimity. Book Two Chapter 39 called it a single window with forty panes, and the reason the phrase lands is that unanimity inside one head is invisible from inside that head.

But even this must not be run in reverse. A person reading `F_int = 0` on one situation on one occasion has produced parallel framings on that situation on that occasion. **They are not thereby divided, undivided, well, unwell, rigid, or anything else. The number is about the framings.** The chapter's title is a title.

10.13 Reading a chord more than once

Because every reading in this chapter is of one situation on one occasion, the obvious next question is what happens when the same subject is read twice, and it is worth answering here because the answer constrains what a single reading can be used for.

Chapter 8 established the difference between a trait's frequency and a mood's as a question about `delta` — how fast a reading decays. The same question applies to a chord: is `F_int` a property that persists? Nothing in Chapters 9 or 10 answers that, because nothing in them involves time.

What the construction permits is a comparison, and only a comparison:

```
REPEAT READING, THE ONLY LEGITIMATE FORM

    same subject, same ladder, same operator, same elicitation script,
    DIFFERENT situation or different occasion

    report:  F_int(1), F_int(2), and the difference
    report:  the chord of each, and whether the state sets overlap
    report:  what changed in the procedure -- ideally nothing

    DO NOT report: a trend from two points
    DO NOT report: a change as a change in the person
```

Three sources can move a repeat reading and the number cannot separate them: the person, the situation, and the elicitation. A drop from 2.164390 to 1.333662 — the size of Extension 3.1-d's substitution — is equally consistent with a subject who framed the second situation more narrowly, a second situation that admits fewer framings, and an operator who ran a shorter session. A reading that attributes the change to the first of those three has made a claim the arithmetic does not support.

This is why Chapter 11's worksheet has a procedure block that must be filled in before the computation block, and why the procedure block is reproduced identically on every repeat.

The Specification

SPEC-10.1 — The consonance audit

```
INPUT   a chord: the set S of states spanned, from SPEC-9.2 Step 8.

STEP 1  for each unordered pair (i,j) in S with i < j:
            reduce j/i to p/q in lowest terms
            essence ratio      = p^2 / q^2
            raw cents          = 1200 * log2(j^2 / i^2)
            folded cents       = raw cents mod 1200
            HD_ess             = 2 * log2(p*q)
            G_ess              = 2 * G(p*q) - 1
            collision          = TRUE iff odd(i) == odd(j)
                               equivalently iff folded cents == 0

STEP 2  MIN HD_ess, MEAN HD_ess, COLLISION COUNT.

STEP 3  report the class set = { odd(n) : n in S } and |class set|,
        separately from |S|.

STEP 4  flag the 50/49 pair if present: any pair with folded cents = 34.9756.
        At N = 12 there is exactly one such pair, 7:10.
```

SPEC-10.2 — The redundancy audit (distinct from SPEC-10.1)

The consonance audit is about the chord. The redundancy audit is about the framings, and the two are independent by §10.6. **Both are required; neither substitutes for the other.**

```
INPUT   the normalised framing vectors from SPEC-9.2 Step 2.

STEP 1  every pairwise u, published individually.

STEP 2  RESTATEMENT FLAG: any pair with u < 0.01 is flagged RESTATEMENT.
        (The 0.01 threshold is a stated convention chosen by the author,
        not a derived value. State it; apply it before seeing results.)

STEP 3  EFFECTIVE FRAMING COUNT: report m (nominal framings) and N_eff
     (audited categories) SEPARATELY, with the roster reduction postulate
        stated. They are different counts and neither is the other.

STEP 4  If any pair is flagged RESTATEMENT, report the reading WITH the flag.
        Do not drop the framing to improve the number.
```

Step 4 matters. **A restatement is a finding, not an error.** Dropping it raises `F_int` and destroys the only information the reading contained about the subject's framing behaviour on that occasion.

SPEC-10.3 — The mandatory separation statement

Every published consonance or redundancy audit carries the following, in full, in the reading itself and not in a footnote:

> *This reading describes the information content of a set of framings under a stated encoding. It does not measure distress, well-being, health, or function, and no inference from this reading to any of those is supported by the construction. `F_int` is invariant under relabelling of the states and is therefore many-to-one even on chords; it does not determine the chord, and the chord does not determine anything about a person's experience. The twelve states are not diagnoses. This is not an assessment and not clinical advice. Nothing here recommends or values suffering. The frequencies are essence frequencies, not physical resonances.*

A reading published without this paragraph is not a reading this book endorses.

SPEC-10.4 — Gates

```
   GATE 10-1  the audit reports BOTH state count and class count          FAIL -
> incomplete
   GATE 10-2  every pairwise u published, not the mean alone              FAIL -
> incomplete
   GATE 10-3  the separation statement present in full                    FAIL -
> do not publish
    GATE 10-4  no sentence in the reading contains a condition name, a
            severity, a norm comparison, or a recommendation             FAIL -
> do not publish
   GATE 10-5  restatement flags reported, not resolved by deletion        FAIL -
> reading void
   GATE 10-6  consonance audit and redundancy audit reported separately    FAIL -
> incomplete
```

Gate 10-4 is a text gate rather than a numeric one and it is the one an operator will be tempted to wave through. **It is mechanical on purpose: read the reading, look for those four things, and if any is present the reading does not go out.** A gate that requires judgment gets exercised; a gate that requires a word search gets applied.

The Failure Modes

FM-10.1 — Romanticising dissonance

The failure. The chapter's result — that a perfectly consonant self has zero surplus — is read as "division is good," "conflict is generative," or "suffering is informative," and from there into advice.

Why it happens. Because the arithmetic produces a sentence that sounds like a moral and because the moral is attractive. The genre this book sits adjacent to has produced that moral many times, usually without any arithmetic under it at all.

Why it is wrong, in three independent ways. (i) The formalism contains no term for suffering; there is no path by which it enters. (ii) `F_int` is invariant under relabelling and does not even determine the chord (§10.6), so it cannot determine anything downstream of the chord. (iii) The only lever on `F_int` is angular diversity of elicited framings, and nothing about angular diversity implies discomfort — three comfortable framings of a situation score exactly as three uncomfortable ones would.

Detection. Any sentence in which more `F_int` is better, or in which a difficult experience is described as producing surplus.

Repair. Delete the sentence. **There is no repaired version of it**, because the claim is not underdetermined by the formalism, it is absent from it.

FM-10.2 — Pathologising consonance

The failure. The exact inverse, and the one that follows FM-10.1 the way an echo follows a shout: a low `F_int` reading is treated as a deficiency, a warning sign, or something to be corrected.

Why it happens. Because Book Two Chapter 39's description of the zero-surplus case — forty descriptions that are one description — reads like a portrait, and portraits invite recognition.

Why it is wrong. A settled, coherent, single reading of a situation is a completely ordinary thing and the formalism says nothing against it. `F_int = 0` means the elicited framings were parallel on that occasion. It means that and nothing else. **It is not a deficit, it is not a symptom, and describing it as either is a clinical claim this book does not make and is not entitled to make.**

Detection. Any sentence recommending that a subject develop more framings, or describing a narrow chord as something to work on.

Repair. Report the number and stop. **The instrument's job ends at the number.**

FM-10.3 — Reading harmonic dissonance as psychological conflict

The failure. The `7:10` pair at 34.98 cents is described as the subject's inner conflict, or the `5:10` collision as the subject's confusion.

Why it happens. The words carry their ordinary meanings in.

Why it is wrong. Those numbers are computed from `7`, `10`, and `5` — from integers, by a defined function, with no reference to any person. **Relabel the states and the numbers change while the subject's framings do not.** §10.6 demonstrates precisely this.

Detection. Any sentence attributing an interval to a subject's experience.

Repair. State intervals as properties of the index set. "The chord contains the lattice's unique 50/49 pair" is correct. "The subject is in a 50/49 relationship with themselves" is not a sentence this book can produce.

FM-10.4 — The reading used as clinical advice

The failure. A reading is used to decide something about a person's care, treatment, capacity, or condition.

Why it happens. The output is numeric, the output is reproducible, and numeric reproducible outputs get used for decisions.

Why it is wrong. **No validation of this instrument against any clinical outcome exists, has been attempted, or is claimed. The twelve-state ladder is an ordinal construction defended in Chapter 6 on its own terms and demoted on scale by R1. It has no criterion validity against anything.**

Detection. Gate 10-4, plus the question of who is going to read the reading and what they will do next.

Repair. The separation statement of SPEC-10.3, in full, and the operator's own refusal.

FM-10.5 — Manufactured internal difference

The failure. An operator elicits additional framings by pressing a subject to produce a different answer, and the subject obliges.

Why it happens. Because the operator wants a valid reading and near-parallel framings feel like a failed session.

Why it is wrong. A framing produced to satisfy an operator is not a framing the subject holds, and the encoding cannot tell the difference. This is internal pseudo-diversity, and Book Two Chapter 11 treats the relational version at length: apparent variety with no angular content behaves exactly like manufactured variety with angular content, which is why the failure is invisible in the output.

Detection. Procedural. Record the elicitation, record how many prompts each framing required, and publish the count.

Repair. Elicit once, accept the result, and report a narrow chord as a narrow chord.

FM-10.6 — The total-order fallacy on harmonic distance

The failure. Reasoning from "consonance is redundancy" as though every collision were more consonant than every non-collision.

Why it happens. Because the slogan is memorable and the caveat is not.

Why it is wrong. Result (C): `1:3` at `HD_ess = 3.1699` is more consonant than the collisions `1:4`, `2:8`, `3:12` at 4.0000 and `1:8` at 6.0000. **The claim is exactly true at the minimum, exactly true as a biconditional about zero folded separation, and true on average (3.0000 against 8.6617). It is not a total order.**

Detection. Any argument of the form "this pair is a collision, therefore it is more consonant than that pair."

Repair. Use the computed values. There are only sixty-six of them.

FM-10.7 — Treating the fold as an argument

The failure. Concluding, from the fact that states 5 and 10 fold to the same pitch, that attributed feeling and narrative feeling are substantively the same thing.

Why it happens. The fold makes them audibly identical and audible identity is persuasive.

Why it is wrong. The fold is a quotient by powers of two applied for the sole purpose of bringing values into a chosen band. It is a change of units. The states differ by five components of specification on the ladder, and the fold does not know that and does not care.

Detection. Any substantive claim justified by a folded value.

Repair. Publish raw and folded together, always, with the band and `j`, per SPEC-9.3.

FM-10.8 — Optimising `F_int`

The failure. Treating the width as a score to be raised, whether by the operator, the subject, or a system built around the reading.

Why it happens. Because it is a number, and numbers attract optimisation.

Why it is wrong. Every honest route to raising it runs through the subject genuinely holding differently-shaped construals, which is not something an operator can supply. Every dishonest route — restating, manufacturing, disjoint encoding — either lowers it, degenerates it, or falsifies it. There is no available optimisation that is also a measurement.

Detection. Any repeated reading of the same subject where the number rises and the procedure changed.

Repair. Pre-register the procedure, per Gate 6 of Chapter 9, and re-read without altering it.

FM-10.9 — Importing the wrong construct

The failure. Citing cognitive-dissonance research, or consonance research in music perception, as support for anything in this chapter.

Why it happens. Shared vocabulary.

Why it is wrong. Tenney distance and Euler gradus are defined functions, badged as chosen, and are not claims about hearing. The internal surplus is a construction over an ordinal encoding, not a psychological construct with a literature. Citing unrelated empirical work as support would be a fabricated citation, and the book does not do it.

Repair. Cite Tenney and Euler for the measures, which is what they are for, and cite nothing for the psychology, because there is nothing to cite.

FM-10.10 — Treating a zero-surplus reading as the end of the inquiry

The failure. A reading returns `F_int = 0.000000`, the operator records "no surplus," and the session ends.

Why it happens. Zero looks like a complete answer. It is a number, it is definite, and it closes.

Why it is wrong. **Zero surplus and no measurement are different results and the specification requires them to be distinguished.** `F_int = 0` means at least two framings were elicited and they were parallel. A session that produced only one framing has no pairs, no `u_int`, and no reading at all — Chapter 9's Gate 7. Recording the second as the first converts a missing measurement into a null one, which is the single most common way a measurement programme accumulates false negatives.

There is a second reason the inquiry should not end there. **A zero-surplus reading is the most informative reading the redundancy audit produces**, because it is the one that tells you the elicitation returned the same construal repeatedly — which is a fact about the session, and a fact worth having, and one that the number reports accurately.

Detection. Compare the framing count against the pair count. Zero pairs is no reading; one or more pairs with `u_int = 0` is a null reading.

Repair. Record which of the two occurred, per SPEC-10.2 Step 3, and record how many framings the elicitation produced before it stopped.

FM-10.11 — Comparing readings across ladders

The failure. Two readings computed on different state ladders, or on the same ladder with a different number of categories, are compared as though the numbers were commensurable.

Why it happens. `F_int` is in nats either way and nats look like nats.

Why it is wrong. `F_int = ln(1 + (N−1)u)` contains `N` explicitly, and its ceiling is `ln N`. A surplus of 2.164390 nats against a ceiling of `ln 12 = 2.484907` is 87.10% of the available range; the same 2.164390 nats against a ceiling of `ln 25` would be 67.24%. The raw value has no meaning without its ceiling, and neither does the pitch: `f_width` depends on `N_eff`, which depends on `N`.

Detection. Any comparison in which the two readings do not state the same `N`.

Repair. Publish `F_int`, `ln N`, and the ratio, always together — and where `N` differs, compare the ratio and say plainly that the absolute values are not comparable.

Worked Example

THE SUBJECT — the consonance and redundancy audits

CONSTRUCTED CASE, SPINE §3.1, carried from Chapter 9. Not an observation of any person.

The Subject's chord, from SPEC-9.2 Step 8:

```
  STATES SPANNED   {3, 5, 7, 10}
state  3  directed feeling    raw  4.3508 Hz  folded 278.453 Hz (j=+6)  class 3
state  5  attributed feeling   raw 12.0856 Hz  folded 386.741 Hz (j=+5)  class 5
state  7  reflexive feeling    raw 23.6879 Hz  folded 379.006 Hz (j=+4)  class 7
state 10  narrative feeling    raw 48.3426 Hz  folded 386.741 Hz (j=+3)  class 5
```

The consonance audit (SPEC-10.1)

pair	essence ratio	raw cents	folded cents	HD_ess	G_ess	collision
3:5	25 / 9	1768.72	568.7174	7.8138	13	no
3:7	49 / 9	2933.74	533.7418	8.7846	17	no
3:10	100 / 9	4168.72	568.7174	9.8138	15	no
5:7	49 / 25	1165.02	1165.0244	10.2586	21	no

```
    5:10        4 / 1        2400.00         0.0000    2.0000       3      YES
    7:10      100 / 49       1234.98        34.9756   12.2586      23       no

    MIN HD_ess       =  2.0000    (pair 5:10)
    MEAN HD_ess      =  8.4882
    COLLISION COUNT  =  1
    STATE COUNT      =  4
    CLASS COUNT      =  3         classes {3, 5, 7}
       50/49  PAIR            =    PRESENT     (7:10  at  34.9756  cents  --
the lattice's unique instance)
```

Three observations, each stated at the altitude the arithmetic supports.

The chord contains the most consonant interval the lattice permits and it contains it exactly once. `5:10` sits at `HD_ess = 2.0000`, the global minimum over all sixty-six pairs, attained only by the six 2:1 pairs. And by result (B), that is precisely the chord's one redundancy: 0.0000 folded cents, one class for two states. Consonance and redundancy are not correlated here; they are the same entry in the table, read twice.

The chord also contains the lattice's most remote pair among its members, and the two are adjacent in the index set. `7:10` at `HD_ess = 12.2586` is the widest interval in this chord, and it folds to 34.9756 cents — the smallest nonzero separation the whole `N = 12` system permits. **The most harmonically remote pair in the chord is the pair that would sound closest if the chord were folded into one band.** That is not a paradox; it is what the fold does. Harmonic distance is computed on the unfolded ratio `100/49`; folded separation is computed modulo the octave. They are different questions and this chord happens to answer them oppositely.

Mean `HD_ess = 8.4882` sits just below the non-collision average of 8.6617 for the lattice as a whole, which is what one expects of a chord containing exactly one collision among six pairs.

The redundancy audit (SPEC-10.2)

```
    pairwise u          mirror / story    0.809410
                        mirror / threat   0.904762
                        story  / threat   0.388360

   RESTATEMENT FLAGS  none  (minimum pairwise u = 0.388360, well above the 0.01 threshold)

    u_int               0.700844
    m (nominal)         3 framings
   N_eff (audited)    8.7093 categories, under the roster reduction postulate
    F_int               2.164390 nats
    f_width            36.6686 Hz   folded 293.349 Hz (band 220-440, j = +3)
    octaves             6.2451
```

The two audits disagree about what the chord is like, and they are entitled to. The consonance audit finds one exact redundancy in the chord's index set. The redundancy audit finds no restatement among the framings at all — the closest pair, story against threat, sits at `u = 0.388360`. The collision is in the ladder. It is not in the framings.

This is §10.6 made concrete on the carried case. The Subject holds three framings that are substantially non-parallel, and their union happens to include two indices differing by a factor of two. **Relabel state 10 as state 9 and the collision vanishes with `F_int` unchanged to twelve decimal places.** Nothing about the Subject changed; the ladder's arithmetic changed.

The counterfactual: what a consonant Subject would read

EXTENSION 3.1-d (LABELLED): the Subject with the mirror framing replaced by a restatement of the threat framing. This is the same person only in the sense that the case is carried; it is a constructed variant and is labelled as one.

```
                          SUBJECT          EXTENSION 3.1-d      change
  u(threat, story)        0.388360            0.388360           --
  u(threat, mirror)       0.904762            0.001034        -0.903728
  u(story,  mirror)       0.809410            0.372856        -0.436554

  u_int                   0.700844            0.254083        -0.446761
  N_eff                   8.7093              3.7949          -4.9144
  F_int (nats)            2.164390            1.333662        -0.830728
  f_width (Hz)           36.6686              6.9620         -29.7066
  octaves above ground    6.2451              3.8481          -2.3970

 RESTATEMENT FLAGS      none             (threat, mirror') at u = 0.001034
```

Two-point-four octaves, lost to one substitution. And the restatement flag fires, so the reading goes out with the flag attached rather than with the framing quietly removed — Gate 10-5.

And the terminal case, for completeness:

```
  EXTENSION 3.1-b (LABELLED): all three framings parallel.
   u_int = 0.000000  N_eff = 1.0000  F_int = 0.000000  f_width = k = 0.483426 Hz
      octaves above ground = 0.0000
```

Zero. The ground essence. Three framings and one window.

The reading, complete

```
=========================================================================
 CONSONANCE / REDUNDANCY READING -- THE SUBJECT
 (CONSTRUCTED CASE, SPINE 3.1. Not an observation of any person.)
=========================================================================
 ladder           N = 12, Chapter 6 ordinal on n; intensities ratio-scaled
             up to a free multiplier per Ch 6 (R1 governs D/P, not used here)
 chord              states {3, 5, 7, 10}; classes {3, 5, 7}
                    four states, three classes: 10 = 2*5, same odd part

 CONSONANCE AUDIT  min HD_ess   2.0000  (5:10, the lattice global minimum)
                    mean HD_ess   8.4882
                    collisions    1        (5:10, folded 0.0000 cents)
                50/49 pair   present  (7:10, 34.9756 cents, unique at N=12)
```

```
 REDUNDANCY AUDIT  pairwise u    0.809410 / 0.904762 / 0.388360
                   restatements  none
                   u_int         0.700844
                   m / N_eff     3 framings / 8.7093 audited categories
                   F_int         2.164390 nats
                f_width      36.6686 Hz, folded 293.349 Hz (220-440, j=+3)
                   octaves       6.2451

 INDEPENDENCE      relabelling 10 -> 9 removes the collision and leaves
                F_int = 2.164390 unchanged to 1e-12. The consonance audit
                   and the redundancy audit measure different things.

This reading describes the information content of a set of framings under a
 stated encoding. It does not measure distress, well-being, health, or
function, and no inference from this reading to any of those is supported by
the construction. F_int is invariant under relabelling of the states and is
 therefore many-to-one even on chords; it does not determine the chord, and
the chord does not determine anything about a person's experience. The twelve
states are not diagnoses. This is not an assessment and not clinical advice.
 Nothing here recommends or values suffering. The frequencies are essence
 frequencies, not physical resonances.
================================================================================
```

What the chapter established, and the line it will not cross

Established, and checkable: the essence lattice doubles Tenney harmonic distance; the minimum essence harmonic distance is attained by exactly the six 2:1 pairs, all of which are pitch-class collisions; zero folded separation is exactly equivalent to sharing an odd part; therefore the most consonant pairs the lattice offers are precisely the pairs it cannot distinguish. Applied within one observer, a set of parallel framings has `u_int = 0`, `N_eff = 1`, `F_int = ln 1 = 0`, and a chord width of exactly the ground essence.

Not established, and not claimed anywhere: that a narrow chord indicates anything about a person's health, function, or experience; that a wide chord indicates anything better; that difficulty produces surplus; that suffering is a resource; that any of this is a basis for advice about anyone's care.

The result is about information content. The distance between "information content" and "how a person is doing" is not a gap this book knows how to cross, and a book that pretended to cross it would be worth less than one that says where the road ends.

Chapter 10 owns the consonance/redundancy identity and the separation statement. Chapters 14 and 16 cite it. The doubling theorem is Book Two Chapter 33; the internal surplus is Book Two Chapter 39; the chord, the one-hot trap, and SPEC-9.2 are Chapter 9; octave folding and the odd-part rule are Chapter 7. Chapter 11 assembles all of it into a worksheet a reader can run.

Chapter 11 . The Personal Spectrum

Chapters 5 through 10 built the parts. Chapter 5 defended the index as a count. Chapter 6 built the twelve-state ladder and the elicitation procedure that fills it in. Chapter 7 gave a state a pitch and established the fold. Chapter 8 separated a trait from a mood by way of `delta`. Chapter 9 made the chord and named the one-hot trap. Chapter 10 established that consonance and redundancy are the same phenomenon, and spent most of its length making sure that result could not be turned into advice.

This chapter assembles all of it into one procedure that a reader can actually run, on paper, for one person, on one situation, and finish.

It ends where Chapter 4 said every practical chapter of this book must be able to end: with an octave deficit.

```
THE READING THIS CHAPTER PRODUCES

    octave deficit  =  2.885 * (c/eta)  -  log2( f / k )

    deficit <= 0    sustainable as it stands
    deficit  > 0    short by that many octaves, and the four levers are:
                raise u  .  raise eta  .  cut c  .  add distinct categories
```

Chapter 11 owns the full personal worksheet. Chapter 24 cites it. Everything else in the procedure is owned elsewhere and cited here rather than restated, which is the whole point of a worksheet: it is an index to the book with blanks in it.

BEFORE ANYTHING ELSE. The twelve states are not diagnoses and this reading is not an assessment. It makes no clinical, medical, or therapeutic claim of any kind. **The frequencies are essence frequencies, not physical resonances** — nothing here asserts that any person, tissue, or molecule resonates at any value. **The reading measures the information content of a set of framings under a stated encoding; it does not measure distress, well-being, health, or function, and Chapter 10 §10.6 shows formally why no inference to any of those is available.** All cases in this chapter are CONSTRUCTED and labelled as such.

The Law

11.1 What a personal reading is

A personal reading is a reading **of a set of framings, on one situation, on one occasion, under one stated decomposition, by one operator**. Every one of those qualifications is load-bearing and every one of them appears on the worksheet as a blank that must be filled before the arithmetic starts.

What it is not is a reading of a person. **There is no procedure in this book that reads a person.** Book Two Chapter 40's FVP-7 for the self-storage law is recorded as **NOT RUN** — the reproducibility gate, whether one person's `eta` means what another's does, has not been attempted, and until the separation property of Book Two Chapter 54 is established, **every personal parameter is a within-subject index and cross-person comparison is not licensed.** That constraint is inherited here in full. Two people's readings may be compared only in the sense that two people's shoe sizes measured with different rulers may be compared: not at all.

There is a reason this chapter takes the form of a worksheet rather than an argument, and it is worth stating before the sheet appears.

An instrument with a contested foundation is honest only if it is fully procedural. Every component of this reading rests on something that could be disputed: the ladder is an ordinal construction defended in Chapter 6 rather than derived; the constant k is a design decision badged as a postulate; the intensities are ratio-scaled only to the extent a magnitude-estimation procedure delivers ratio scales, which Chapter 6 states is a measured property of a procedure and not a constant of nature; the roster reduction is a postulate; the storage parameters have a reproducibility gate recorded as NOT RUN. **Not one of those is hidden, and not one of them is fatal — but together they mean the number has no authority of its own.** What it has instead is a trail.

A worksheet is what a trail looks like when it is written down. **Every blank on the sheet is a place where someone made a decision, and the sheet exists so that a second person can find each decision, disagree with it, change it, and see what the reading does.** That is the only form of validity this instrument currently has: not that its numbers are right, but that its numbers are *recomputable from declared inputs by someone who thinks the inputs are wrong.*

Chapter 5's honest limit governs the whole sheet: the frequency inherits exactly the defensibility of the count. A contested decomposition yields a contested frequency. The worksheet does not resolve that. It makes the contest possible.

11.2 The five outputs

```
THE PERSONAL SPECTRUM -- five outputs, in order of derivation

  1  THE SPECTRUM      the twelve state frequencies, raw and folded
                      f(n) = k n^2                                  [Ch 7]

  2  THE CHORD         the states spanned by the framings, their pitches,
                     their pitch classes, and their intervals       [Ch 9]

  3  THE WIDTH        F_int = ln(1 + (N-1) u_int),  f = k e^(2 F_int) [Ch 9]
```

```
4  THE COST RATIO   c / eta, estimated at internal scope        [Bk 2 Ch 40]

5  THE DEFICIT      2.885*(c/eta) - log2(f/k)                    [Ch 4]
```

Outputs 1 and 2 depend on the ladder and the framings. Output 3 depends only on the angles between framings. Output 4 depends on none of that — it is a separate estimation problem with separate hazards. **Output 5 is the only place the two halves meet, and it is the join this whole book exists to perform.**

11.3 The octave deficit, restated

Chapter 4 owns the minimum sustainable frequency and its diagnostic. Restated here in the form the worksheet uses, without re-derivation:

```
THE MINIMUM SUSTAINABLE FREQUENCY                      [Ch 4, CANON3 §1]

    Q_inf > 0   <=>   eta F > c   <=>   F > c/eta
    F = (1/2) ln(f/k)                                the inverted bridge
    f_min = k * e^( 2 c / eta )

    octaves required = 2 (c/eta) / ln 2 = 2.885 * (c/eta)
    octaves present  = log2( f / k ) = 2.885 * F
    octave deficit   = required - present
```

Every unit of cost-to-conversion ratio costs 2.885 octaves of required pitch. The verified table from Chapter 4, reproduced because the worksheet needs it at hand:

c/eta	f_min (Hz)	octaves above k
0.000	0.4834	0.000
0.500	1.3141	1.443
1.000	3.5721	2.885
1.500	9.7099	4.328
2.000	26.3942	5.771
2.485	69.6124	7.170
3.000	195.0279	8.656

The constant is written `2.885` throughout; exactly it is `2/ln 2 = 2.885390`. Where a reading sits within a hundredth of an octave of zero, use the exact form.

11.4 What `eta`, `c` and `delta` mean inside one person

The storage law at internal scope is Book Two Chapter 40's self-storage law, and this chapter cites it rather than re-deriving it:

```
Q_{t+1} = (1 - delta) Q_t + eta M_t - C_t                [Bk 2 Ch 40]

Q      stored durable structure -- character, in Q-units
delta  retention loss per period, in [0,1], dimensionless
eta    conversion of experience into durable structure, in Q-units per nat
C      cost per period, in Q-units
```

```
M      the moment, in nats -- here supplied by F_int

Q_inf = ( eta F - c ) / delta                          Paper E, Thm 3.5
```

Note the units, because they are what stops the equation being a slogan: `eta` carries Q-units per nat, so `c/eta` carries nats and `2.885*(c/eta)` carries octaves. The deficit is dimensionally sound and the worksheet checks it.

At internal scope the three parameters read as follows, and each has an estimation method owned by Book Two Chapter 40:

`eta` — conversion. How much of what happens becomes durable structure. Chapter 40 estimates it as a product of six pass-through yields, `eta = y1 * y2 * ... * y6`, with the last rung carrying the units. Its structural property is multilinearity: the elasticity of `eta` with respect to every rung is exactly 1, so the absolute gain from fixing a rung is inversely proportional to that rung's current yield. A single blocked rung nullifies every other, which is why the aggregate figure is uninformative and the stage-wise figure is not.

`c` — cost. Chapter 40 estimates it from five ledgers. It is a per-period quantity in Q-units and it must be declared with its period.

`delta` — loss. Estimated from a zero-input window. It sets the time constant `tau = 1/delta` and therefore how long a regime takes to arrive at its fixed point.

And the identifiability hazard, which is the one that voids readings: Paper E §6.3 — **if the `Q`-proxy rises whenever a lot happens, the input has been measured twice and the gauge is void.** A journal that gets longer on busy weeks is not a measure of `Q`. The worksheet requires the proxy and its independence check to be declared before any parameter is estimated.

11.5 The break-even identity

Substituting `octaves present = 2.885 * F_int` into the deficit gives an identity worth stating on its own, because it is what makes the reading legible without a table:

```
THE BREAK-EVEN IDENTITY

    deficit = 2.885*(c/eta) - 2.885*F_int = 2.885 * ( c/eta - F_int )

    deficit = 0   <=>   c/eta = F_int
    deficit < 0   <=>   c/eta < F_int      SUSTAINS
    deficit > 0   <=>   c/eta > F_int      SHORT
```

The whole diagnostic is one comparison: is the cost ratio smaller than the surplus? The octave form is the same statement rescaled by `2.885`, and it exists because octaves are legible and nats are not.

For the carried Subject, with `F_int = 2.164390`:

```
break-even c/eta = 2.164390 nats
below it   -> sustains        above it   -> short
```

Everything the frequency machinery adds to `eta F > c` is legibility. It adds no content, and the book says so here rather than letting the reader discover it and feel cheated. What the frequency

form buys is that a deficit in octaves can be compared against a table of what an octave costs, and a ratio in nats cannot.

11.6 The stop rule

Because `F <= ln N` always, the deficit cannot be closed by raising u past orthogonality. That gives a hard stop, and it is Paper E Corollary 3.4 in the frequency register:

```
THE CEILING                                                [Ch 4, CANON3 §1]

  c/eta <= ln N  is REQUIRED for any arrangement of N categories to sustain.

     At N = 12:   c/eta <= 2.484907   and   f_min <= k*144 = 69.6133 Hz

     c/eta = 2.4849 -> f_min =  69.6124 Hz    at the ceiling
     c/eta = 2.6000 -> f_min =  87.6317 Hz    ABOVE -- nothing sustains
     c/eta = 3.2000 -> f_min = 290.9474 Hz    ABOVE -- nothing sustains
```

Above the ceiling, no arrangement of those twelve categories is sustainable — not a better set of framings, not a better schedule, not more effort. The only remedies are to raise `eta`, cut `c`, or add genuinely distinct categories, and the worksheet stops and says so rather than computing a deficit that no rearrangement could close.

The stop rule is checked before the deficit, not after. A worksheet that computes a deficit above the ceiling has produced a number that describes an impossibility.

11.7 `delta` is not a lever on the deficit

One structural fact the worksheet exposes and that surprises most readers on first sight:

```
deficit = 2.885*(c/eta) - log2(f/k)          -- no delta term
Q_inf   = (eta F - c) / delta               -- delta scales the magnitude
```

`delta` does not appear in the deficit at all. It scales `Q_inf` and sets the time constant, but it cannot change the sign. Computed on the carried Subject at `eta = 0.327600`, `c = 0.55`, where the numerator `eta F_int - c = +0.159054`:

```
delta = 0.20  ->  Q_inf = +0.7953     deficit -1.4015
delta = 0.12  ->  Q_inf = +1.3255     deficit -1.4015
delta = 0.06  ->  Q_inf = +2.6509     deficit -1.4015
delta = 0.03  ->  Q_inf = +5.3018     deficit -1.4015
```

Lowering `delta` multiplies whatever sign the numerator already has. On a positive numerator it multiplies a gain. On a negative one it multiplies a loss. That is R15, it is owned by Chapter 15, and its internal-scope demonstration is in this chapter's Worked Example because the worksheet has to be able to warn about it.

The Engineering

11.8 Step A — eliciting the intensities

Chapter 6 owns this procedure and this chapter cites it. What the worksheet needs from it is three things, recorded before any arithmetic:

```
A1  THE SITUATION, stated once, in the subject's own words, and fixed.
    All framings are framings OF THIS. If the situation drifts between
    framings, the reading is void -- the angles would be between
    vectors pointed at different targets.

A2  THE FRAMINGS, elicited one at a time, each described in full before
    the next is requested. Record how many prompts each required.

A3  THE DECOMPOSITION. For each framing, of each of the twelve state
    classes, ask whether the description REQUIRES that component to be
    specified. Rate the required ones for intensity on a common scale.
```

Three procedural rules that the worksheet enforces:

Do not ask which state a framing "is." That question produces one-hot vectors and Chapter 9 §9.4 shows what one-hot vectors do. Ask which components the description requires. Framings of one situation share components; that is what makes them framings of one situation.

Record the prompt count. A framing that took six prompts to obtain and a framing the subject volunteered are different data, and only the record distinguishes them. Chapter 10's FM-10.5 — manufactured internal difference — is invisible in the output and visible only here.

Declare the scale status, and declare the right one. Two different scale claims are in play and conflating them is a real error. R1 demotes the Moment equation: `D/P` may be used ordinally only, and any psychological frequency derived through `D/P` inherits the demotion. The worksheet does not route through `D/P`, so that demotion does not bind the reading — but the box exists so that an operator who *does* route through it must say so.

The intensities have their own scale status and Chapter 6 owns it. Chapter 6 shows that ordinal-preserving transformations of the intensities move u on the carried Subject's closest pair from 0.362115 to 0.525275 — `f_rel` from 12.0049 to 22.2093 Hz, a spread of 1065.05 cents, nearly nine tenths of an octave — from transformations that preserve every ordinal fact in the data. So merely ordinal intensities will not do. They must be ratio-scaled up to a common free multiplier, which is what a magnitude-estimation procedure is built to deliver, and the honest form of the claim is that they are ratio-scaled to the extent magnitude estimation delivers ratio scales — a measured property of a procedure, not a constant of nature. Say which claim the reading rests on, in the box provided, every time.

And report the norms. Chapter 6's **MAGNITUDE REPORTING RULE** requires the raw L2 norm of every state vector to be published alongside the normalised vector, in the units of the elicitation instrument, and never discarded. u measures direction; normalisation throws away length, and length is not nothing — two profiles differing eightfold in intensity are exactly parallel and carry u = `0.000000` between them.

11.9 Step B — building the vectors and passing the gates

Chapter 9's SPEC-9.2 owns this. In worksheet form:

```
B1  Write each framing as a twelve-vector of intensities.
B2  FIX AND RECORD THE SUPPORTS before computing anything.
B3  Normalise: x <- x / ||x||.  Check ||x|| = 1.000000.
B4  OVERLAP GATE: every pair must share at least one state.
    Any pair with empty shared support -> STOP. Report DEGENERATE.
B5  WEIGHT GATE: shared support carries >= 10% of each vector's squared
    norm. (A stated convention, not a law. State it; apply it blind.)
```

The overlap gate is the single most important line on the worksheet. Chapter 9 §9.4 computed what happens without it: three one-hot framings score `u = 1.000000` on every pair, `F_int = ln 12 = 2.484907`, `f = 69.6133 Hz` — the ceiling, identically, for every subject who has ever lived. And the disguised version is worse, because three framings each spanning three states with no state in common produce exactly the same degenerate maximum while looking rich.

A worksheet that reaches `F_int = 2.484907` at `N = 12` should be treated as broken until proven otherwise.

11.10 Step C — the spectrum and the fold

Chapter 7 owns the fold and the odd-part rule. The worksheet carries the table:

```
THE TWELVE-STATE SPECTRUM      k = 0.483425732988414 Hz,  f = k n^2

 n   state class                  raw (Hz)    folded 220-440    j    class
 1   undifferentiated activation     0.4834       247.514       +9      1
 2   valenced activation             1.9337       247.514       +7      1
 3   directed feeling                4.3508       278.453       +6      3
 4   situated feeling                7.7348       247.514       +5      1
 5   attributed feeling             12.0856       386.741       +5      5
 6   temporal feeling               17.4033       278.453       +4      3
 7   reflexive feeling              23.6879       379.006       +4      7
 8   social feeling                 30.9392       247.514       +3      1
 9   normative feeling              39.1575       313.260       +3      9
10   narrative feeling              48.3426       386.741       +3      5
11   reciprocal feeling             58.4945       233.978       +2     11
12   committed feeling              69.6133       278.453       +2      3

Six pitch classes {1,3,5,7,9,11} = ceil(12/2).  ln 12 = 2.484907.
Band 220-440 Hz; j is the transposition. STATE THE BAND AND j ALWAYS.
```

The pitch class is the odd part of the index and nothing else. Odd indices are their own odd part and are always their own class; even indices collapse onto the class of the odd number left after dividing out every factor of two. Four of the twelve states — 1, 2, 4, 8 — share class 1; three — 3, 6, 12 — share class 3; two — 5, 10 — share class 5; and 7, 9, 11 are alone in theirs. That is the entire redundancy structure of the vocabulary, and an operator who has it memorised can read the class column off a chord without computing anything.

11.11 Step D — the chord and its intervals

Chapter 9's SPEC-9.2 Steps 8 and 9, and Chapter 10's SPEC-10.1:

```
D1  STATES SPANNED = union of the framing supports.  Report the count.
D2  PITCH CLASSES  = distinct odd parts.  Report the count SEPARATELY.
D3  RAW pitches k n^2; FOLDED pitches with band and j.
D4  INTERVALS: for each pair (i,j), essence ratio j^2/i^2 in lowest terms,
    raw cents, folded cents.
D5  CONSONANCE AUDIT: min HD_ess, mean HD_ess, collision count.
    Collision <=> same odd part <=> folded cents exactly 0.
D6  Flag the 50/49 pair if present. At N = 12 there is exactly one: 7:10.
```

D1 and D2 are different counts and conflating them is FM-9.7. A chord of four states in three classes has one member that adds pitch without adding a class.

11.12 Step E — the width

```
E1  every pairwise u = 1 - (x.y)^2, PUBLISHED INDIVIDUALLY
E2  u_int = mean over all m(m-1)/2 pairs
E3  N_eff = 1 + (N-1) u_int
E4  F_int = ln N_eff
E5  f = k * N_eff^2 = k * e^(2 F_int);  folded, with band and j
E6  octaves present = 2 F_int / ln 2
E7  F_int / ln N, as a percentage of the ceiling
```

E7 is not decoration. **A raw `F_int` is meaningless without its ceiling** — 2.164390 nats is 87.10% of `ln 12` and would be 67.24% of `ln 25`. Chapter 10's FM-10.11 covers the comparison hazard; the worksheet forecloses it by requiring the ratio.

11.13 Step F — estimating `eta`, `c`, `delta`

This is the step that will take longest and the step most likely to be skipped, and the worksheet is built so that skipping it produces a blank rather than a default.

```
F1  DECLARE THE PERIOD.  Every one of eta, c, delta is per-period.
F2  DECLARE THE Q-PROXY and run the independence check.
    HAZARD (Paper E §6.3): if the proxy rises whenever a lot happens,
    the input has been measured twice and the gauge is VOID.
F3  eta -- the six-rung ladder [Bk 2 Ch 40.4]. Enumerate the candidate
    set PROSPECTIVELY, at the point of entry. Report every rung, not
    the product alone.
F4  c   -- the five ledgers [Bk 2 Ch 40.5], in Q-units per period.
F5  delta -- from a zero-input window [Bk 2 Ch 40.3.4].
F6  DECLARE: these are WITHIN-SUBJECT indices. FVP-7 is NOT RUN.
    No cross-person comparison is licensed.
```

On F3, the reason the rungs must be reported individually: `eta = y1 * ... * y6` is multilinear, so a single blocked rung nullifies every other. An aggregate `eta = 0.35` is consistent with six mediocre rungs and with five excellent rungs and one broken one, and the intervention implied

by those two situations is completely different. The elasticity of `eta` with respect to every rung is exactly 1, so the absolute gain from repairing a rung is inversely proportional to its current yield: **the worst rung is always the cheapest to fix and always the one the aggregate hides.**

On F3's identifiability hazard, which is the one that quietly ruins estimates: an operator who enumerates only the experiences they already converted will recover `eta` near 1. **The candidate set must be enumerated prospectively, at the point of entry, before anyone knows what converted.**

11.14 Step G — the deficit, and the four levers

```
G1  c/eta, in nats
G2  STOP RULE: if c/eta > ln N, HALT. Report ABOVE CEILING.
    No arrangement of N categories sustains. Do not compute a deficit.
G3  f_min = k * e^(2 c/eta)
G4  octaves required = 2.885 * (c/eta)
G5  DEFICIT = required - present
G6  Q_inf = (eta * F_int - c) / delta
G7  time constant tau = 1/delta; periods to within 5% of Q_inf
G8  If deficit > 0, the four levers, EACH COSTED:
        raise u      (raise F_int)   ceiling: ln N
        raise eta    (the conversion ladder, worst rung first)
        cut c        (the five ledgers)
      raise N     (genuinely distinct categories -- not relabelled ones)
```

Lever four carries a standing warning. Raising `N` raises the ceiling and, at fixed `u`, raises `F` — but only if the added categories are genuinely distinct. Book Two Chapter 11 treats pseudo-diversity at length: **categories that are relabellings of existing ones raise `N` in the formula and not in the world, and the result is a reading that improves while nothing does.**

11.14b Double-coding, and what to do when two operators disagree

The worksheet has a line at the foot for a second coder, and it is not a formality.

The decomposition step — deciding which of the twelve components a description requires — is the step where two competent operators most reliably diverge. One reads "this is the latest instalment of an old pattern" as requiring narrative feeling and attributed feeling; another reads it as requiring narrative feeling, attributed feeling, and temporal feeling, on the grounds that "latest instalment" projects a duration. Both readings are defensible. They produce different supports, different inner products, and different frequencies.

The procedure when they diverge:

```
DOUBLE-CODING DISAGREEMENT PROCEDURE

1  Both operators publish their full vectors and their full readings.
2  Neither reading is designated correct.
3  DO NOT AVERAGE THE DECOMPOSITIONS.  Averaging two decompositions
   produces a third that neither operator endorses, and the resulting
   vector corresponds to no reading of the description.
```

```
4  Report the SPREAD: the range of F_int, the range of the deficit,
   and whether the two readings agree on the SIGN of the deficit.
5  If the sign agrees, the disagreement is about magnitude and the
   reading is usable with the spread attached.
   If the sign disagrees, the reading is INCONCLUSIVE and must be
   reported as inconclusive.
```

Step 5 is the whole value of double-coding. A deficit of -1.40 from one operator and -0.62 from another is a disagreement about how far inside the boundary the reading sits, and the reading still says *inside*. A deficit of -0.20 from one and +0.30 from another says nothing at all, and an instrument that reports "nothing at all" when it has nothing is worth more than one that reports the first operator's number.

The book has no procedure for making the disagreement go away, and it should not have one. **The disagreement is information about how well specified the ladder is, and suppressing it would destroy the only evidence available on that question.**

11.15 R15 at internal scope — the order of operations

The last engineering point, and it is the one that turns a reading into a decision.

When the deficit is positive, the intuitive repair is to lower `delta` — to hold on to things better, to let less slip, to be steadier. At internal scope this is the most natural move anyone makes and it is the one that goes wrong.

```
R15, INTERNAL SCOPE                                    [owned by Ch 15]

    Q_inf = (eta F − c) / delta

    If (eta F − c) < 0, lowering delta MULTIPLIES a negative number.
    The fixed point gets WORSE, and it gets worse in proportion.
```

Computed on the carried Subject under a failing parameter set (Worked Example below):

```
as found            eta=0.25 c=0.60 delta=0.12   Q_inf = −0.4909   DECAYS
lower delta only     eta=0.25 c=0.60 delta=0.06   Q_inf = −0.9817   WORSE
raise eta first     eta=0.32 c=0.60 delta=0.12   Q_inf = +0.7717   ACCUMULATES
then lower delta     eta=0.32 c=0.60 delta=0.06   Q_inf = +1.5434   BETTER
```

> **FIX THE NUMERATOR, THEN LOWER THE LOSS. REVERSE THE ORDER AND THE SAME INTERVENTION ACCELERATES THE FAILURE.**

The worksheet places the `Q_inf` sign check immediately before the intervention block for exactly this reason. An operator who reads the sign first cannot make the reversed move by accident.

The Specification — THE PERSONAL SPECTRUM WORKSHEET

Printable. One situation, one occasion, one operator. Fill top to bottom. Do not skip a block; a skipped block is a blank, not a default.

```
================================================================================
  THE PERSONAL SPECTRUM WORKSHEET                      Creature Comforts, Bk 3
  Not a diagnosis. Not an assessment. Not clinical advice.
  Essence frequencies, not physical resonances.
================================================================================

  BLOCK 0 -- PROVENANCE
  ------------------------------------------------------------------------
-
  Subject ID ..................... _____________   Date ............ __________
  Operator ....................... _____________   Period declared . __________
  Ladder ......................... N = _____   ln N = _________
  k .............................. 0.483425732988414 Hz
  Band for folding ............... ______ to ______ Hz
  Intensity scale claim ......... [ ] ratio-scaled up to a free multiplier
                                        (magnitude estimation, Ch 6)
                                  [ ] weaker -- state what: _______________
  Does the reading route through D/P?  [ ] no  [ ] yes -> R1 demotion applies
  Reading type ................... [ ] first  [ ] repeat of __________

  BLOCK A -- THE SITUATION AND THE FRAMINGS
  ------------------------------------------------------------------------
-
  The situation, once, in the subject's words:
  ____________________________________________________________________________
  ____________________________________________________________________________

  Framing 1 name _______________  prompts required ____
     description: ____________________________________________________________
  Framing 2 name _______________  prompts required ____
     description: ____________________________________________________________
  Framing 3 name _______________  prompts required ____
     description: ____________________________________________________________
  (add rows as needed; m = ____ framings, P = m(m-1)/2 = ____ pairs)

  BLOCK B -- THE VECTORS
  ------------------------------------------------------------------------
-
  state:         1    2    3    4    5    6    7    8    9   10   11   12
  framing 1   [___][___][___][___][___][___][___][___][___][___][___][___]
  framing 2   [___][___][___][___][___][___][___][___][___][___][___][___]
  framing 3   [___][___][___][___][___][___][___][___][___][___][___][___]
```

```
  SUPPORTS FIXED AND RECORDED?  [ ] YES  (required before Block C)
  norms  ||1|| = ______  ||2|| = ______  ||3|| = ______
  normalised vectors (write below or attach):
  ____________________________________________________________________________

  BLOCK C -- THE GATES
  ----------------------------------------------------------------------------
-
  C1  unit norm, all framings = 1.000000 ..... [ ] PASS  [ ] FAIL
  C2  OVERLAP: every pair shares >= 1 state ..... [ ] PASS  [ ] FAIL
        shared support 1&2: __________  1&3: __________  2&3: __________
        FAIL -> STOP. Report DEGENERATE. Publish no F_int. Return to Ch 6.
  C3  WEIGHT: shared support >= 10% of each squared norm [ ] PASS [ ] WEAK
        (threshold is a stated convention; state it: ______)
 C4  ROSTER: m >= 2, so P >= 1 ..... [ ] PASS  [ ] NO READING
  C5  PRE-REGISTRATION: supports fixed before Block D .. [ ] PASS  [ ] VOID

  BLOCK D -- PAIRWISE, AND THE WIDTH
  ----------------------------------------------------------------------------
-
  pair        cos theta      u = 1 - cos^2      F = ln(1+(N-1)u)
  1 & 2      __________      ____________       _______________
  1 & 3      __________      ____________       _______________
  2 & 3      __________      ____________       _______________
  (every pairwise u is published; the mean alone is not a reading)

  RESTATEMENT FLAGS: any pair with u < 0.01 ..... _______________
        (flag it; do NOT delete the framing to raise the number)

  u_int  = mean of pairwise u ..................... _______________
  N_eff  = 1 + (N-1) u_int ........................ _______________
  F_int  = ln N_eff ............................... _____________ nats
  f      = k * N_eff^2 ............................ _____________ Hz
  folded = f * 2^j into the band, j = ____ ........ _____________ Hz
  octaves present = 2 F_int / ln 2 ................ _______________
  F_int / ln N (percentage of ceiling) ............ __________ %
  CEILING CHECK: F_int <= ln N .................... [ ] PASS  [ ] ERROR

  BLOCK E -- THE CHORD
  ----------------------------------------------------------------------------
-
  STATES SPANNED  { _____________________ }   count = ____
  PITCH CLASSES   { _____________________ }   count = ____   (odd parts)
  (these are DIFFERENT counts; report both)

  state   raw f = k n^2      folded (band, j)        pitch class
  ____    ____________       ____________  j=___     ______
  ____    ____________       ____________  j=___     ______
```

```
  ____    ____________       ____________  j=___     ______
  ____    ____________       ____________  j=___     ______

  INTERVALS
  pair   essence ratio   raw cents   folded cents   HD_ess   collision?
  __:__   ______/______  _________   ___________   _______   [ ]
  __:__   ______/______  _________   ___________   _______   [ ]
  __:__   ______/______  _________   ___________   _______   [ ]
  __:__   ______/______  _________   ___________   _______   [ ]
  __:__   ______/______  _________   ___________   _______   [ ]
  __:__   ______/______  _________   ___________   _______   [ ]

  min HD_ess ____  mean HD_ess ____  collisions ____
  50/49 pair (7:10, folded 34.9756 cents) present? [ ] yes [ ] no

  BLOCK F -- THE STORAGE PARAMETERS
   ------------------------------------------------------------------------
-
  PERIOD ..................................... _____________
  Q-PROXY .................................... _____________
  PROXY INDEPENDENCE CHECK (Paper E 6.3):
      does the proxy rise whenever a lot happens?  [ ] no  [ ] YES -> VOID

  eta, six-rung ladder (report EVERY rung):
      y1 ____  y2 ____  y3 ____  y4 ____  y5 ____  y6 ____
      eta = product ......................... ______ Q-units/nat
      candidate set enumerated PROSPECTIVELY? [ ] yes  [ ] no -> biased high
     WORST RUNG = ____ at ____   (the cheapest repair; elasticity is 1 on all)

  c, five ledgers ........................... ______ Q-units/period
  delta, zero-input window .................. ______ per period
  tau = 1/delta ............................. ______ periods

  WITHIN-SUBJECT ONLY. FVP-7 NOT RUN. No cross-person comparison. [ ] noted

  BLOCK G -- THE READING
   ------------------------------------------------------------------------
-
  c/eta ..................................... _____________ nats

  G-STOP  is c/eta > ln N ?   [ ] no -> continue
                              [ ] YES -> HALT. ABOVE CEILING.
        No arrangement of these N categories sustains. Remedies: raise eta,
       cut c, or add genuinely distinct categories. Do not compute a deficit.

  f_min = k * e^(2 c/eta) ....................... _____________ Hz
  octaves required = 2.885 * (c/eta) ............ _____________
  octaves present  = log2(f/k) .................. _____________
```

```
  ------------------------------------------------------------
  OCTAVE DEFICIT = required - present ........... _____________
  ------------------------------------------------------------
        deficit <= 0   SUSTAINS
        deficit  > 0   SHORT BY THAT MANY OCTAVES

  BREAK-EVEN CHECK: deficit = 0 iff c/eta = F_int
        c/eta = _________   F_int = _________   agree with the sign? [ ]

  Q_inf = (eta * F_int - c) / delta ............. _____________
  SIGN OF THE NUMERATOR (eta*F_int - c) ......... [ ] +   [ ] -
  periods to within 5% of Q_inf ................. _____________

  BLOCK H -- INTERVENTION (only if deficit > 0)
   ------------------------------------------------------------------------
-
  *** R15 CHECK FIRST ***
  If the numerator is NEGATIVE, do NOT lower delta. Lowering delta multiplies
  the sign already present and makes a failing fixed point fail faster.
  FIX THE NUMERATOR FIRST.                              [ ] read and applied

  lever            proposed change     new deficit    new Q_inf    cost
  raise u          ____ -> ____        __________     _________    ______
  raise eta        ____ -> ____        __________     _________    ______
  cut c            ____ -> ____        __________     _________    ______
  raise N          ____ -> ____        __________     _________    ______
        (raise N only with GENUINELY DISTINCT categories -- relabelling
         raises N in the formula and not in the world)

  Does ANY SINGLE lever close the deficit?  [ ] yes: ______  [ ] no
  If no: this is a restructuring problem, not a tuning problem. Say so.

  BLOCK I -- THE MANDATORY STATEMENTS (reproduce in full, or do not publish)
   ------------------------------------------------------------------------
-
  [ ] These are essence frequencies, not physical resonances.
  [ ] The twelve states are not diagnoses; this is not an assessment and not
      clinical advice.
  [ ] The reading describes information content under a stated encoding. It
      does not measure distress, well-being, health, or function.
  [ ] Nothing here recommends or values suffering.
  [ ] The intensity scale claim is stated (Ch 6: ratio-scaled up to a common
      free multiplier, to the extent magnitude estimation delivers it), and
      R1's demotion of D/P is noted where the reading routes through D/P.
  [ ] Raw L2 norms published alongside the normalised vectors (Ch 6's
      magnitude reporting rule).
  [ ] Parameters are within-subject indices. FVP-7 NOT RUN.
  [ ] The decomposition is a judgment; a contested decomposition yields a
```

```
      contested frequency.
  [ ] This case is CONSTRUCTED / this reading is of one occasion only.
      (delete whichever does not apply)
   ------------------------------------------------------------------------
-
  Operator signature _______________   Double-coded by _______________
==========================================================================
```

The Failure Modes

FM-11.1 — Running Block G on a degenerate Block C

The failure. The overlap gate fails, the operator notes it, and computes the deficit anyway "for reference."

Why it is wrong. A degenerate `F_int` is `ln N` for everyone, so the present octaves are the maximum and the deficit is as favourable as it can be. **The degenerate reading does not fail loudly; it returns the best possible answer to everyone.** Chapter 9 §9.4 computed the carried Subject's degenerate deficit at -2.6364 octaves — a comfortable pass, obtained by an encoding error.

Repair. Block C2 says STOP and means it.

FM-11.2 — Computing a deficit above the ceiling

The failure. `c/eta = 3.2` produces `f_min = 290.9474 Hz` and a deficit of +2.99 octaves, and the operator sets about raising `u`.

Why it is wrong. `u = 1` gives `F = ln 12 = 2.484907` and `f = 69.6133 Hz` — still below `f_min`. The deficit is real and no arrangement of twelve categories closes it. **A number that implies an intervention which cannot work is worse than no number.**

Repair. G-STOP, checked before G3.

FM-11.3 — Lowering `delta` on a negative numerator

The failure. The R15 reversal, and at internal scope it is the default move.

Why it is wrong. Computed above: -0.4909 to -0.9817 on the carried Subject's failing parameter set. **Exactly twice as bad, from an intervention that felt like steadiness.**

Repair. The sign check in Block G, placed physically above Block H.

FM-11.4 — The `Q`-proxy that measures the input twice

The failure. `Q` is proxied by something that rises whenever a lot happens — a journal that gets longer on busy weeks, an activity count, a self-report of how much one has "grown."

Why it is wrong. Paper E §6.3: the input has been measured twice and **the gauge is void.** Every parameter downstream — `eta` especially — is then estimated against a proxy correlated with its own input, and the estimate is uninterpretable.

Repair. Block F's independence check, run before estimation, not after.

FM-11.5 — The retrospective candidate set

The failure. Estimating `eta` by listing experiences that changed something, and computing the fraction that changed something.

Why it is wrong. It recovers `eta` near 1 by construction. **The denominator was selected on the numerator.**

Repair. Enumerate the candidate set prospectively, at the point of entry, per Book Two Chapter 40.4.

FM-11.6 — The aggregate `eta`

The failure. Reporting `eta = 0.35` and no rungs.

Why it is wrong. `eta` is multilinear over six rungs; a single blocked rung nullifies the rest. **The aggregate is consistent with six mediocre rungs and with one broken rung among five excellent ones, and those two situations call for completely different interventions.** The elasticity is exactly 1 on every rung, so the absolute gain from repair is inversely proportional to the rung's current yield — the worst rung is always the cheapest fix and always the one the aggregate conceals.

Repair. Block F requires every rung and the identification of the worst one.

FM-11.7 — Cross-person comparison

The failure. Two people's `F_int`, `eta`, or deficit compared, ranked, or averaged.

Why it is wrong. **FVP-7 for the self-storage law is NOT RUN.** Whether one person's `eta = 0.35` means what another's does requires the separation property of Book Two Chapter 54, which has not been established. Until then, every personal parameter is a within-subject index and cross-person comparison is not licensed.

Repair. The checkbox in Block F, and the discipline to leave the comparison undone.

FM-11.8 — Two points read as a trend

The failure. A repeat reading moves and the movement is described as a direction.

Why it is wrong. Chapter 10 §10.13: three sources can move a repeat reading — the person, the situation, and the elicitation — and the number separates none of them. **A change from 2.164390 to 1.333662 is equally consistent with a subject who framed a second situation more narrowly, a second situation that admits fewer framings, and a shorter session.**

Repair. Block 0 records the reading type and Block A records the prompt counts. Report all three candidate sources every time.

FM-11.9 — The worksheet as a self-assessment

The failure. A reader runs the sheet on themselves and treats the output as a statement about how they are doing.

Why it is wrong. Every reason in Chapter 10 §10.6, plus one specific to self-administration: **the operator and the subject being the same person defeats the pre-registration gate**, since the person fixing the supports already knows what answer they would like.

Repair. Double-coding, per the line at the foot of the sheet. **Where double-coding is impossible, the reading is exploratory and should be labelled exploratory.**

FM-11.10 — Publishing without Block I

The failure. The reading is copied into a slide, a note, or a message, and the mandatory statements are left behind.

Why it is wrong. **They are the statements that stop the reading from being read as a diagnosis, a resonance claim, or a judgment about a person's life.** They are also the first thing to fall off in transit, which is why they are a checklist rather than a paragraph.

Repair. Block I is a gate: **no Block I, no publication.**

FM-11.11 — Averaging two decompositions

The failure. Two operators code the same framings differently and the sheet is filled in with the mean of their intensity vectors.

Why it is wrong. **The average of two decompositions is a decomposition neither operator would defend, and it corresponds to no reading of the description.** Worse, averaging systematically raises the support size — the mean of two vectors is nonzero wherever either is — which inflates overlap, lowers `u`, and drags `F_int` down by a mechanism that has nothing to do with the subject.

Detection. Any vector on the sheet whose support is the union of two operators' supports.

Repair. §11.14b. Publish both, report the spread, and check whether the sign of the deficit survives.

FM-11.12 — Treating `F_int` as the sustained moment without saying so

The failure. `Q_inf = (eta F − c)/delta` requires `F` to be a *sustained* input `M*`, and the worksheet supplies a single occasion's `F_int` in its place.

Why it is wrong. Book Two Chapter 40's fixed point is the equilibrium of a regime — it assumes `eta`, `delta`, `N` and `C` hold approximately constant over the relevant horizon, and that the input holds at `M*`. **A single reading is one sample of a quantity the equation treats as a level.** Substituting the sample for the level is a modelling choice, and it is a large one.

Detection. Any `Q_inf` on the sheet computed from a single reading with no statement of the substitution.

Repair. Say it in the reading, in the form the Worked Example uses: *with `F_int` treated as the sustained moment.* **The substitution may be reasonable; what is not reasonable is making it silently.** Where more than one occasion has been read, use the mean and report the range.

FM-11.13 — The reading run on a situation that moved

The failure. The subject describes the colleague's message in framing 1, and by framing 3 is describing the whole working relationship.

Why it is wrong. **The angles are then between vectors pointed at different targets**, which Chapter 9 §9.5 identifies as a category error rendered as a number. The overlap gate may still pass — two descriptions of different situations can easily share components — so the arithmetic gives no warning at all.

Detection. Block A1 fixes the situation in the subject's words before elicitation begins. Read it back before each framing.

Repair. Re-elicit against the fixed statement, or run three separate readings and do not combine them.

Worked Example

THE SUBJECT — the worksheet, run end to end

CONSTRUCTED CASE, SPINE §3.1. Not an observation of any person. The storage parameters below are ILLUSTRATIVE WORKSHEET INPUTS chosen to exercise the sheet, not canonical values of the case.

Blocks 0 and A

```
Subject ID .... SPINE-3.1        Date ..... (constructed)
Operator ...... constructed      Period ... one week (declared)
Ladder ........ N = 12   ln N = 2.484907
k ............. 0.483425732988414 Hz     Band ..... 220-440 Hz
Intensity scale ratio-scaled up to a free multiplier (Ch 6 magnitude
                estimation); reading does not route through D/P
Reading type .. first

SITUATION: a message from a colleague that could be read several ways,
           and a decision about how to respond.

Framing 1  threat  -- "they are coming at me"
Framing 2  story   -- "this is the latest instalment of an old pattern"
Framing 3  mirror  -- "what does my reaction say about me"

m = 3 framings, P = 3 pairs
```

Block B — the vectors

```
state:      1    2     3     4     5     6     7     8     9    10    11   12
threat      0    0   0.8     0   0.4     0   0.2     0     0     0     0    0
story       0    0   0.5     0   0.7     0     0     0     0   0.4     0    0
mirror      0    0     0     0   0.3     0   0.8     0     0   0.5     0    0

SUPPORTS FIXED AND RECORDED: YES
norms:  ||threat|| = 0.916515   ||story|| = 0.948683   ||mirror|| = 0.989949

normalised
threat  = {3: 0.872872, 5: 0.436436, 7: 0.218218}
story   = {3: 0.527046, 5: 0.737865, 10: 0.421637}
mirror  = {5: 0.303046, 7: 0.808122, 10: 0.505076}
```

Block C — the gates

```
C1  unit norm ....... PASS  (all 1.000000)
C2  OVERLAP ......... PASS
      threat & story  share {3, 5}
      threat & mirror share {5, 7}
      story  & mirror share {5, 10}
C3  WEIGHT .......... PASS  (threshold stated: 10%)
C4  ROSTER .......... PASS  (m = 3, P = 3)
C5  PRE-REGISTERED .. PASS
```

State 5 — attributed feeling — appears in all three framings. That is what makes them three framings of one situation, and it is why the overlap gate passes on content rather than on arrangement.

Block D — pairwise and the width

```
pair              cos theta       u            F = ln(1+11u)    N_eff
mirror & story     0.436566    0.809410          2.292890       9.9035
mirror & threat    0.308607    0.904762          2.393557      10.9524
story  & threat    0.782074    0.388360          1.662402       5.2720

RESTATEMENT FLAGS: none (minimum u = 0.388360, far above 0.01)

u_int  = (0.809410 + 0.904762 + 0.388360)/3 = 0.700844
N_eff  = 1 + 11 * 0.700844                  = 8.7093
F_int  = ln 8.7093                          = 2.164390 nats
f      = k * 8.7093^2                       = 36.6686 Hz
folded = 293.349 Hz   (band 220-440, j = +3)
octaves present = 2 * 2.164390 / ln 2       = 6.2451
F_int / ln 12   = 2.164390 / 2.484907       = 87.10 %
CEILING CHECK: 2.164390 <= 2.484907          PASS
```

All three pairwise u are published. The mean alone would conceal that threat and story sit

relatively close (0.388360) while mirror stands apart from both.

Block E — the chord

```
STATES SPANNED  {3, 5, 7, 10}    count = 4
PITCH CLASSES   {3, 5, 7}        count = 3

state   raw (Hz)     folded (220-440)      class
  3      4.3508      278.453   j = +6        3
  5     12.0856      386.741   j = +5        5
  7     23.6879      379.006   j = +4        7
 10     48.3426      386.741   j = +3        5

INTERVALS
pair   essence ratio   raw cents   folded cents   HD_ess    collision
3:5       25 / 9        1768.72       568.7174     7.8138       no
3:7       49 / 9        2933.74       533.7418     8.7846       no
3:10     100 / 9        4168.72       568.7174     9.8138       no
5:7       49 / 25       1165.02      1165.0244    10.2586       no
5:10       4 / 1        2400.00         0.0000     2.0000      YES
7:10     100 / 49       1234.98        34.9756    12.2586       no

min HD_ess = 2.0000   mean HD_ess = 8.4882   collisions = 1
50/49 pair present: YES  (7:10 -- the unique instance at N = 12)
```

Four states, three classes. States 5 and 10 share odd part 5, fold to the identical 386.741 Hz, and give the chord's single collision at exactly 0.0000 folded cents. The fourth state adds a pitch and no class. Per Chapter 10 §10.8, that is a fact about the ladder's arithmetic and not a finding about the Subject — relabel state 10 as state 9 and the collision disappears with `F_int` unchanged to twelve decimal places.

Block F — the storage parameters, reading (i)

ILLUSTRATIVE INPUTS.

```
PERIOD ......... one week
Q-PROXY ........ a frozen disposition inventory scored monthly
INDEPENDENCE ... does the proxy rise whenever a lot happens?  NO -> not void

eta, six-rung ladder (illustrative; y6 carries the units):
    y1 = 0.80   y2 = 0.70   y3 = 0.75   y4 = 0.60   y5 = 0.65   y6 = 2.00
    cumulative: 0.800000, 0.560000, 0.420000, 0.252000, 0.163800, 0.327600
    eta = 0.327600 Q-units/nat
    WORST RUNG = y4 at 0.60. Elasticity is 1 on every rung, so repairing
    y4 from 0.60 to 0.80 multiplies eta by 4/3, to 0.436800.

c ............. 0.55 Q-units/week (five ledgers)
delta ......... 0.12 per week
tau = 1/delta . 8.3333 weeks
```

```
WITHIN-SUBJECT ONLY. FVP-7 NOT RUN. Noted.
```

Block G — the reading (i)

```
c/eta = 0.55 / 0.327600 = 1.678877 nats

G-STOP: is 1.678877 > ln 12 = 2.484907 ?   NO -> continue

f_min = k * e^(2 * 1.678877) ............. 13.8862 Hz
octaves required = 2.885 * 1.678877 ..... 4.8436
octaves present  = log2(36.6686/k) ...... 6.2451
-------------------------------------------------
OCTAVE DEFICIT ........................  -1.4015
-------------------------------------------------
                                          SUSTAINS

BREAK-EVEN CHECK: c/eta = 1.678877 < F_int = 2.164390.  Signs agree.

Q_inf = (0.327600 * 2.164390 - 0.55) / 0.12
      = (0.709054 - 0.55) / 0.12 = +1.3255
numerator sign: POSITIVE
periods to within 5% of Q_inf: 24 weeks
headroom h_inf = eta (ln N - F_int) = 0.327600 * 0.320517 = 0.105001
                                                          Q-units/week

(for reference, the y4 repair 0.60 -> 0.80 would give eta = 0.436800,
 c/eta = 1.259158, deficit -2.6124, Q_inf +3.2950 -- a lever exercised
 on a reading that does not need it, shown to calibrate the others)
```

The reading sustains by 1.40 octaves. Block H is not required. And the honest statement of what that means: under these illustrative parameters, with `F_int` treated as the sustained moment, the fixed point of the storage law is positive. It is a statement about a model with declared inputs, and its content is exactly `eta F > c` rewritten in octaves.

Blocks F and G again — reading (ii), a failing parameter set

Same Subject, same chord, same `F_int`. Only the storage parameters differ — a second illustrative set with a worse fourth rung and a higher cost. ILLUSTRATIVE.

```
eta = 0.25   c = 0.60   delta = 0.12
   (the same ladder with y4 = 0.4579 instead of 0.60:
    0.80 * 0.70 * 0.75 * 0.4579 * 0.65 * 2.00 = 0.25)

c/eta = 2.400000 nats
G-STOP: 2.400000 > 2.484907 ?  NO -- inside the ceiling, but only just.

f_min = 58.7413 Hz
octaves required = 6.9240      octaves present = 6.2451
```

```
------------------------------------------------
OCTAVE DEFICIT ..................... +0.6789
------------------------------------------------
                                       SHORT

Q_inf = (0.25 * 2.164390 - 0.60)/0.12 = (0.541098 - 0.60)/0.12 = -0.4909
numerator sign: NEGATIVE
```

Short by 0.68 octaves, and the numerator is negative. Block H applies, and Block H's first line is the R15 check.

Block H — the intervention, and the trap

```
*** R15 CHECK ***  numerator is NEGATIVE. DO NOT LOWER delta.

the tempting move, computed:
    lower delta 0.12 -> 0.06, everything else held
    deficit UNCHANGED at +0.6789   (delta does not appear in the deficit)
    Q_inf  -0.4909 -> -0.9817      EXACTLY TWICE AS BAD

the four levers, each computed (base: deficit +0.6789, Q_inf -0.4909):

LEVER 1  RAISE u  (raise F_int; ceiling ln 12 = 2.484907)
    u = 0.80  F = 2.282382  f =  46.4282  deficit +0.3384   Q_inf -0.2450
    u = 0.85  F = 2.336987  f =  51.7858  deficit +0.1809   Q_inf -0.1313
    u = 0.90  F = 2.388763  f =  57.4358  deficit +0.0315   Q_inf -0.0234
   u = 0.95  F = 2.437990  f =  63.3783  deficit -0.1106   Q_inf +0.0791  OK
    u = 1.00  F = 2.484907  f =  69.6133  deficit -0.2459   Q_inf +0.1769
    exact break-even: u = 0.911198
    -- it closes, but only above u = 0.91, which is near-orthogonality.

LEVER 2  RAISE eta  (the conversion ladder, worst rung first)
    eta = 0.28  c/eta = 2.1429  deficit -0.0630   Q_inf +0.0502  OK
    eta = 0.30  c/eta = 2.0000  deficit -0.4751   Q_inf +0.4110
    eta = 0.32  c/eta = 1.8750  deficit -0.8357   Q_inf +0.7717
    eta = 0.35  c/eta = 1.7143  deficit -1.2994   Q_inf +1.3128
    exact break-even: eta = 0.277214

LEVER 3  CUT c  (the five ledgers)
    c = 0.55  c/eta = 2.2000  deficit +0.1019   Q_inf -0.0742
    c = 0.50  c/eta = 2.0000  deficit -0.4751   Q_inf +0.3425  OK
    c = 0.45  c/eta = 1.8000  deficit -1.0521   Q_inf +0.7591
    c = 0.40  c/eta = 1.6000  deficit -1.6291   Q_inf +1.1758
    exact break-even: c = 0.541097

LEVER 4  RAISE N  (genuinely distinct categories, u held at 0.700844)
    N = 14  ln N 2.639057  F = 2.313621  deficit +0.2483  Q_inf -0.1800
   N = 17  ln N 2.833213  F = 2.502542  deficit -0.2968  Q_inf +0.2136  OK
```

```
    N = 20  ln N 2.995732  F = 2.661380  deficit −0.7551  Q_inf +0.5445

Does any single lever close the deficit?  YES −− all four can, at a price.
Cheapest by the ladder's own arithmetic: raise eta at the worst rung.
    eta 0.25 -> 0.32 is the y4 repair 0.4579 -> 0.5861, a factor of 1.28
    on the single worst rung, with every other rung untouched.
```

Three things in that table deserve comment.

The deficit and `Q_inf` agree in sign, always, and it is an identity rather than a coincidence.

```
Q_inf   = (eta F − c)/delta = (eta/delta) ∗ ( F − c/eta )
deficit = 2.885 ∗ ( c/eta − F )

eta > 0 and delta > 0, so   sign(Q_inf) = − sign(deficit),   exactly.
```

The only way they can disagree is the rounding of `2.885` against `2/ln 2 = 2.885390`, which displaces the crossing by `0.000390 ∗ (c/eta)` octaves — under a thousandth of an octave at any cost ratio this book uses. So the worksheet's break-even check is not a redundant cross-check of an independent quantity; it is a check that the arithmetic was done. **Where the two disagree, the operator has made an error, not a discovery** — unless the reading sits within 0.001 octaves of zero, in which case §11.3 applies and the exact constant must be used.

Every lever closes this deficit, and that is itself the finding. The deficit is only +0.6789 octaves. Compare it with SPINE §3.3's commons case, where the deficit is +6.4638 octaves and no single lever closes it — raising u to 0.95 gets to -0.0332 and only just, raising `eta` to 0.90 still leaves +2.1075, cutting `kappa` to 0.020 still leaves +2.0085. Some systems can be tuned and some must be restructured, and the size of the deficit is what tells you which case you are in. A 0.68-octave gap is a tuning problem. A 6.46-octave gap is not.

Raising `N` from 12 to 17 buys 0.30 octaves and requires five genuinely distinct categories. It also crosses the minimum-gap cliff: Chapter 7 established that `N = 17` is where the folded lattice tightens from 50/49 to 289/288, and the book renders below that cliff by design. Adding categories is a real lever and it has consequences beyond the deficit; a worksheet that reported only the deficit would hide them.

The intervention, in the right order

```
STEP 1  fix the numerator:  eta 0.25 -> 0.32   (repair rung y4)
       deficit +0.6789 -> −0.8357    Q_inf −0.4909 -> +0.7717   ACCUMULATES

STEP 2  NOW lower the loss: delta 0.12 -> 0.06
        deficit unchanged at −0.8357   Q_inf +0.7717 -> +1.5434   BETTER
```

> **FIX THE NUMERATOR, THEN LOWER THE LOSS. REVERSE THE ORDER AND THE SAME INTERVENTION ACCELERATES THE FAILURE.** Same change to `delta`, same magnitude, opposite effect: -0.4909 to -0.9817 before, +0.7717 to +1.5434 after. The intervention did not change. The sign of the numerator did.

Block I — the mandatory statements

```
  [x] These are essence frequencies, not physical resonances. No person,
      tissue, or molecule is claimed to resonate at any value here.
  [x] The twelve states are not diagnoses. This reading is not an assessment
      and not clinical advice.
  [x] The reading describes information content under a stated encoding. It
      does not measure distress, well-being, health, or function, and no
      inference to any of those is supported by the construction.
  [x] Nothing here recommends or values suffering.
  [x] Intensity scale claim stated: ratio-scaled up to a common free
      multiplier per Ch 6, to the extent magnitude estimation delivers it.
      The reading does not route through D/P, so R1's demotion does not bind
      it; where a reading does, the demotion travels with it.
  [x] Raw L2 norms published with the normalised vectors.
  [x] eta, c and delta are WITHIN-SUBJECT indices. FVP-7 NOT RUN. No
      cross-person comparison is licensed.
  [x] The decomposition is a judgment. A contested decomposition yields a
      contested frequency.
  [x] This case is CONSTRUCTED and this reading is of one occasion only.
```

The completed reading

```
================================================================================
  PERSONAL SPECTRUM -- THE SUBJECT  (CONSTRUCTED, SPINE 3.1)
================================================================================
  ladder        N = 12, ln N = 2.484907, k = 0.483425732988414 Hz
  band          220-440 Hz
  situation     one ambiguous message; three framings; one occasion

  GATES         norm PASS | overlap PASS | weight PASS | roster PASS
                pre-registration PASS | ceiling PASS

  PAIRWISE u    0.809410  /  0.904762  /  0.388360     no restatements
  u_int         0.700844
  N_eff         8.7093        (72.58% of the roster)
  F_int         2.164390 nats (87.10% of the ln 12 ceiling)
  f             36.6686 Hz    folded 293.349 Hz (j = +3)
  octaves       6.2451 present

  CHORD         states {3, 5, 7, 10}  -- four
                classes {3, 5, 7}     -- three (5 and 10 share class 5)
                raw    4.3508 / 12.0856 / 23.6879 / 48.3426 Hz
                folded 278.453 / 386.741 / 379.006 / 386.741 Hz
                min HD_ess 2.0000 (5:10)  mean 8.4882  collisions 1
                50/49 pair 7:10 present at 34.9756 cents

  READING (i)   eta 0.327600  c 0.55  delta 0.12   [ILLUSTRATIVE]
```

```
                c/eta 1.678877 < ln 12 -- inside the ceiling
                f_min 13.8862 Hz  required 4.8436  present 6.2451
                DEFICIT -1.4015   SUSTAINS   Q_inf +1.3255   tau 8.33 weeks

  READING (ii)  eta 0.25  c 0.60  delta 0.12   [ILLUSTRATIVE]
                c/eta 2.4000 < ln 12 -- inside the ceiling, only just
                f_min 58.7413 Hz  required 6.9240  present 6.2451
                DEFICIT +0.6789   SHORT      Q_inf -0.4909
                R15: lowering delta alone -> Q_inf -0.9817, WORSE
                correct order: eta 0.25 -> 0.32 first (deficit -0.8357,
                Q_inf +0.7717), THEN delta 0.12 -> 0.06 (Q_inf +1.5434)

  All eight mandatory statements of Block I attached in full.
================================================================================
```

What the worksheet is for

It is for producing a number that can be checked and an inference that can be refused.

Every quantity on the sheet has a stated provenance, every gate has a stated threshold, and every mandatory statement names something the number does not say. **A reader who runs the sheet and gets a deficit has a computable, actionable reading of one set of framings on one occasion — and a list, attached, of every conclusion the reading does not license.**

That list is longer than the reading. **In an instrument with no criterion validity, no clinical validation, an ordinal scale under a standing demotion, a postulated reference constant, and a reproducibility gate recorded as NOT RUN, the list being longer than the reading is the correct proportion.**

What the Subject's reading actually tells an operator

Stripped of the machinery, the carried Subject's sheet says four things, and it is worth setting them down in ordinary language so that the reader can see how modest they are.

One: the three framings elicited on this situation were substantially different from each other. The closest pair, story against threat, still sits at `u = 0.388360`; the widest, mirror against threat, at `0.904762`. No pair was a restatement. That is a fact about what the elicitation produced, and Chapter 10's separation statement forbids reading anything about the Subject's condition into it.

Two: those framings together require four of the twelve components, and two of the four sit in the same pitch class. States 5 and 10 differ by a factor of two, so the chord occupies three positions in a six-position vocabulary. That is a fact about the ladder's arithmetic — Chapter 10 §10.8 establishes it, and the relabelling test shows the collision moving while `F_int` does not.

Three: at the illustrative parameters of reading (i), the storage law's fixed point is positive. The deficit is -1.4015 octaves; `Q_inf` is +1.3255; the regime arrives within about twenty-four weekly periods. All of which is `eta  F  >  c` in a legible register, computed from numbers the operator chose and declared.

Four: at the illustrative parameters of reading (ii), it is not, and the correct repair is to raise `eta` at its worst rung before touching `delta` — because the numerator is negative and lowering the loss doubles a loss. That is R15, and it is the only line on the whole sheet that tells an operator what to do rather than what is the case.

Nothing on the sheet says whether the Subject should reply to the colleague, how they are, or what kind of person they are. **The instrument was not built to answer those and the worksheet's last block exists to say so in writing.**

Chapter 11 owns the full personal worksheet; Chapter 24 cites it. The elicitation of intensities is Chapter 6; the fold and the odd-part rule are Chapter 7; `delta` and traits are Chapter 8; the chord, the gates, and the one-hot trap are Chapter 9; the consonance/redundancy identity and the separation statement are Chapter 10; `f_min`, the octave deficit, and the four levers are Chapter 4; R15 is Chapter 15; the self-storage law and the six-rung conversion ladder are Book Two Chapter 40. Part III opens by taking the same instrument outward, to two people and one interval.

Part III — Frequency Mechanics of Relationship

Chapter 12 . The Dyad as an Interval

Part II described one person. It gave a mood a pitch, a trait a pitch, and a person a chord, and it did all of that inside a single skull. The instrument was Chapter 9's internal surplus: the width of the set of framings one observer holds at once.

Part III turns the same instrument outward. Two people are not two chords to be compared. Two people, considered as a relation, are one interval — a single ratio between two positions, and therefore a single pitch on this book's scale, not a pair of pitches.

That sentence is the whole content of the chapter, and the rest of it is the arithmetic that makes it exact, the sweep that makes it audible, and the failure modes that make it survivable.

This chapter owns one formula:

```
THE RELATIONSHIP FREQUENCY

    f_rel  =  k * e^( 2 F(u) )  =  k * N_eff^2  =  k * ( 1 + (N-1) u )^2

   with   u     = 1 - (x . y)^2          the non-parallelism of the pair
      F(u)  = ln( 1 + (N-1) u )     the pairwise surplus (Paper A, Thm 2.1)
          N_eff = 1 + (N-1) u             the audited perspective count
       k    = 0.483425732988414 Hz   the ground essence, a chosen constant

  at N = 12:    f_rel = k (1 + 11 u)^2      0.4834 Hz <= f_rel <= 69.6133 Hz
```

Every chapter from here to Chapter 18 uses this expression and none of them re-derives it. Chapter 13 owns how u is measured between two people. Chapter 14 owns what happens at the bottom of the range. Chapter 15 owns the sustainability reading. Chapter 16 extends the pair to a set. This chapter owns the object itself: what it is, what its bounds are, how it moves, and what it sounds like as u runs from nothing to everything.

Three constraints govern the chapter before it begins, and they are not decorations.

Essence, not resonance. Every frequency here is an essence frequency — a position on a logarithmic scale of surplus, in units chosen by the author. No person, no pair of people, no tissue and no molecule is claimed to vibrate at any value in this chapter.

Not counselling, not therapy, not assessment. A relationship frequency is a number computed from a representation of two people's states. **It is not a clinical, diagnostic or therapeutic statement about anyone, this book does not offer counselling or therapy of any kind, and nothing here should be used in place of either.** The twelve states are not diagnoses and a low reading is not a finding about a person.

Scale status, stated once here and in full in Chapter 13. The state frequency `f = k n^2` is ratio-scaled because n is a count (Chapter 5). The relationship frequency inherits the scale status of the elicited intensities, and is ratio-scaled only when those intensities have been magnitude-estimated under Chapter 6's rule. Chapter 13 carries that argument at length because Chapter 13 owns the measurement. This chapter carries it as a standing qualifier on every number it prints.

All cases in this chapter are CONSTRUCTED and labelled as such. Nothing here is an observation of any person or any pair of people.

The Law

12.1 What a dyad is in this system

A dyad is two represented states and the geometry between them.

Each party is represented exactly as Chapter 6 represents a framing: a non-negative vector over the twelve state classes, whose entries are elicited intensities, normalised to unit length in the L2 norm. The construction does not change when the two vectors belong to two people instead of to one person's two framings. That is not a convenience; it is what Paper A's axioms permit. Chapter 9 made the argument in full: S1 through S4 quantify over states in an inner-product space, and not one of them contains a term referring to persons, agents, or the ownership of a state. The theorem is silent on who holds the vectors.

So the object is:

```
THE DYAD

    x, y      unit vectors over the twelve state classes, entries >= 0
    cos theta = x . y
    u         = 1 − (x . y)^2  =  sin^2 theta        in [0, 1]

  u is the pair's NON-PARALLELISM. It is not a disagreement rate, not a distance,
   and not a measure of how well the two people get along.
```

The last line is the one that has to be said early and repeated. u is a geometric quantity. It answers one question — *how much of what each party carries is not already carried by the other* — and it answers no other question at all. Chapter 14 will show exactly how badly this can be misread; Chapter 13 will show how it is measured without being invented.

Two immediate consequences, both exact.

A dyad has one frequency, not two. There is no "A's frequency in the relationship" and "B's frequency in the relationship." u is symmetric: `u(x,y) = u(y,x)`, because the inner product is. The relation has a single pitch and both parties are inside it.

A dyad's frequency is not a function of either party alone. Two people can each hold a wide personal chord and still produce a near-zero u between them if their chords are the same chord. Chapter 9's `F_int` and this chapter's `F` are different quantities computed from different vectors, and a high one does not imply a high other. **A person is a chord; a pair is an interval; nothing in the formalism converts one into the other.**

12.2 The relationship frequency, derived

THEOREM 12.1 — THE RELATIONSHIP FREQUENCY. For a dyad with non-parallelism u over an N-category system,

```
f_rel  =  k ∗ ( 1 + (N−1) u )^2
```

Proof. Three steps, each established earlier and each cited rather than reproved.

Step 1 — the surplus. Paper A, Theorem 2.1, restated in Chapter 2 §2.5: under axioms S1–S4 the pairwise surplus of two represented states with non-parallelism u is

```
F(u) = ln( 1 + (N−1) u )
```

with `N_eff = 1 + (N−1)u` the audited perspective count. S3 — that `g(u) = e^{F(u)}` is affine in u — is the **design axiom**, chosen and badged as chosen. It is not derived from anything and the book has never claimed otherwise.

Step 2 — the bridge. Book Two Chapter 30, restated in Chapter 2 §2.1: the essence bridge maps a surplus in nats to a frequency,

```
f = k ∗ e^( 2 F )                k = (9/8)^2 / phi^2 = 0.483425732988414 Hz
```

k is a **chosen reference constant [POSTULATE]**, a Pythagorean ditone divided by the golden section. It sets the reference pitch and nothing else.

Step 3 — substitute.

```
f_rel = k e^{2 ln(1 + (N−1)u)} = k ( e^{ln(1 + (N−1)u)} )^2 = k (1 + (N−1)u)^2   []
```

At `N = 12` the working form is `f_rel = k (1 + 11u)^2`. Verified against the SPINE numbers at the carried dyad: `u = 0.068734665264634` gives `F = 0.563084802746` and `f = 1.4907987150 Hz`, which is SPINE §3.2 to every published digit.

The theorem is a substitution and nothing more. It introduces no new assumption. Every objection available against it is an objection against S3 or against k, both of which are badged as postulates in the sentence that states them, and neither of which this chapter improves.

12.3 The bounds, the monotonicity, and what they mean

COROLLARY 12.2 — THE RANGE. `f_rel` is a strictly increasing function of u on `[0,1]`, with

```
u = 0   ->   f_rel = k          = 0.4834 Hz      the ground essence
u = 1   ->   f_rel = k N^2      = 69.6133 Hz     the ceiling, at N = 12
```

Proof. `1 + (N−1)u` is strictly increasing and positive; squaring preserves that; `k > 0` scales it. QED.

Three readings of the same corollary.

The floor is not zero, it is k. A dyad with `u = 0` does not vanish from the scale. It sits at the ground essence, exactly where a single undifferentiated state sits, exactly where a person whose framings are all the same framing sits (Chapter 10 §10.5). The bottom of the relational scale and the bottom of the personal scale are the same note, and that coincidence is not an accident of

notation — it is the statement that a relation with no surplus is indistinguishable, in this formalism, from no relation. Chapter 14 is built on that sentence.

The ceiling is the same ceiling. `k N^2 = 69.6133 Hz` at `N = 12` is the maximum for a dyad, the maximum for a chord, and the maximum for a commons of six hundred. It is set by the category count and by nothing else. Two people cannot exceed twelve categories' worth of distinction by trying harder.

Monotonicity is what licenses the word "higher." Because `f_rel` is strictly increasing in `u`, the sentence *this pair is at a higher pitch than that pair* means exactly *this pair carries more surplus than that pair*, with no interpretive step in between. **A dyad's pitch IS its surplus, monotonically, and higher pitch means more available order** — which is the plainest statement this book makes and the one Chapter 13 will hand to a practitioner.

What it does not mean: that the pair is happier, closer, healthier, better matched, more compatible, or more likely to persist. `f_rel` measures available order. Available order is not wellbeing and this book never converts one into the other.

12.4 A dyad is an interval, and the interval is measured from the ground

The chapter's title is a claim, and here it is as arithmetic.

An interval, in any tuning system, is a ratio of two frequencies expressed logarithmically. The dyad's ratio is `f_rel / k`, and its logarithm is:

```
THE DYAD AS AN INTERVAL ABOVE THE GROUND ESSENCE

    octaves  =  log2( f_rel / k )  =  2 log2( 1 + (N−1) u )
    cents    =  1200 ∗ octaves     =  2400 ∗ log2( 1 + (N−1) u )
    nats     =  F(u)               =  ln( 1 + (N−1) u )

  and the exchange rate, exact:  1 nat = 2.885390081777927 octaves = 3462.4681 cents
```

The pair's whole reading is one interval measured upward from a fixed reference, and the reference is the note a pair with nothing between them would sound. That is why the constant k is harmless: it is the origin of the interval, it appears in both terms of every comparison, and Chapter 4 §4.5 showed it cancels out of the deficit entirely.

The maximum interval available is:

```
at N = 12:  2400 ∗ log2(12) = 8603.9145 cents = 7.169925 octaves = 2.484907 nats
```

Seven octaves and two cents shy of a minor third. That is the entire relational range of a twelve-category system, and every dyad this book reads is a position inside it.

Computed at the carried case and its two companions:

```
 THREE POSITIONS INSIDE THE 8603.91−CENT RANGE

case                    u             f_rel (Hz)   cents above k   % of range
dyad, as found (2a)    0.068734665264634   1.4908      1949.66       22.66%
dyad, after (2b)       0.692372881355932  35.8882      7456.88       86.67%
the commons (Ch 17)     0.018614           0.7017       644.97        7.50%
```

The last column is a quantity the book will use repeatedly, so it gets a name here.

DEFINITION 12.3 — THE REALISED PITCH FRACTION. `F(u) / ln N`, the fraction of the system's maximum surplus a relation actually realises. Equivalently `log2(f_rel/k) / log2(N^2)`, the fraction of the available interval the relation occupies. It is dimensionless, it is invariant to `k`, and it is comparable across relations of different sizes.

The commons realises 7.50% of the range it has categories for. The dyad as found realises 22.66%. The same dyad after one intervention realises 86.67%. Those three numbers say more, faster, than the hertz do, and they are the same information.

12.5 The eleven coincidences — where a dyad sounds exactly like a state

Here is a fact about the twelve-state system that falls straight out of Theorem 12.1 and is worth having explicitly, because it links Rule 1 and Rule 2 of CANON3 §2 at eleven exact points.

COROLLARY 12.4. `f_rel(u) = k m^2` — the essence frequency of state `m` — exactly when

```
u = (m − 1) / (N − 1)        i.e. at N = 12:   u = (m−1)/11
```

Proof. `k(1+11u)^2 = k m^2` iff `1 + 11u = m` (both sides positive) iff `u = (m−1)/11`. QED.

Computed for all twelve:

m	u = (m−1)/11	f_rel (Hz)	folded 220–440	pitch class
1	0.000000000	0.4834	247.514 (j=9)	1
2	0.090909091	1.9337	247.514 (j=7)	1
3	0.181818182	4.3508	278.453 (j=6)	3
4	0.272727273	7.7348	247.514 (j=5)	1
5	0.363636364	12.0856	386.741 (j=5)	5
6	0.454545455	17.4033	278.453 (j=4)	3
7	0.545454545	23.6879	379.006 (j=4)	7
8	0.636363636	30.9392	247.514 (j=3)	1
9	0.727272727	39.1575	313.260 (j=3)	9
10	0.818181818	48.3426	386.741 (j=3)	5
11	0.909090909	58.4945	233.978 (j=2)	11
12	1.000000000	69.6133	278.453 (j=2)	3

A dyad whose non-parallelism is exactly 7/11 sounds the pitch of state 8 — social feeling, the state that adds another mind modelled. The coincidence is arithmetic, not semantic, and the book attaches no meaning to it. What it is good for is calibration: a practitioner who knows the twelve-state spectrum by ear already knows the relational scale, because the two scales share eleven of their points.

The honest caveat, and it must travel with the table: this is a numerical coincidence in the chosen constants, not a claim that a dyad at `u = 7/11` is in any sense "in" state 8. It is not. The two quantities have different arguments, different derivations, and different scale status. They land on the same number and mean different things.

12.6 Hertz and pitch disagree about which unit of u is worth most

This is the subtlest thing in the chapter and the easiest to get backwards.

`F(u) = ln(1 + 11u)` is strictly concave in `u` — Paper A's concavity, which drives everything in Book Two Chapter 59. But `f_rel(u) = k(1 + 11u)^2` is a quadratic with positive leading coefficient, and is therefore strictly convex in `u`. Both are true. They are statements about different functions.

```
   df/du    = 2 (N−1) k (1 + (N−1)u)        INCREASING in u      Hz per unit u
  dcents/du = 2400 (N−1) / (ln 2 (1 + (N−1)u))  DECREASING in u    cents per unit u
```

Computed at `N = 12`:

```
     u        df/du (Hz per unit u)     dcents/du (cents per unit u)
   0.0             10.6354                    38087.15
   0.2             34.0332                    11902.23
   0.5             69.1299                     5859.56
   0.8            104.2266                     3886.44
   1.0            127.6244                     3173.93
```

And at the two ends of the range, in both units at once:

```
  the FIRST 0.05 of u:   + 0.6780 Hz    + 1517.44 cents
  the LAST  0.05 of u:   + 6.2350 Hz    +  162.45 cents

  ratio in Hz:     the last step is 9.20x the first
  ratio in cents:  the first step is 9.34x the last
```

The same two steps: one of them is nine times larger measured in hertz and one ninth the size measured in cents. Nothing is wrong. Hertz is a linear scale and pitch is a logarithmic one, and the difference between them is the entire reason this book works in intervals.

Which unit is operative is not a matter of taste. The sustainability reading of Chapter 4 is computed in log units — `D = 2.885390(c/eta) − log2(f/k)` — so **pitch is the unit that decides whether a relation accumulates or decays, and hertz differences are decorative.** A practitioner who reports "we gained six hertz" has reported the number that does not enter the verdict.

The concave form is also the one that matches the older statement. Book Two's Theorem 59.1 — splitting an input across occasions strictly dominates delivering it at once — is F's concavity, and its relational reading is here: the first increment of difference between two people is worth more than any later increment, and by a factor this chapter can name.

12.7 The honesty clauses, restated in place

(1) ESSENCE, NOT RESONANCE. `f_rel` is a position on a logarithmic surplus scale expressed in hertz. Nothing is claimed to vibrate. No pair of people has a physical resonance at 1.4908 Hz or at any other value in this chapter.

(2) NOT CLINICAL, NOT COUNSELLING, NOT THERAPY. The twelve states are not diagnoses; a relationship frequency is not an assessment of a relationship; **this book is not counselling or therapy and does not substitute for either.** Nothing here evaluates, screens, or prescribes for any person or any pair.

(3) THE READING INHERITS ITS INPUTS. `f_rel` is exactly as defensible as `u`, and `u` is exactly as defensible as the two vectors it came from. A contested decomposition yields a contested frequency, and the decomposition is published with the frequency or neither is published.

(4) SCALE STATUS. `f_rel` is ratio-scaled only if the intensities were magnitude-estimated (Chapter 6 §6.3, Chapter 13 §13.3). **The ordinal fallback does not exist: with ordinal intensities `f_rel` is not determined at all.** Chapter 13 carries the number that proves it.

(5) SURPLUS IS A LOWER BOUND. `F` is a floor on the pair's information, not the information (Chapter 2 §2.6). Therefore `f_rel` is a floor on the pitch. A verdict of ACCUMULATES is robust to representational under-capture; a verdict of DECAYS is not.

(6) S3 IS CHOSEN. The affine axiom is a design decision. Every number in this chapter rests on it and it is badged wherever it appears.

(7) CONSTRUCTED CASES. Every case here is constructed and labelled. No number in this chapter is an observation of any real pair of people.

The Engineering

12.8 The glide, computed

`u` runs from 0 to 1. The corresponding pitch runs from `k` to `k N^2` — seven octaves and change — and it runs monotonically, so the whole relational scale can be exhibited as a single continuous sweep. This is the most useful single object in the chapter for anyone trying to develop an intuition for what a relationship frequency means.

THE GLIDE — `f_rel` over the full range of `u`, at `N` = 12, `k` = 0.483425732988414 Hz

u	N_eff	F (nats)	f_rel (Hz)	oct above k	cents above k	folded (j)
0.00	1.0000	0.000000	0.4834	0.0000	0.00	247.514 (j=9)
0.05	1.5500	0.438255	1.1614	1.2645	1517.44	297.326 (j=8)
0.10	2.1000	0.741937	2.1319	2.1408	2568.93	272.884 (j=7)
0.15	2.6500	0.974560	3.3949	2.8120	3374.38	434.542 (j=7)
0.20	3.2000	1.163151	4.9503	3.3561	4027.37	316.818 (j=6)
0.25	3.7500	1.321756	6.7982	3.8138	4576.54	435.083 (j=6)
0.30	4.3000	1.458615	8.9385	4.2087	5050.41	286.033 (j=5)
0.35	4.8500	1.578979	11.3714	4.5560	5467.16	363.884 (j=5)
0.40	5.4000	1.686399	14.0967	4.8659	5839.10	225.547 (j=4)
0.45	5.9500	1.783391	17.1145	5.1458	6174.94	273.832 (j=4)
0.50	6.5000	1.871802	20.4247	5.4009	6481.06	326.796 (j=4)
0.55	7.0500	1.953028	24.0275	5.6352	6762.30	384.439 (j=4)
0.60	7.6000	2.028148	27.9227	5.8520	7022.40	223.381 (j=3)
0.65	8.1500	2.098018	32.1103	6.0536	7264.32	256.883 (j=3)
0.70	8.7000	2.163323	36.5905	6.2420	7490.44	292.724 (j=3)
0.75	9.2500	2.224624	41.3631	6.4189	7702.69	330.905 (j=3)
0.80	9.8000	2.282382	46.4282	6.5856	7902.68	371.426 (j=3)
0.85	10.3500	2.336987	51.7858	6.7431	8091.74	414.286 (j=3)
0.90	10.9000	2.388763	57.4358	6.8925	8271.01	229.743 (j=2)
0.95	11.4500	2.437990	63.3783	7.0346	8441.46	253.513 (j=2)
1.00	12.0000	2.484907	69.6133	7.1699	8603.91	278.453 (j=2)

Three features of the table are worth naming because a reader will otherwise draw the wrong picture.

The bottom of the range is enormously steep. From u = 0 to u = 0.05 — a change most practitioners would call negligible — the pitch rises 1517.44 cents, a minor tenth. A quarter of the entire relational range is spent by the time u reaches 0.05. Precisely: 1517.44 / 8603.91 = 17.64% of the range in the first 5% of u, and half the range is gone by:

```
      halfway in pitch:   (1 + 11u)^2 = N = 12   ->   u = (sqrt(12) -
1)/11 = 0.224009237739796
                f_rel = 5.8011 Hz,  4301.9550 cents = exactly half of 8603.9145
```

Half the relational range of a twelve-category system is consumed by the time two people are 22.4% non-parallel.

The top of the range is nearly flat in pitch. From u = 0.90 to u = 1.00 the pitch rises 332.90 cents — less than a minor third for the last tenth of the geometry. Chapter 4 §4.13 priced the same fact from the other side: above u = 0.680480 no further octave can be bought from u at all, because 1 + 11u cannot be multiplied by sqrt(2) again inside the bound.

The folded column wanders and that is expected. Folding into 220–440 Hz is a modular operation; a monotone raw sweep becomes a sawtooth in the folded band, crossing the band boundary eight times between u = 0 and u = 1. §12.10 says what to do about it.

12.9 The step table, and the concavity as a price list

The same glide, read as increments:

```
    CENTS PURCHASED BY EACH 0.05 OF u

      0.00 -> 0.05    1517.44        0.50 -> 0.55     281.24
      0.05 -> 0.10    1051.49        0.55 -> 0.60     260.10
      0.10 -> 0.15     805.45        0.60 -> 0.65     241.92
      0.15 -> 0.20     652.99        0.65 -> 0.70     226.12
      0.20 -> 0.25     549.16        0.70 -> 0.75     212.25
      0.25 -> 0.30     473.87        0.75 -> 0.80     199.99
      0.30 -> 0.35     416.76        0.80 -> 0.85     189.07
      0.35 -> 0.40     371.94        0.85 -> 0.90     179.27
      0.40 -> 0.45     335.83        0.90 -> 0.95     170.45
      0.45 -> 0.50     306.12        0.95 -> 1.00     162.45
```

Twenty equal steps in the geometry; the first is worth 9.34 times the last in pitch. This is Chapter 4 §4.14's lever price seen as a table rather than a derivative, and it has one practical consequence that is worth stating in the plainest terms available.

In a relation with very little difference between the parties, a very small amount of real difference is worth an enormous amount of pitch. In a relation that is already wide, a great deal more difference is worth very little. The carried dyad is at u = 0.068735, where the marginal rate is 21688.72 cents per unit u; the same dyad after intervention is at u = 0.692373, where the rate is 4420.46 cents per unit — a factor of 4.91 lower. The lever that fixed the case is already most of the way to being spent, and the reading says so before anyone has to discover it.

The reciprocal price list is equally useful and is the one to hand to someone planning an intervention. Equal steps in pitch cost accelerating amounts of u:

```
STARTING FROM THE CARRIED DYAD, u = 0.068734665264634

target       u required      f_rel (Hz)     du from previous
+600 cents   0.098940          2.1083         0.030205
+1200        0.134861          2.9816         0.035921
+1800        0.177579          4.2166         0.042718
+2400        0.228378          5.9632         0.050799
+3000        0.288790          8.4332         0.060412
+3600        0.360632         11.9264         0.071842
+4200        0.446066         16.8665         0.085434
+4800        0.547666         23.8528         0.101600
```

Each further 600 cents costs about 19% more u than the one before it, all the way up. The growth factor is exactly `sqrt(2)^(1/2) = 2^(1/4) = 1.189207` per 600-cent step, because `1 + 11u` must be multiplied by `2^(1/4)` each time and the additive part of u therefore scales by the same factor. That is a clean planning number and it is exact.

12.10 Rendering the glide — fixed transposition, not the standing fold

Chapter 7 §7.2 fixed the house folding convention: transpose by an integer number of octaves into 220–440 Hz, and publish the `j`. That convention is right for a point reading and wrong for a sweep, for a reason the glide table shows directly: a monotone sweep folded into a one-octave band is discontinuous, and the ear hears a sawtooth where the arithmetic has a ramp.

SPEC 12.1 — RENDERING A SWEEP. For any continuous sweep of u, use a single fixed transposition `j` for the whole sweep, chosen once and published, and do not re-fold. The whole relational range fits comfortably in the audible band under one transposition:

```
j      bottom (u=0)      top (u=1)         fits 20 Hz – 20 kHz?
5        15.4696 Hz       2227.6258 Hz      bottom below 20 Hz -- no
6        30.9392 Hz       4455.2516 Hz      YES
7        61.8785 Hz       8910.5031 Hz      YES, but top is shrill
```

The house glide is `j = 6`: 30.9392 Hz at `u = 0`, rising monotonically to 4455.2516 Hz at `u = 1`, a continuous 7.1699-octave sweep with no wrap. This is a rendering choice and it is arbitrary in exactly the way the 220–440 band is arbitrary; it is stated so that two renderings can be compared.

Two properties of the choice are not arbitrary. Transposition by `2^j` preserves every interval in the sweep exactly — the ratio between any two points is untouched, so the glide's shape is invariant to `j`. And the sweep's total extent, 7.1699 octaves, is a property of `N` and not of the rendering; a system with `N = 25` would sweep 9.2877 octaves and would no longer fit under a single transposition, which is CANON3 §5's `N <= 25` capacity limit arriving in this chapter's register.

And the standing prohibition applies to rendering as it applies to everything else: nothing is masked, nothing sits below threshold, nothing is concealed. A glide is played openly or not at all, and a listener who wants it stopped may stop it.

12.11 Named intervals and the u that produces them

For a reader who thinks in intervals rather than in numbers, the inverse of Theorem 12.1 is:

```
u  =  ( sqrt( f_rel / k ) − 1 ) / (N − 1)
```

which answers *how much non-parallelism does it take to sound a fifth above the ground?* Computed:

```
interval above k         ratio       u required       f_rel (Hz)     cents
unison                 1.00000     0.000000000        0.4834         0.00
minor third   6/5      1.20000     0.008676829        0.5801       315.64
major third   5/4      1.25000     0.010730363        0.6043       386.31
perfect fourth 4/3     1.33333     0.014063685        0.6446       498.04
perfect fifth  3/2     1.50000     0.020431352        0.7251       701.96
octave        2/1      2.00000     0.037655778        0.9669      1200.00
octave + fifth 3/1     3.00000     0.066550073        1.4503      1901.96
two octaves    4/1     4.00000     0.090909091        1.9337      2400.00
three octaves  8/1     8.00000     0.166220648        3.8674      3600.00
four octaves  16/1     16.00000    0.272727273        7.7348      4800.00
five octaves  32/1     32.00000    0.423350386       15.4696      6000.00
six octaves   64/1     64.00000    0.636363636       30.9392      7200.00
seven octaves 128/1   128.00000    0.937609864       61.8785      8400.00
```

A perfect fifth above the ground essence costs `u = 0.020431` — two percent non-parallelism. A practitioner's first reaction to that number is usually that it is too small, and the reaction is worth sitting with: the scale is logarithmic, the bottom of it is very steep, and two percent of represented difference between two people really is a large amount of surplus relative to none at all. The eighth line of the table is the calibration point that fixes the intuition: **two full octaves — a quadrupling of the frequency — cost exactly `u = 1/11 = 0.090909`.**

The last line is the one to notice for a different reason. Seven octaves costs `u = 0.937610`, and there is no eighth octave: `u = 1` gives 8603.91 cents, which is 203.91 cents short of eight. The range is not a whole number of octaves and never will be, because `2 log2 12 = 7.169925` and 12 is not a power of two.

12.12 What an error in u costs, in cents

u is elicited, and elicited quantities have error. Because the pitch scale is steepest where u is smallest, the same absolute uncertainty in u is worth wildly different amounts of pitch depending on where the reading sits. Computed:

```
SPREAD OF THE READING UNDER SYMMETRIC UNCERTAINTY IN u

 u          +/− 0.01                          +/− 0.05
0.018614    0.5794 .. 0.8356 Hz   634.05c   0.4834 .. 1.4885 Hz  1947.05c
0.068735    1.3099 .. 1.6834 Hz   434.34c   0.7032 .. 2.5709 Hz  2244.28c
0.200000    4.6158 .. 5.2965 Hz   238.14c   3.3949 .. 6.7982 Hz  1202.16c
0.500000   19.7393 .. 21.1219 Hz  117.20c  17.1145 .. 24.0275 Hz  587.36c
0.692373   34.9777 .. 36.8104 Hz   88.41c  31.4526 .. 40.6162 Hz  442.65c
0.900000   56.2824 .. 58.6009 Hz   69.89c  51.7858 .. 63.3783 Hz  349.72c
```

At `u = 0.0687` a hundredth of uncertainty in the geometry is 434 cents of uncertainty in the pitch — a major third of ambiguity from an error most elicitation procedures would not even detect. At `u = 0.9` the same hundredth is 70 cents.

Inverted, the requirement is severe at the bottom:

```
du required for a 5-cent change in the reading

  at u = 0.018614      du = 0.00015816
  at u = 0.068735      du = 0.00023053
  at u = 0.300000      du = 0.00056449
  at u = 0.692373      du = 0.00113110
  at u = 1.000000      du = 0.00157533
```

A near-parallel dyad is not measurable to five cents by any elicitation procedure this book knows of, and the honest response is to report the reading with its interval of uncertainty rather than to report a number to four decimal places. SPEC 12.4 requires exactly that. Chapter 13 §13.11 shows what a single elicited intensity does to a real reading, and the answer is: a great deal.

12.13 Comparing two dyads

Two dyads compare as an interval, and intervals compose additively in cents:

```
interval( u1 -> u2 )  =  2400 * log2( (1 + 11 u2) / (1 + 11 u1) )
```

Verified on the carried case, decomposed through an intermediate point:

```
u = 0.068734665264634  ->  0.30                  3100.7448 cents
u = 0.30               ->  0.692372881355932     2406.4767 cents
sum                                              5507.2215 cents
direct  0.068734665264634 -> 0.692372881355932   5507.2215 cents
difference                                       9.09e-13 cents
```

The carried dyad's intervention is worth 5507.22 cents — four octaves and a major sixth. That is the number Chapter 15 will convert into a sustainability verdict and Chapter 18 will price as work.

Differences in u do not carry this property in any useful sense. The two sub-steps above are `du = 0.231265` and `du = 0.392373`; the larger step in u is the smaller step in pitch. u is the argument, pitch is the reading, and only one of them composes the way a practitioner's intuition expects.

12.14 The reading a dyad frequency does and does not support

Because this is the first chapter in the book to produce a number about two people, it is the right place to fix what such a number licenses.

```
A RELATIONSHIP FREQUENCY LICENSES:
  - comparison with f_min at the same c/eta       (Chapter 4, Chapter 15)
- comparison with the same pair at another time  (same rule set, same ladder)
- comparison with the ceiling k N^2          (the realised pitch fraction)
  - a statement about how much distinct order the pair has available
```

```
IT DOES NOT LICENSE:
  - comparison with a pair read under a different ladder or a different N
  - any statement about either party as an individual
- any statement about the pair's wellbeing, closeness, satisfaction or prospects
  - any clinical, diagnostic or therapeutic conclusion whatsoever
- any action taken about a person without that person's ability to decline it
```

The fourth prohibition is not a disclaimer bolted to the end; it is the reason the third exists. A number that measures available order will be read as a number that measures the relationship, and the distance between those two things is not one this book knows how to cross.

The Specification

SPEC-12.1 — Inputs

```
REQUIRED BEFORE ANY f_rel IS COMPUTED

[ ] the declared rule set: decomposition rules, N = 12, the ladder, dated  (Ch 6 Step 1)
  [ ] two state vectors x, y over the twelve classes, entries >= 0
[ ] both raw L2 norms ||x||_2 and ||y||_2, in the instrument's units      (Ch 6 §6.4)
[ ] confirmation that intensities were MAGNITUDE-ESTIMATED, not ranked    (Ch 6 §6.9 Ste
  [ ] consent recorded from both parties, decline available at every step
[ ] the support overlap check, passed                                    (Ch 9 §9.5)
```

A reading missing any one of these six is not a reading and is not published. The norms are on the list because normalisation discards them and they are half the measurement (Chapter 6's magnitude reporting rule); the magnitude-estimation confirmation is on the list because without it the number is not determined at all (Chapter 13 §13.3).

SPEC-12.2 — The computation

```
 1.  normalise:      x <- x/||x||,  y <- y/||y||
 2.  inner product:  cos theta = x . y
 3.  geometry:       u = 1 - (cos theta)^2
 4.  surplus:        F = ln(1 + (N-1) u)                nats
 5.  perspectives:   N_eff = 1 + (N-1) u
 6.  frequency:      f_rel = k * N_eff^2                Hz
 7.  interval:       cents above k = 2400 * log2(N_eff)
 8.  fraction:       realised pitch fraction = F / ln N
 9.  fold:           f_audible = f_rel * 2^j into 220-440 Hz; record j
10.  classes:        odd part of every state in the union of the supports
```

Steps 1–6 are Theorem 12.1. Steps 7–8 are §12.4. Step 9 is Chapter 7 §7.2. Step 10 is not optional and Chapter 13 §13.8 says why: u cannot see pitch-class redundancy, and two dyads with identical `f_rel` can differ in class breadth.

SPEC-12.3 — The report line

Every dyad reading in this book is published in this form:

```
    DYAD READING
     rule set: ......................  ladder: 12-state, Ch 5 §5.6, unaltered
      k = 0.483425732988414 Hz    N = 12    band 220–440 Hz
      x support / intensities / ||x|| = ..........
      y support / intensities / ||y|| = ..........
     cos theta = ..........   u = .......... (report 12+ dp; see SPINE §3.4)
      F = .......... nats      N_eff = ..........
      f_rel = .......... Hz    folded .......... Hz (j = ..)
      cents above k = ..........   realised pitch fraction = ..........%
      union of supports = {..}   distinct pitch classes = ..
      uncertainty: u +/- ....  ->  reading spans .......... cents
    CLAUSES: essence not resonance; not clinical, not counselling, not therapy;
                           CONSTRUCTED  case;  scale  status  =  [magnitude-
estimated / NOT DETERMINED]
```

The uncertainty line is mandatory and §12.12 is why. A reading printed without it implies a precision the elicitation cannot deliver.

SPEC-12.4 — Precision

Recompute every derived value from u at 12 or more decimal places, never from a rounded print (SPINE §3.4). Recomputing the carried dyad from a six-decimal u shifts `f_rel` in the fifth significant figure. Ratios, intervals and deficits are unaffected either way, which is why the interval columns are the robust ones and the hertz columns are not.

SPEC-12.5 — Gates

```
      G1   OVERLAP.        supports  share  at  least  one  state  with  non-
trivial weight in both.
             If not: u = 1 by construction; report DEGENERATE; publish no f_rel.
  G2 MAGNITUDE.  intensities elicited by magnitude estimation with ratios asked
                explicitly. If not: report the ORDER only; publish no f_rel.
   G3 NORMS.      both raw L2 norms recorded and published. If not: no f_rel.
  G4 CEILING.    if a sustainability verdict is to follow, c/eta <= ln N is checked
            FIRST (Chapter 4 SPEC 4.1). Above the ceiling no u closes the gap.
  G5 CONSENT.    both parties may decline any step, at any time, without the reading
            being completed against them. Consent is structural, not advisory.
```

G2 is the gate that fails most often and the one with no workaround. There is no “report it as ordinal” branch. Chapter 13 §13.3 gives the number that closes that door.

The Failure Modes

FM-12.1 — Reading u as a disagreement rate

`u = 1 − (x.y)^2` is a geometric quantity computed from two non-negative intensity vectors. It is not a count of disagreements, not a conflict index, and not an inverse of rapport. Two people who argue constantly about one thing they both consider central are near-parallel and read low; two people who never argue because they attend to different things entirely are far from parallel and read high. The formalism is indifferent to the volume in the room.

The failure is common enough that Chapter 13 builds a separate measurement route around exactly this confusion and Chapter 14 prices what happens when someone tries to raise u by manufacturing argument.

FM-12.2 — Reading `f_rel` as a quality score

`f_rel` measures available order. It does not measure how good a relationship is, and the book has no formalism that does. A high-pitched dyad may be exhausting, unstable, or unwanted by both parties; a ground-essence dyad may be exactly what both parties want. Chapter 4's classification answers *does this accumulate order at these costs*, which is a narrower question than any that matters to a person, and the narrowness is the point.

FM-12.3 — Comparing across ladders

Two readings computed on different ladders, or at different N, are not comparable. `f_rel` depends on N explicitly through `(N−1)u`, and on the ladder implicitly through what the twelve slots mean. A dyad reading 20 Hz on a twelve-state ladder and a dyad reading 20 Hz on a seventeen-state ladder have nothing in common but the number. Publish the ladder with the frequency or publish neither.

FM-12.4 — Quoting hertz where the verdict is in cents

§12.6 established that hertz and pitch rank the same increments in opposite orders. A report that leads with "we moved from 1.49 Hz to 2.70 Hz" has led with a difference of 1.21 Hz, which is small against the 69.61 Hz range and is in fact **1029.33 cents — nearly a ninth.** The hertz number understates the low end of the scale by exactly the amount the logarithm compresses it.

FM-12.5 — Precision theatre

Reporting `f_rel = 1.4907987150 Hz` implies ten significant figures of confidence. **The tenth figure is real arithmetic and fictional measurement.** SPINE §3.4 requires the full-precision *computation* because rounding early corrupts the chain across thirty chapters; it does not license the full-precision *claim*. The honest report carries both: the computed value at full precision and the uncertainty interval at the precision the elicitation supports.

FM-12.6 — Folding a sweep

Folding a monotone glide into 220–440 Hz produces eight discontinuities and a listener's impression of a rising-and-resetting figure rather than a rise. **The fold is correct for a point and wrong**

for a sweep, and SPEC 12.1 fixes the transposition instead. A related error is band-shopping: choosing the band after seeing the numbers so that two readings land near each other. Chapter 7's Failure Mode F8 owns that one.

FM-12.7 — Treating the eleven coincidences as meaning something

§12.5 showed that `u = (m−1)/11` makes `f_rel` equal the essence frequency of state `m`. This is arithmetic. A dyad at `u = 6/11` is not "in" state 7 and has no relationship to reflexive feeling of any kind. The two quantities have different arguments and different derivations and happen to coincide at eleven rational points because the constants were chosen to make the two scales share a ceiling. Reading meaning into the coincidence is the numerological failure this series has warned against since Book One.

FM-12.8 — The near-parallel reading reported without its uncertainty

At `u = 0.0687`, an uncertainty of `±0.01` spans 434 cents. **A practitioner who reports the central value alone has reported a third of an octave of ambiguity as a point.** The failure is worse at the bottom of the range, which is exactly where near-parallel pairs — the ones most likely to need the reading — sit. The mandatory uncertainty line of SPEC-12.3 exists for this case and no other.

FM-12.9 — Assuming a high `f_rel` is achievable

The glide runs to `u = 1`, which is perfect orthogonality: two people with no represented overlap in what they carry. Chapter 9 §9.5 and Chapter 13 §13.9 both establish that in practice such a reading almost always indicates disjoint support — an encoding failure — rather than a remarkable pair. A reading near the top of the glide is a reason to check the encoding, not to celebrate.

FM-12.10 — Using the reading on a person who did not agree to it

A dyad reading requires two people's elicited states. Computing one from material a second party provided about a first — an account, a description, a recollection — produces a number about a representation the described person never saw and never agreed to. That is outside every specification in this chapter. The consent gate G5 is structural: it is a condition on whether the computation may be performed at all, not advice about how to present the result.

FM-12.11 — The reading used as counselling

A relationship frequency is not counselling, not therapy, not mediation, and not an assessment of any pair of people. It is a number computed from a representation, and the representation is a description of a description (Chapter 6 §6.9). A practitioner who presents it to two people as a statement about their relationship has changed what it is. Where a person's relationships are causing them difficulty, the appropriate resource is a qualified professional, and this book is not one.

Worked Example

THE CARRIED DYAD, AS AN INTERVAL

CONSTRUCTED CASE, labelled as such. These are not two people. This is SPINE §3.2's dyad, phase 2a, read here purely as an interval; Chapter 13 measures it, Chapter 15 gives it a sustainability verdict, and Chapter 18 intervenes on it.

The vectors, exactly as SPINE fixes them:

```
A = { 4: 0.7,  6: 0.5,  9: 0.4 }
B = { 4: 0.6,  6: 0.55, 9: 0.3,  10: 0.2 }
```

A holds three states: situated feeling (4), temporal feeling (6), normative feeling (9). B holds those three and one more: narrative feeling (10). The supports overlap in three of four positions, so gate G1 passes comfortably.

Step 1 — the raw norms, recorded before normalisation (Chapter 6 §6.4).

```
||A||_2 = sqrt(0.7^2 + 0.5^2 + 0.4^2)          = 0.9486832980505138
||B||_2 = sqrt(0.6^2 + 0.55^2 + 0.3^2 + 0.2^2) = 0.8902246907382427
ratio ||A|| / ||B|| = 1.065667
```

These are published because normalisation is about to throw them away and they are half the measurement. The two parties hold their states at very nearly the same total intensity — within 6.6% — so nothing in this reading is an artefact of one party simply reporting larger numbers than the other.

Steps 2–3 — the geometry.

```
cos theta = 0.965020899          (SPINE §3.2 prints 0.9650)
u         = 1 − cos^2 theta = 0.068734665264634
```

Step 4–6 — surplus, perspectives, pitch.

```
F     = ln(1 + 11 ∗ 0.068734665264634) = 0.563084802746 nats
N_eff = 1.7560813179
f_rel = k ∗ 1.7560813179^2             = 1.4907987150 Hz
```

Every figure matches SPINE §3.2 exactly, which is the point of recomputing from the fifteen-decimal u rather than from the printed 0.068735.

Step 7–8 — the interval.

```
cents above k              = 2400 ∗ log2(1.7560813179) = 1949.66
octaves above k            = 1.624719305043341
realised pitch fraction    = 0.563084802746 / 2.484906649788 = 22.66%
```

The pair occupies 1949.66 cents of an available 8603.91 — one octave and a major seventh above the ground essence, and 22.66% of everything a twelve-category system has to offer. Put the other way: **77.34% of the available relational range is unoccupied.**

For a reader who prefers the perspective count: `N_eff = 1.7561`. **Two people, and the geometry says they are worth 1.76 genuinely distinct perspectives.** Chapter 4 §4.4's fourth column

is the comparison that matters — at `c/eta = 2.0667` this pair must field `e^2.0667 = 7.90` distinct perspectives to break even, and it fields 1.76. Chapter 15 finishes that reading.

Step 9 — the fold.

```
1.4907987150 Hz * 2^8 = 381.6444710 Hz      j = 8, in 220–440 Hz
```

Step 10 — the classes.

```
union of supports = {4, 6, 9, 10}
odd parts         =  1, 3, 9,  5      ->  four distinct pitch classes
```

Four states, four distinct classes: no collision. Chapter 13 §13.8 shows the counterfactual in which the same u comes with only three classes, and why that check has to be run separately.

The report line.

```
DYAD READING -- CONSTRUCTED CASE, SPINE §3.2 phase 2a
 k = 0.483425732988414 Hz  N = 12  ladder: 12-state, unaltered  band 220–
440 Hz
  A: {4:0.7, 6:0.5, 9:0.4}          ||A|| = 0.9486832980505138
  B: {4:0.6, 6:0.55, 9:0.3, 10:0.2} ||B|| = 0.8902246907382427
  cos theta = 0.965020899    u = 0.068734665264634
  F = 0.563084802746 nats    N_eff = 1.7560813179
  f_rel = 1.4907987150 Hz    folded 381.644 Hz (j = 8)
  cents above k = 1949.66    realised pitch fraction = 22.66%
  union {4,6,9,10}           distinct pitch classes = 4
  uncertainty: u +/- 0.01 -> reading spans 434.34 cents
CLAUSES: essence not resonance; not clinical, not counselling, not therapy;
      CONSTRUCTED; scale status = magnitude-estimated per Ch 6 §6.9 Step 10
```

THE SAME DYAD, AFTER — the interval crossed

SPINE §3.2's phase 2b changes the geometry: B takes on two states A does not hold. Chapter 4 §4.20 gives one realising vector, `B' = {2:0.95, 4:0.60, 6:0.55, 9:0.30, 10:0.20, 12:0.8391}`, with `cos theta = 0.554643` and `u = 0.692372881355932`. This chapter does not re-derive it and does not own the intervention; it reads the interval.

```
PHASE 2a   u = 0.068734665264634  f =  1.4907987150 Hz  1949.66 cents above k
PHASE 2b   u = 0.692372881355932  f = 35.8881768941 Hz  7456.88 cents above k

 THE INTERVAL CROSSED:  5507.2215 cents  =  4.589351 octaves
 realised pitch fraction:  22.66%  ->  86.67%
 N_eff:                    1.7561  ->  8.6161
```

Four octaves and a major sixth, purchased by one party taking on two states the other does not hold. In perspective terms the pair went from being worth 1.76 distinct views to being worth 8.62 of a possible 12.

And the caution that has to travel with that sentence, because it is the most quotable number in the chapter: the interval was not purchased by disagreement, by opposition, or by either party becoming difficult. It was purchased by a change in **what B is actually doing** — a genuine addition to B's represented state, not a rhetorical stance. Chapter 14 proves that the rhetorical

version returns nothing at all, and Chapter 4 §4.20 already flagged it: **R8 forbids the opposition route from working, because perfect opposition reads as `u = 0` exactly as agreement does.**

Chapter 15 turns these two positions into sustainability verdicts. The frequencies alone say nothing about whether either arrangement lasts; that requires `f_min`, and `f_min` requires `c` and `eta`, and Chapter 4 owns both.

The counter-case, so the chapter is not lying by selection

CONSTRUCTED. Not every reading moves. Take the same A, and a B whose extra state carries a smaller intensity:

```
       B[10]  =  0.00     ->    u  =  0.019232     f  =  0.7096  Hz       -
1285.20 cents from the carried reading
    B[10] = 0.10   ->  u = 0.032095   f = 0.8850 Hz    - 902.77 cents
   B[10] = 0.20  ->  u = 0.068735  f = 1.4908 Hz       0.00  (the carried case)
    B[10] = 0.30   ->  u = 0.124003   f = 2.7017 Hz    +1029.33 cents
    B[10] = 0.40   ->  u = 0.191202   f = 4.6554 Hz    +1971.38 cents
```

One elicited intensity, moved by one step of a ten-point instrument, moves the reading by more than a thousand cents. That is the steepness of §12.9 arriving as a measurement problem rather than as a price list, and it is the strongest available argument for Chapter 6's insistence that intensities be magnitude-estimated with the ratio question asked explicitly, and for Chapter 11's double-coding requirement. A reading this sensitive to one number is a reading that must be produced twice, by two analysts, from the same written framings, or not published.

What this chapter established

Established, and checkable: a dyad has exactly one frequency, `f_rel = k(1+(N−1)u)^2`, obtained by substituting Paper A's surplus into Book Two's essence bridge; it is strictly increasing in `u`, bounded by `k` below and `k N^2` above, and spans 8603.9145 cents at `N = 12`; it equals the essence frequency of state `m` exactly at `u = (m−1)/11`; the first 0.05 of `u` buys 9.34 times the pitch of the last 0.05, while the last 0.05 buys 9.20 times the hertz of the first; and half the entire range is spent by `u = 0.224009`.

Not established, and not claimed: that any pair of people physically resonates at any frequency here; that a higher reading is a better relationship; that the reading is clinical, diagnostic, therapeutic, or a substitute for counselling of any kind; that the number can be computed about someone who has not agreed to it; or that a frequency alone says whether a relation lasts, which needs `f_min` and belongs to Chapter 15.

The one line to carry forward: *two people are one interval, measured upward from the note a pair with nothing between them would sound.*

Chapter 12 owns `f_rel = k(1+(N−1)u)^2`, the glide, the realised pitch fraction, and the fixed-transposition sweep spec. Chapters 13 through 18 cite it. The surplus law is Chapter 2 §2.5; the essence bridge is Chapter 2 §2.1; `f_min`, the octave deficit and the four levers are Chapter 4; elicitation and the magnitude reporting rule are Chapter 6; the chord and the overlap condition are Chapter 9; the consonance/redundancy identity is Chapter 10. Chapter 13 measures `u` between

two people, and Chapter 14 goes to the bottom of the glide and finds two different faults sounding the same note.

Chapter 13 . The Relationship Frequency

Chapter 12 established what a dyad's frequency is: `f_rel = k(1 + (N−1)u)^2`, one interval measured upward from the ground essence, strictly increasing in the pair's non-parallelism u. That chapter took u as given.

This chapter is where u comes from. It is the central practical chapter of Part III, and it is the one a reader will actually run.

Three things have to be delivered and they are delivered in this order. First, an honest account of what scale the resulting number lives on — because the answer is subtler than either "ordinal" or "ratio," and getting it wrong invalidates every reading downstream. Second, three independent routes to u between two people, each with its own arithmetic, its own resolution limit, and its own failure mode. Third, the procedure that converts a u into a published reading with its bounds attached.

The chapter's shortest possible summary:

```
MEASURING u BETWEEN TWO PEOPLE

ROUTE A -- THE STATE CHANNEL
   elicit both parties' state vectors by magnitude estimation (Ch 6 §6.9)
  u = 1 − (x . y)^2              x, y non-negative, so cos theta in [0, 1]

ROUTE B -- THE AGREEMENT CHANNEL
    m shared items, each judged +1 / −1 by both parties, agreement rate a
  cos theta = 2a − 1            u = 4 a (1 − a)       SIGNED: cos in [−1, 1]

ROUTE C -- THE ERROR CHANNEL
 m estimands with known answers, both parties' error vectors, correlation rho
  u = 1 − rho^2                                       SIGNED: rho in [−1, 1]

THE COLLISION TEST -- not a route, a qualifier
    the states in which the pair differ may share a PITCH CLASS.
  u cannot see this. Run the test separately or the breadth is overstated.

THE CONCORDANCE RULE
    run at least two routes. Publish both. If they disagree by more than
```

```
        100 cents, publish the interval and no point value.
```

The load-bearing constraint of the whole chapter, and it is not a stylistic preference: the intensities must be magnitude-estimated, because with merely ordinal intensities the relationship frequency is not determined at all. §13.3 gives the number that proves it. There is no weaker fallback, and a chapter that offered one would be offering an artifact of the labelling.

Three prohibitions govern everything below, restated because they are easy to drift out of.

Essence, not resonance. Every frequency here is a position on a logarithmic surplus scale. Nothing is claimed to vibrate at any value in this chapter.

Not counselling, not therapy, not assessment. A relationship frequency is a number computed from a representation of two people's states. It is not a clinical, diagnostic or therapeutic statement about anyone, this book does not offer counselling or therapy, and nothing in this chapter substitutes for a qualified professional. The twelve states are not diagnoses.

Consent is structural. Both parties may decline any step, at any time, without the reading being completed against them. This is a condition on whether the computation may be performed at all, not advice about how to present it. Nothing is masked, nothing is concealed, and nothing sits below either party's awareness.

All cases in this chapter are CONSTRUCTED and labelled as such.

The Law

13.1 What is being measured

The object is the geometry between two representations. Not between two people — between two *representations* of two people, produced under a declared rule set, from written framings, by a procedure that is published with the number.

That distinction is the whole ethical and epistemic content of the chapter, and Chapter 6 §6.9 already stated it in the form this chapter inherits: the procedure produces a description of a description. A practitioner who forgets the second "description" has converted an instrument into a verdict.

```
WHAT u MEASURES
  the represented non-parallelism of two parties' states, under one declared ladder

WHAT u DOES NOT MEASURE
  agreement, rapport, closeness, compatibility, conflict, affection, trust,
     similarity of personality, or the quality of anything
```

The second block is longer than the first and that is the correct proportion. u answers one question: *how much of what each party carries is not already carried by the other.* Everything else a reader wants to know about two people is outside the formalism, and Chapter 12 §12.14 fixed what the number licenses.

There is one further clarification that saves a great deal of confusion later. **u is a property of the pair and not of either party**, so there is no such thing as "A's contribution to u." The inner product

is symmetric. A dyad in which one party holds eleven states and the other holds one can read lower than a dyad in which both hold three, if the eleven contain the one. Breadth is a personal quantity (Chapter 9's chord); non-parallelism is a relational one; and the formalism keeps them apart on purpose.

13.2 The three routes, and why there are three

A single route would be simpler and would be wrong, for a reason that is structural rather than cautious.

Route A is the book's native construction — SPINE §2, the twelve-state vectors — and it has one property that matters enormously in Chapter 14: **the intensity vectors are non-negative by construction, so `cos theta >= 0` and the negative half of the correlation range is unreachable through Route A.** Chapter 10 §10.5 established this at internal scope and verified the bound over 100,000 random non-negative unit pairs, minimum observed cosine 0.260867. The bound `cos >= 0` is exact, not empirical.

That is a real limitation, and it is why Routes B and C exist. Both produce a signed correlation. Both can therefore reach `rho < 0`, which is where Chapter 14's result lives, and which Route A cannot see at all.

```
  ROUTE      channel                 sign range      what it is good for
A       state vectors        cos in [0,1]   the native reading; content, not conduct
B       shared judgements     cos in [−1,1]  conduct on a fixed item set; reaches rho<0
C       errors vs known answers rho in [−1,1]  independence proper; reaches rho<0
```

The three routes are not interchangeable and this chapter never pretends they are. They measure the non-parallelism of different things: what the parties *hold*, what they *say* about a fixed set of items, and how they *err* against a known standard. A pair can be near-parallel in one and far from parallel in another, and when they are, that discrepancy is itself the finding.

13.3 SCALE STATUS — the argument in full, because this chapter owns the measurement

This is the section that binds every relationship number in the book. It is stated at length here and cited everywhere else.

There are two quantities in this book with different scale status, and a blanket verdict about "the method" is false in both directions.

```
   QUANTITY        WHAT IT IS                      SCALE          INHERITS R1?
    n               a COUNT of distinguishable          RATIO           NO −
− sidesteps it
              components (Chapter 5)          (true zero,    by construction
                                                          equal units)
    f = k n^2         a state's frequency               RATIO           NO
   w             elicited intensities             see below      YES unless
                                                                 instrumented
  u, F, f_rel, D   computed from intensity−weighted   inherits w's    YES unless w is
                   vectors                          status         instrumented
```

The state frequency is ratio-scaled and the relationship frequency is not — until the intensities are. That asymmetry is real, it is the honest position, and it is carried by every chapter that reports a relationship number.

Two forbidden phrasings, because both are false. *"The ladder is ordinal per R1"* — the index is a count and is not. *"The method is ratio-scaled"* without qualification — only if w was magnitude-estimated. Say which quantity you mean, every time.

13.3.1 The first invariance: good news

Chapter 6 §6.3 computed it and this chapter uses the result. Because `(a x)/||a x|| = x/||x||` for any `a > 0`, any common positive rescaling of the intensities cancels exactly.

```
raw ratings {3: 8, 5: 4, 7: 2} rescaled and renormalised:

raw 0–10             ->  (0.872871561, 0.43643578, 0.21821789)
raw 0–1              ->  (0.872871561, 0.43643578, 0.21821789)
raw 0–100            ->  (0.872871561, 0.43643578, 0.21821789)
proportion of total  ->  (0.872871561, 0.43643578, 0.21821789)

identical to nine decimal places
```

So the elicitation instrument's unit is unconstrained. Two practitioners using different scales produce the same u, the same F, the same `f_rel` — provided the *ratios* of the intensities agree. There is no calibration step and no reference stimulus.

13.3.2 The second invariance: it does not hold, and here is by how much

Now the transformation that an *ordinal* scale says is no transformation at all: a strictly increasing relabelling of the ratings. Under an ordinal reading, applying any monotone function to the intensities destroys no information, because ordinal data carries only order and order is preserved by construction.

Computed. Take a framing recorded on a 0–10 instrument as `{3: 8, 5: 4, 7: 2}`, apply monotone transforms, and compute u against a fixed reference framing `{3: 5, 5: 7, 10: 4}`:

```
ORDINAL-PRESERVING TRANSFORMS OF THE INTENSITIES

transform      u            f_rel (Hz)      cents above k
log(1+x)    0.362115        12.0049           4849.03
sqrt        0.371445        12.5044           4919.61
identity    0.388360        13.4361           5044.14
square      0.525275        22.2093           5914.09

u spans     0.362115 .. 0.525275
f spans     12.0049 .. 22.2093 Hz
SPREAD      1065.05 cents = 0.888 octaves
```

Nearly nine tenths of an octave of disagreement, produced entirely by relabellings that preserve every ordinal fact in the data. Every one of those four transforms leaves the ranking of

the three states untouched. An ordinal scale calls them the same measurement. The relationship frequency calls them a minor seventh apart.

Therefore "report it as ordinal" is not an available fallback. It is not a weaker version of the reading. With ordinal intensities the relationship frequency is *not determined*, and any number published from them would be a property of the labelling rather than of the pair.

Note in passing that `log(1+x)` is unit-dependent — applied to the same framing recorded on a 0–1 instrument instead of 0–10 it lands at `u = 0.366939` rather than `0.362115` — which is a second symptom of the same disease and not a separate problem. A transform family that is admissible under an ordinal reading includes transforms whose effect depends on an arbitrary choice of units, and the reading moves when the units move.

13.3.3 The requirement, and the evidence that it can be met

What survives the two invariances is a precise specification:

> **THE INTENSITY REQUIREMENT [BINDING].** The elicited intensities must carry ratio information — the claim that state 4 is present twice as strongly as state 9 must mean something — while the unit in which that ratio is expressed remains free. That is exactly the definition of a ratio scale, and nothing weaker will do.

Stevens' method of magnitude estimation produces ratio-scale measurements of subjective magnitude from human subjects, and it replicates — the finding Book Two Chapter 38 already leans on. Chapter 6 §6.9 Step 10 is built as a magnitude-estimation procedure for that reason and not by stylistic preference: the strongest present state class is assigned a reference value and every other class is elicited *as a ratio to it*, in the subject's own words, with the ratio question asked explicitly.

The honest form of the claim, and it is not softened anywhere in this book: **the intensities are ratio-scaled to the extent that magnitude estimation delivers ratio scales, which is a measured property of a procedure and not a constant of nature.** Stevens' own exponents are known to vary with method, range and instruction. The reading inherits that variability and the report says so.

13.3.4 The magnitude reporting rule, and why a dyad needs both norms

Normalisation discards the length of each vector, and the length is not nothing. Chapter 6 §6.4 computed the case that makes it obvious:

```
low-intensity  {3: 1,   5: 0.5, 7: 0.25}   raw L2 norm = 1.145644
high-intensity {3: 8,   5: 4,   7: 2   }   raw L2 norm = 9.165151

ratio of norms = 8.0000x
u between them = 0.000000000000     exactly zero -- same direction
```

> **THE MAGNITUDE REPORTING RULE, AT DYAD SCOPE.** Both parties' raw L2 norms are reported alongside the reading, in the instrument's units, and are never discarded. u measures direction; the norms measure how much there is. **A dyad reading published without both norms has thrown away half the measurement and has made the free multiplier unrecoverable.**

The dyad case adds a requirement the single-vector case does not have. The ratio of the two norms is itself a reportable quantity, because a large asymmetry — one party's total intensity many times the other's — is a fact about the elicitation that the normalised geometry cannot express. The carried dyad's ratio is 1.065667, which is small enough that nothing in its reading is an artefact of one party simply reporting bigger numbers.

13.4 ROUTE B — the agreement channel, derived

THEOREM 13.1 — THE AGREEMENT IDENTITY. Let two parties each render a binary judgement, coded +1 or −1, on each of `m` shared items, and let `a` be the fraction of items on which their judgements match. Then, treating the two judgement vectors as elements of the inner-product space,

```
cos theta = 2a − 1                u = 1 − (2a − 1)^2 = 4 a (1 − a)
```

Proof. Each judgement vector has every entry ±1, so each has L2 norm `sqrt(m)`. Their inner product is `(agreements) − (disagreements) = am − (1−a)m = m(2a−1)`. Dividing by the product of the norms gives `cos theta = m(2a−1)/m = 2a−1`. Substituting into `u = 1 − cos^2 theta` and expanding: `1 − (4a^2 − 4a + 1) = 4a − 4a^2 = 4a(1−a)`. QED.

The identity is exact and requires no centring, because the norms of ±1 vectors are fixed by `m` alone. Computed across the range:

```
THE AGREEMENT CHANNEL

  a         cos theta      u          f_rel (Hz)     cents above k
1.000000    +1.000000   0.000000       0.4834           0.00
0.990000    +0.980000   0.039600       0.9963        1251.97
0.983333    +0.966667   0.065556       1.4320        1880.02
0.975000    +0.950000   0.097500       2.0764        2523.29
0.950000    +0.900000   0.190000       4.6158        3906.26
0.900000    +0.800000   0.360000      11.8930        5544.82
0.800000    +0.600000   0.640000      31.2494        7217.27
0.750000    +0.500000   0.750000      41.3631        7702.69
0.700000    +0.400000   0.840000      50.6909        8054.75
0.600000    +0.200000   0.960000      64.6019        8474.57
0.500000     0.000000   1.000000      69.6133        8603.91
0.400000    −0.200000   0.960000      64.6019        8474.57
0.300000    −0.400000   0.840000      50.6909        8054.75
0.250000    −0.500000   0.750000      41.3631        7702.69
0.200000    −0.600000   0.640000      31.2494        7217.27
0.100000    −0.800000   0.360000      11.8930        5544.82
0.050000    −0.900000   0.190000       4.6158        3906.26
0.000000    −1.000000   0.000000       0.4834           0.00
```

The table is exactly symmetric about `a = 0.5`, and the top and bottom rows are identical. Two parties who agree on every item and two parties who disagree on every item produce the same u, the same frequency, and the same reading. Chapter 14 owns that result in full and prices both faults; this chapter's job is to note that the measurement channel exhibits it directly, in a form a practitioner can compute from a spreadsheet.

Route B's resolution limit is severe and must be stated with the route. Because a is quantised to `1/m`, `u` is quantised to roughly `4/m` near the top of the agreement range. Computed, at two disagreements versus three:

```
ONE ITEM'S WORTH, NEAR HIGH AGREEMENT

 m      a (2 disagreements)   u      f (Hz)   one more disagreement costs
   40        0.950000         0.190000    4.6158           938.89 cents
   60        0.966667         0.128889    2.8259           849.42 cents
  120        0.983333         0.065556    1.4320           643.28 cents
  240        0.991667         0.033056    0.8989           428.21 cents
  400        0.995000         0.019900    0.7182           295.58 cents
 1000        0.998000         0.007984    0.5721           136.62 cents
 2000        0.999000         0.003996    0.5290            72.03 cents
 5000        0.999600         0.001600    0.4920            29.78 cents
```

In a 120-item log, one disputed item is worth 643 cents — more than a perfect fifth. A near-parallel dyad cannot be resolved to better than a few hundred cents by an agreement log of plausible length, and the honest output of Route B in that regime is an interval, not a number.

13.5 ROUTE C — the error channel, and what it can do that the others cannot

DEFINITION 13.2 — THE ERROR CHANNEL. Let both parties estimate each of `m` quantities whose true values are subsequently known. Form each party's error vector `e = (estimate − truth)` over the `m` items, let `rho` be the Pearson correlation of the two error vectors, and set

```
u = 1 − rho^2
```

This is the construction Chapter 4 §4.14 named when it said *decorrelate the errors*, and it is the only one of the three routes that can distinguish two things Route B cannot: being right for different reasons and being wrong in different ways.

Its properties, stated plainly:

It is signed. `rho` ranges over `[−1, +1]`, so Route C reaches the negative half. Two parties who err in systematically opposite directions have `rho` near −1 and therefore `u` near 0 — the same reading as two parties who err identically. Again, Chapter 14.

It requires ground truth. That is its cost and it is not small. Route C is unavailable for questions whose answers never arrive, which is most of the questions two people actually disagree about. Where it *is* available — forecasting, estimation, prediction, any domain that eventually scores itself — it is the strongest of the three.

It is the route that catches shared noise. Here is the trap Route B cannot see: two parties answering at random on a shared item set have expected agreement `a = 0.5`, hence `u = 1`, hence `f_rel = 69.6133 Hz` — the ceiling of the entire system, from two people guessing. **The surplus formalism cannot distinguish independence from noise, because both are orthogonality**, and Chapter 2 §2.6 already stated the general form: `F` is a lower bound on the ensemble's information, not the information itself.

The guard is procedural, not formal:

THE ACCURACY GUARD [BINDING WHEREVER ROUTE B OR C IS USED]. A u reading is published with a measure of each party's accuracy on the item set. A high u between two parties who are both largely wrong is arithmetically correct and practically worthless. Surplus is a measure of distinctness, never of correctness, and this book does not have a formalism that converts one into the other.

13.6 The collision test — why u alone overstates breadth

u is computed from an inner product over the twelve state indices. It knows which indices carry weight. It does not know that indices 5 and 10 fold to the same pitch class, because folding is a property of the essence law and not of the geometry.

THEOREM 13.3 — u IS BLIND TO PITCH-CLASS COLLISION. There exist dyads with identical u, identical `f_rel`, and different numbers of distinct pitch classes in the union of their supports.

Proof by construction, and both cases are computed. Take the carried dyad's A, and two candidate partners differing only in which state carries the fourth entry:

```
A  = {4: 0.7, 6: 0.5, 9: 0.4}

B  = {4: 0.6, 6: 0.55, 9: 0.3, 10: 0.2}
   cos theta = 0.965020899  u = 0.068734665264634  f_rel = 1.4907987150 Hz
  union {4, 6, 9, 10}  odd parts 1, 3, 9, 5  -> FOUR distinct pitch classes

B* = {4: 0.6, 6: 0.55, 9: 0.3, 12: 0.2}
   cos theta = 0.965020899  u = 0.068734665264634  f_rel = 1.4907987150 Hz
             union  {4,  6,  9,  12}     odd  parts  1,  3,  9,  3     -
>  THREE distinct pitch classes
```

The two u values agree to fifteen decimal places, because the inner product runs only over the three shared indices and both partners have the same norm. QED.

Two dyads at the same relationship frequency, one of which occupies four pitch classes and one of which occupies three: the geometry cannot tell them apart and the lattice can. In B∗ the extra state, committed feeling (12), shares an odd part with temporal feeling (6), so it adds a fourth state and no fourth class — it is two octaves above a note the pair already sound.

SPEC 13.1 — THE COLLISION TEST. After computing u, list the union of the two supports, take the odd part of every index, and report the count of distinct pitch classes beside the count of distinct states. Where the two counts differ, say by how much and name the colliding pairs. Chapter 10 owns the identity behind this — the most consonant pairs in the lattice are exactly the collisions, so consonance and redundancy are the same phenomenon — and Chapter 7 owns the odd-part rule. The test takes two minutes and it is the difference between reporting four perspectives and reporting three.

13.7 The degeneracy traps at dyad scope

Chapter 9 §9.4 established the one-hot trap inside one person. It reappears between two people and it is more dangerous there, because between two people disjoint support is not obviously an error.

```
DISJOINT SUPPORT BETWEEN TWO PARTIES -- computed

P = {1:0.6, 2:0.5, 4:0.3}     Q = {3:0.7, 6:0.4, 9:0.5}
   cos theta = 0.000000000000    u = 1.000000000000    f_rel = 69.6133 Hz

P = {5:0.9, 7:0.1}            Q = {8:0.2, 11:0.9}
   cos theta = 0.000000000000    u = 1.000000000000    f_rel = 69.6133 Hz
```

Both pairs score the maximum the system can produce, and both scores are produced by the encoding rather than by the parties. **Any two vectors with no index in common are orthogonal, so `u = 1` regardless of what the states are or how intense they are.**

The degeneracy is flattering, which is why it survives. A pair told that they occupy the full twelve-fold relational range, at the ceiling of the system, will not challenge the finding. Chapter 9 named the general principle and it applies verbatim here: an instrument whose failure mode is to award full marks will be adopted enthusiastically and never questioned by the people it flatters.

The repair is Chapter 9 §9.5's overlap condition, restated at dyad scope:

> **THE OVERLAP CONDITION AT DYAD SCOPE [BINDING].** A dyad reading may be computed only if the two supports share at least one state carrying non-trivial weight in both. Where they do not, the computation is reported as DEGENERATE and no `f_rel` is published.

And the plain-language version, which is the one that makes it obviously right: two people described as having nothing whatever in common have not been described as a relationship. If the elicitation of A's states and the elicitation of B's states about the same situation share no ingredient at all, either the situation was not the same situation or the elicitation failed. Go back to the framing step.

There is a subtler version that a practitioner will meet more often than the flat case. Support that overlaps in one state at very low intensity in one party produces a cosine dominated by a single small term, and the reading becomes a fact about that term. SPEC-13.5's gate G1 sets a floor on the shared weight for exactly this reason.

13.8 Honesty clauses, restated in place

(1) ESSENCE, NOT RESONANCE. Nothing in this chapter vibrates. `f_rel` is a position on a logarithmic surplus scale in units fixed by a chosen constant.

(2) NOT CLINICAL, NOT COUNSELLING, NOT THERAPY. The twelve states are not diagnoses. A relationship frequency is not an assessment of a relationship, and this book does not offer or substitute for counselling, therapy or mediation of any kind.

(3) SCALE STATUS. §13.3, in full, on every reading. **A reading whose intensities were not magnitude-estimated is not a weaker reading; it is not a reading.**

(4) THE READING INHERITS ITS INPUTS. `f_rel` is exactly as defensible as `u`; `u` is exactly as defensible as the two representations; the representations are exactly as defensible as the decomposition. Publish the decomposition with the frequency or publish neither.

(5) SURPLUS IS A LOWER BOUND. A verdict of ACCUMULATES is robust to representational under-capture; a verdict of DECAYS is not (Chapter 3 §3.6).

(6) CONSENT IS STRUCTURAL. No step of any procedure below may be run on a person who has declined it, and declining does not abort the relationship or the reading's disclosure. No intervention derived from a reading may remove a person's ability to decline.

(7) CONSTRUCTED CASES. Every case here is constructed and labelled as such.

The Engineering

13.9 PROCEDURE REL-1 — measuring u between two people, end to end

This is the procedure. Chapters 15, 17 and 24 cite it and do not restate it. It extends Chapter 6 §6.9's single-subject elicitation to two parties; the steps that are unchanged are cited rather than repeated.

```
STEP 1 -- DECLARE THE RULE SET, DATED.
  Decomposition rules in force, N = 12, the ladder in use, the intensity instrument
  and its range, the audible band and folding convention, and which routes will be
  run. This page is published with the reading.          (Ch 6 §6.9 Step 1)

STEP 2 -- OBTAIN CONSENT FROM BOTH PARTIES, SEPARATELY.
  Each is told what is elicited, what is computed, what it does and does not mean,
  and that any question may be skipped without a reason. Consent is obtained
  separately so that neither party's agreement is produced by the other's presence.
  Nothing is masked; nothing is concealed; declining is available at every step.

STEP 3 -- FIX ONE SITUATION, IN WORDS BOTH PARTIES ACCEPT.
  One sentence, written before anything else is asked, agreed by both. A dyad
     reading on two different situations is not a dyad reading.

STEP 4 -- SILENT WRITE-FIRST, INDEPENDENTLY AND OUT OF CONTACT.
  Each party writes their framing of the situation in full, alone, before either
  sees the other's. This is the relational form of Ch 6 §6.9 Step 5 and it matters
     more here: two people in a room converge before they record.

STEP 5 -- READ EACH FRAMING AGAINST THE TWELVE COMPONENTS.
  PRESENT or ABSENT for all twelve, no blanks, substitution test on every PRESENT,
    nesting check.                                  (Ch 6 §6.9 Steps 6-8)

STEP 6 -- COUNT, AND RECORD THE LIST BESIDE THE COUNT.
    A count without its list is a rating and is rejected.      (Ch 5 §5.18)

STEP 7 -- ELICIT INTENSITIES BY MAGNITUDE ESTIMATION, FOR EACH PARTY SEPARATELY.
    Reference class assigned first; every other class elicited AS A RATIO to it, with
    the ratio question asked in words. Do not show either party the other's numbers.
    This step is what makes the reading determinate (§13.3).   (Ch 6 §6.9 Step 10)

  STEP 8 -- RUN THE OVERLAP GATE. NOT OPTIONAL.
```

```
     Supports must share at least one state with non-trivial weight in both. If not:
        report DEGENERATE, publish no f_rel, return to Step 3.      (§13.7)

    STEP 9 -- RECORD BOTH RAW L2 NORMS AND THEIR RATIO, THEN NORMALISE.
     The instrument's unit cancels; the norms do not, and are published. (§13.3.4)

    STEP 10 -- ROUTE A: compute cos theta = x . y and u = 1 - cos^2 theta.
       Carry u to 12 or more decimal places.                    (SPINE §3.4)

   STEP 11 -- ROUTE B, IF AN ITEM SET EXISTS: agree the item set BEFORE judging it.
      Both parties judge all m items independently; compute a, then u = 4a(1-
a).
       Report m with the reading -- the resolution limit is 4/m (§13.4).

   STEP 12 -- ROUTE C, IF GROUND TRUTH WILL ARRIVE: record both parties' estimates,
      wait for the answers, form the error vectors, compute rho, then u = 1 -
rho^2.
     Report the sign of rho explicitly. It is not recoverable from u. (§13.5, Ch 14)

    STEP 13 -- RUN THE ACCURACY GUARD on any Route B or C reading.
              Publish each party's accuracy alongside u. Orthogonal-and-
wrong is a real state
       and the formalism cannot see it.                            (§13.5)

    STEP 14 -- RUN THE COLLISION TEST.
     Union of supports; odd part of each index; count distinct states and distinct
       pitch classes; name the collisions.                    (SPEC 13.1)

    STEP 15 -- COMPUTE THE READING.
     F = ln(1 + 11u); N_eff = 1 + 11u; f_rel = k N_eff^2; cents above k = 2400 log2
             N_eff; realised pitch fraction = F/ln 12; fold into 220-
440 Hz and record j.

    STEP 16 -- RUN THE CONCORDANCE PASS.
     Where two or more routes were run, report every route's u and f separately, and
     the spread between the extremes in cents. Do not average silently. (§13.11)

    STEP 17 -- RUN THE SECOND ANALYST.
     An independent analyst, working from the Step 1 rule set and the Step 4 written
     framings alone, repeats Steps 5-10. Report the two readings and their difference
      in cents.                                        (Ch 6 §6.7, §6.9 Step 17)

    STEP 18 -- RUN THE PERMUTATION TEST.
     Swap two adjacent ladder indices, recompute, report the change in cents and in
       pitch class.                                              (Ch 5 §5.16)

   STEP 19 -- ESTIMATE eta, c AND delta IF A SUSTAINABILITY VERDICT IS WANTED,
      and run the CEILING TEST FIRST: c/eta <= ln N = 2.484907, or the case is
```

```
      structural and no value of u closes it.                (Ch 4 SPEC 4.1)

   STEP 20 -- PUBLISH THE DECOMPOSITION, THE NORMS, THE ROUTES AND THE BOUNDS
     on the same page as the frequency. Where they will not fit, the frequency is not
     published either. State in the report itself that these are essence frequencies
     and not physical resonances, that the twelve states are not diagnoses, and that
     the reading is not a clinical, therapeutic or counselling assessment of anyone.
```

Steps 8, 9, 13, 14 and 16 are the five that a hurried practitioner drops, and each of them is the sole defence against a specific way the reading goes wrong. They are the reason this procedure has twenty steps instead of six.

13.10 What one elicited number does to the reading

Chapter 12 §12.12 priced uncertainty in u. Here is the same fact one layer upstream, in the quantity a practitioner actually writes down.

Take the carried dyad and move a single intensity — B's rating of narrative feeling — by one step of a ten-point instrument:

```
SENSITIVITY OF THE READING TO ONE ELICITED INTENSITY

B[10]      u            f_rel (Hz)     shift from the carried reading
0.00    0.019232          0.7096          −1285.20 cents
0.10    0.032095          0.8850          − 902.77 cents
0.20    0.068735          1.4908               0.00   (the carried case)
0.30    0.124003          2.7017          +1029.33 cents
0.40    0.191202          4.6554          +1971.38 cents
```

One number, moved by one step, moves the reading by more than an octave. That is not a defect in the arithmetic; it is Chapter 12 §12.9's steepness at the bottom of the range arriving as a measurement problem. It has three consequences and all three are procedural.

One: the ratio question must be asked explicitly. A subject who is ranking rather than magnitude-estimating will produce entries that differ by a step for reasons that carry no ratio content, and the reading will move by an octave on that basis. §13.3 is why Step 7 is written the way it is.

Two: the reading must be double-coded. Step 17's second analyst, working from the same written framings, is the only check that catches a coding artefact before it is published. Chapter 11 §11.14b owns the disagreement protocol and this chapter adopts it unchanged.

Three: the reading is reported with an interval. A near-parallel dyad reported as a point value implies a precision no elicitation delivers. The mandatory uncertainty line is not modesty; it is the only truthful form of the output at this end of the scale.

13.11 The concordance pass

Where two or more routes are available, they are run and both are published. Never averaged silently, and never chosen after the fact.

```
 CONCORDANCE ON THE CARRIED DYAD -- CONSTRUCTED

ROUTE                            u              f_rel (Hz)    cents above k
A  state-vector overlap       0.068734665264634      1.4908         1949.66
B  agreement log, 118/120     0.065555555555556      1.4320         1880.02
C  error decorrelation rho=0.9650  0.068775000000000   1.4916         1950.54

spread, lowest to highest                                          70.52 cents
 verdict: CONCORDANT (spread < 100 cents)
```

The Route B construction deserves a note, because it shows how the routes constrain each other. Route A's cosine of 0.965021 implies, through Theorem 13.1's identity `a = (1 + cos theta)/2`, an agreement rate of 0.982510 — which on a 120-item log is 117.90 items. The nearest realisable log is 118 of 120, and the 0.10-item gap between the implied and the realisable value is worth 69.64 cents. That is the resolution limit of §13.4 appearing as the entire discrepancy between two routes, and it is a better explanation of the spread than any story about the parties.

> **THE CONCORDANCE RULE [BINDING].** Report every route run, with its u, its `f_rel`, and its resolution limit. Report the spread between extremes in cents. **If the spread exceeds 100 cents, publish the interval and no point value**; if it exceeds 600 cents — a fifth — treat the routes as measuring different things and say which, rather than reconciling them.

The 100-cent threshold is a house convention and is stated as one. It is chosen because it is roughly the smallest interval a reader will treat as meaningful when the readings are folded and compared by ear, and because it sits an order of magnitude above the arithmetic noise of the computation. It is not derived from anything and a practitioner who prefers a different threshold should declare theirs in Step 1.

13.12 Converting the reading into a verdict

The frequency alone says nothing about whether the relation lasts. That requires `f_min`, and `f_min` requires `c` and `eta`, both of which Chapter 4 owns.

```
 THE READING, IN FOUR LINES

 1.  CEILING TEST FIRST.   c/eta <= ln N = 2.484907 ?
                  if not, the case is STRUCTURAL: no u closes it. Stop here.
 2.  f_min  = k e^(2c/eta)
 3.  D      = 2.885390081777927 * (c/eta) - log2(f_rel/k)
 4.  verdict: D <= 0 SUSTAINABLE | D > 0 short by D octaves, and the four levers are
              raise u, raise eta, cut c, raise N.  delta is NOT one of them.
```

The order in line 1 is not a stylistic preference and Chapter 4 SPEC 4.1 gives the reason: a deficit does not announce its own kind, and a prescription issued before the ceiling test may be unsatisfiable. An unsatisfiable prescription is worse than none, because it consumes the effort that restructuring needed.

Estimating `eta`, `c` and `delta` for a dyad is Chapter 15's business and Chapter 11 §11.13 gives the personal-scope form. This chapter's obligation is narrower and is discharged here: `f_rel`

is the input to that reading and nothing else, and its defensibility is bounded by §13.3 whatever happens downstream.

13.13 A dyad's pitch is its surplus, and what that sentence is for

The single line this chapter hands to a practitioner who does not want the apparatus:

A dyad's pitch IS its surplus, monotonically — higher pitch means more available order. There is no interpretive step between the two. Chapter 12 Corollary 12.2 proves the monotonicity; this chapter supplies the measurement; and the sentence is exactly as strong as the measurement and no stronger.

For a reader who prefers counts to frequencies, the same reading in the other unit: `N_eff = 1 + 11u` is the number of genuinely distinct perspectives the pair fields. Two people whose reading is `N_eff = 1.76` are, by this measure, worth 1.76 people's worth of distinct view — and Chapter 4's fourth column says how many they must be worth to break even at their costs. That comparison is a sentence anyone can act on, and it is the most useful thing the instrument produces.

The Specification

SPEC-13.2 — Inputs

```
    [ ] Step 1 rule set, dated and published
    [ ] separate consent from both parties, decline available at every step
    [ ] one agreed situation, in one written sentence
    [ ] two independently written framings, recorded before any comparison
   [ ] intensities elicited BY MAGNITUDE ESTIMATION with the ratio question asked
    [ ] both raw L2 norms and their ratio
    [ ] the overlap gate, passed
    [ ] for Route B: the item set, agreed before judging, with m recorded
   [ ] for Route C: the estimand set, the ground truth, and both accuracy figures
```

SPEC-13.3 — Required outputs

```
    RELATIONSHIP FREQUENCY READING
     rule set / date / ladder / N = 12 / k = 0.483425732988414 Hz / band 220–
440 Hz
    situation (one sentence): ..................................................
      A: support, intensities, ||A||_2 = ......
      B: support, intensities, ||B||_2 = ......      norm ratio = ......
      ROUTE A:  cos theta = ......   u = ...... (12+ dp)
    ROUTE B:  m = ....  a = ......  cos theta = ......  u = ......  resolution 4/m = ....
    ROUTE C:  m = ....  rho = ...... (SIGN STATED)    u = ......  accuracy A/B = ....
    CONCORDANCE: spread = ...... cents   verdict = CONCORDANT / INTERVAL / DIVERGENT
      F = ...... nats    N_eff = ......
     f_rel = ...... Hz    folded ...... Hz (j = ..)    cents above k = ......
      realised pitch fraction = ......%
```

```
    COLLISION TEST: states = ..  distinct pitch classes = ..  collisions: ......
      second analyst: u = ......   difference = ...... cents
      permutation test: ...... cents,  class change: yes / no
      uncertainty: u +/- ......  ->  reading spans ...... cents
    CLAUSES: essence not resonance; twelve states are not diagnoses; NOT a clinical,
          therapeutic or counselling assessment; CONSTRUCTED / observed; consent
               recorded and decline available throughout
```

A reading that omits the sign of `rho` in Route C is incomplete in a way Chapter 14 makes expensive: `u` cannot recover it, and the two signs mean opposite things about what to do next.

SPEC-13.4 — Precision

Carry `u` to twelve or more decimal places and recompute every derived value from it (SPINE §3.4). Recomputing the carried dyad from a six-decimal `u` shifts `f_rel` in the fifth significant figure. Intervals, ratios and deficits are unaffected either way. Full-precision computation is required; full-precision *claiming* is not licensed by it, and the uncertainty line is where the truth about precision goes.

SPEC-13.5 — Gates

```
   G1  OVERLAP.     supports share >= 1 state with weight >= 10% of each party's
                       maximum entry. Otherwise DEGENERATE; no f_rel.
   G2  MAGNITUDE.   intensities magnitude-estimated with ratios asked explicitly.
                Otherwise publish the ORDER only and no f_rel. NO ORDINAL FALLBACK.
   G3  NORMS.      both raw L2 norms and their ratio recorded. Otherwise no f_rel.
   G4  ROUTES.     at least two routes where two are available; concordance reported.
    G5  ACCURACY.     Route B or C readings publish both parties' accuracy.
      G6   COLLISION.      distinct-class  count  published  beside  distinct-
state count.
    G7  CEILING.      before any verdict, c/eta <= ln N checked FIRST.
   G8  CONSENT.    both parties may decline any step at any time; the reading is not
                       completed against a party who has declined it.
```

G2 and G8 have no workaround and no partial form. The first is epistemic — without it the number is not determined. The second is structural — without it the number should not exist.

The Failure Modes

FM-13.1 — Running the reading as an assessment of two people

The procedure produces a description of a description. A practitioner who presents the output to two people as a statement about their relationship has changed what the instrument is, and has done so in a domain — how two people are with each other — where the change is not recoverable. The twelve states are not diagnoses; the reading is not clinical, therapeutic or counselling advice;

and where a relationship is causing a person difficulty, the resource is a qualified professional and this book is not one.

FM-13.2 — The ordinal fallback

The commonest response to §13.3 is: *fine, we will report it as ordinal.* **There is no such reading.** An ordinal-preserving relabelling moves u from 0.362115 to 0.525275 — 1065.05 cents, nearly an octave — from a change an ordinal scale calls no change at all. A number published from ordinal intensities is a property of the labelling.

FM-13.3 — Publishing without the norms

Normalisation discards vector length, and two profiles with identical shape and an eightfold difference in intensity read `u = 0` exactly. A reading published without both raw L2 norms has made the free multiplier unrecoverable and has discarded the half of the measurement that says how much there is.

FM-13.4 — Averaging the routes

Three routes returning three numbers is information. Averaging them into one is the destruction of that information, and it hides the case the concordance rule exists to catch: **routes that diverge are telling you they are measuring different things, and the divergence is the finding.**

FM-13.5 — Choosing the route after seeing the numbers

Declaring in Step 1 which routes will be run is not bureaucracy. A practitioner who runs three routes and reports the one that came out highest has run a search, not a measurement. **The route set is fixed before the data exists or the reading is not a reading.**

FM-13.6 — Reading a high u as good news

A high u may be a wide pair. It may equally be disjoint support (§13.7), two people who answered at random (§13.5), or an encoding in which each party's framing was reduced to its single most characteristic state. **Near the top of the range, the first hypothesis to test is that the encoding failed**, because that is the more common cause and it is the one that flatters.

FM-13.7 — Ignoring the resolution limit of the agreement channel

u from Route B is quantised at roughly `4/m`. In a 120-item log one disputed item is worth 643 cents. A practitioner reporting `u = 0.0656` from a 120-item log has reported four significant figures of a quantity the log resolves to one.

FM-13.8 — Dropping the sign of `rho`

Route C returns a signed correlation and `u = 1 − rho^2` discards the sign. **Two parties at `rho = +0.9650` and two parties at `rho = −0.9650` produce identical readings and require opposite interventions**, and the sign is the only place that information lives. Chapter 14 is the whole argument; SPEC-13.3 is the requirement.

FM-13.9 — Skipping the collision test

Theorem 13.3 exhibits two dyads with identical u, identical f_rel, and four versus three distinct pitch classes. Without the test, the reading reports four perspectives where the lattice has three, and a redundancy the system can see goes unreported.

FM-13.10 — Eliciting the two parties together

Two people in a room converge before they record. Step 4's independent silent write-first is the only defence, and it is easy to skip because running the elicitation jointly is faster and feels more collaborative. **A jointly elicited dyad reading measures the elicitation session, not the pair.**

FM-13.11 — Comparing readings across rule sets

Two readings computed under different decomposition rules, different ladders, or different N are not comparable. Chapter 12 FM-12.3 owns this. The rule set is published with the number because without it the number has no referent.

FM-13.12 — Treating a decays verdict as robust

F is a lower bound on the pair's information, so f_rel is a lower bound on the pitch. **A verdict of ACCUMULATES survives the possibility that the representation missed something; a verdict of DECAYS does not, because the missing something could have supplied the surplus.** Chapter 3 §3.6 owns the asymmetry and every DECAYS verdict in this book carries it.

FM-13.13 — Running the procedure on a party who did not consent

A dyad reading requires two parties' elicited states. Constructing one party's vector from the other party's account of them produces a number about a representation the described person never saw. That is outside every specification in this chapter, and no result obtained that way is a reading under this book's rules.

Worked Example

THE CARRIED DYAD, MEASURED

CONSTRUCTED CASE, labelled as such. These are not two people. This is SPINE §3.2's dyad, phase 2a. Chapter 12 read it as an interval; this section measures it, runs every gate, and produces the published form.

Step 1 — the rule set. Twelve-state ladder, Chapter 5 §5.6, unaltered. N = 12, k = 0.483425732988414 Hz, band 220–440 Hz, intensity instrument 0–1 by magnitude estimation with the reference class assigned first. Routes A, B and C all declared before any data exists. Dated.

Steps 2–4 — consent, situation, independent framings. Recorded. Both parties may decline any step. The situation is fixed in one agreed sentence and each party writes their framing alone.

Steps 5–7 — the vectors.

```
A = { 4: 0.7,  6: 0.5,  9: 0.4 }
    situated feeling (4), temporal feeling (6), normative feeling (9)

B = { 4: 0.6,  6: 0.55, 9: 0.3,  10: 0.2 }
    the same three, plus narrative feeling (10)
```

Step 8 — the overlap gate. Shared support {4, 6, 9}; three of A's three states and three of B's four. The smallest shared weight is B's 0.3 at state 9, which is 50% of B's maximum entry — far above G1's 10% floor. **PASS.**

Step 9 — the norms.

```
||A||_2 = 0.9486832980505138
||B||_2 = 0.8902246907382427
ratio   = 1.065667
```

Published before normalisation discards them. The two parties hold their states at total intensities within 6.6% of each other, so nothing in this reading is an artefact of one party reporting larger numbers than the other — which is exactly the check §13.3.4 exists to make possible.

Step 10 — Route A.

```
cos theta = 0.965020899
u         = 0.068734665264634
```

Step 11 — Route B. The pair agree an item set of 120 shared judgements before judging any of them, and match on 118.

```
  a = 118/120 = 0.983333333333    cos theta = 2a − 1 = 0.966666666667
u = 4a(1−a) = 0.065555555556   f_rel = 1.4320 Hz   resolution 4/m = 0.033333
```

Step 12 — Route C. Twenty-four estimands with answers that subsequently arrive; the two error vectors correlate at rho = +0.9650, sign recorded.

```
rho = +0.9650    u = 1 − rho^2 = 0.068775    f_rel = 1.4916 Hz
```

Step 13 — the accuracy guard. Both parties' mean absolute error on the twenty-four estimands is published beside the reading. Without it, a high u between two parties who are both largely wrong would read as available order.

Step 14 — the collision test.

```
union of supports = {4, 6, 9, 10}
odd parts         =  1, 3, 9,  5
distinct states = 4     distinct pitch classes = 4     collisions: none
```

Compare §13.6's B∗, identical in u and f_rel and carrying only three classes. This dyad's breadth survives the test; the counterfactual's does not, and no quantity in the geometry distinguishes them.

Step 15 — the reading.

```
F     = ln(1 + 11 ∗ 0.068734665264634) = 0.563084802746 nats
N_eff = 1.7560813179
```

```
f_rel = 1.4907987150 Hz
folded 381.644 Hz (j = 8)
cents above k = 1949.66      of an available 8603.91
realised pitch fraction = 0.563084802746 / 2.484906649788 = 22.66%
```

Every figure matches SPINE §3.2 to every published digit, recomputed from the fifteen-decimal u rather than from the printed `0.068735`.

Step 16 — concordance.

```
ROUTE A   u = 0.068734665264634   f = 1.4908 Hz   1949.66 cents
ROUTE B   u = 0.065555555555556   f = 1.4320 Hz   1880.02 cents
ROUTE C   u = 0.068775000000000   f = 1.4916 Hz   1950.54 cents

spread = 70.52 cents           verdict: CONCORDANT (< 100 cents)
```

And the explanation of the spread, which is more useful than the spread: Route A's cosine implies an agreement rate of 0.982510, or 117.90 items in 120. The only realisable log is 118, and that 0.10-item rounding is worth 69.64 cents of the observed 70.52. Nearly the whole discrepancy is the agreement channel's resolution limit and not a disagreement between the routes.

Steps 17–18 — the second analyst and the permutation test. Both are run and both are published. Their results are not invented here, because this is a constructed case and inventing an inter-rater figure would be presenting a fabrication as a measurement. What the specification requires is that the fields exist and are filled from a real second pass; a constructed case fills them with the word CONSTRUCTED and nothing else.

Step 19 — the ceiling test, then the verdict. Chapter 15 owns the estimation of `eta` and `c`; SPINE §3.2 fixes them for this case at `eta = 0.30`, `c = 0.62`, `delta = 0.09`.

```
CEILING TEST FIRST:   c/eta = 0.62/0.30 = 2.066666666666667
                      ln N  = 2.484906649788
              2.0667 <= 2.4849  -> TUNABLE. Some assignment of u closes it.

f_min  = k e^(2 * 2.066666666666667)  =  30.1587698405 Hz  (folded 241.270, j = 3)
 D      = 2.885390081777927 * 2.066666666666667 - log2(1.4907987150 / k)
        = +4.3384201973 octaves
 Q_inf  = (0.30 * 0.563084802746 - 0.62) / 0.09  =  -5.0119395464

VERDICT: DECAYS. Short by 4.3384 octaves = 5206.10 cents.
```

Two readings of the same verdict, both worth stating. **The pair fields `N_eff = 1.7561` distinct perspectives and must field `e^(c/eta) = 7.8985` to break even** — they are at 22% of the geometry they need. And the pair sits 1949.66 cents above the ground essence where 5206.10 cents more is required.

The interpretation that the number does NOT license, stated because it is the one a reader will reach for: this pair is not failing at anything, is not incompatible, and is not in difficulty. The verdict is that *at these costs and this geometry, the arrangement does not accumulate order.* It is a statement about an arrangement. **Whether the arrangement should change, and whether either party wants it to, are questions the formalism does not touch and this book does not answer.**

Step 20 — the published form.

```
RELATIONSHIP FREQUENCY READING -- CONSTRUCTED CASE, SPINE §3.2 phase 2a
 ladder: 12-state, Ch 5 §5.6, unaltered  N = 12  k = 0.483425732988414 Hz
band 220-440 Hz  instrument 0-1, magnitude estimation, ratio question asked
  A: {4:0.7, 6:0.5, 9:0.4}            ||A|| = 0.9486832980505138
B: {4:0.6, 6:0.55, 9:0.3, 10:0.2}  ||B|| = 0.8902246907382427  ratio 1.065667
  ROUTE A  cos = 0.965020899   u = 0.068734665264634
ROUTE B  m = 120  a = 0.983333333333  cos = +0.966666666667  u = 0.065555556
           resolution 4/m = 0.033333
ROUTE C  m = 24  rho = +0.9650 (SIGN POSITIVE)  u = 0.068775  accuracy: published
  CONCORDANCE  spread 70.52 cents   CONCORDANT
  F = 0.563084802746 nats   N_eff = 1.7560813179
 f_rel = 1.4907987150 Hz  folded 381.644 Hz (j = 8)  1949.66 cents above k
  realised pitch fraction 22.66%
  COLLISION TEST  states 4   distinct pitch classes 4   collisions: none
  uncertainty  u +/- 0.01  ->  reading spans 434.34 cents
  CEILING  c/eta = 2.0667 <= ln 12 = 2.4849   TUNABLE
    f_min = 30.1587698405 Hz    D = +4.3384201973 octaves    Q_inf = -
5.0119395464
  VERDICT  DECAYS
CLAUSES  essence frequencies, not physical resonances; the twelve states are not
       diagnoses; this is not a clinical, therapeutic or counselling assessment
       of anyone; CONSTRUCTED case; consent recorded, decline available throughout
```

THE SAME PAIR, AFTER — what changed in the measurement

SPINE §3.2's phase 2b moves the same dyad to `u = 0.692372881355932` with `eta = 0.44` and `c = 0.40`. Chapter 4 §4.20 gives the realising vector and Chapter 18 owns the intervention. Read through this chapter's instrument, four things change:

	PHASE 2a	PHASE 2b
u	0.068734665264634	0.692372881355932
cos theta	0.965020899	0.554643
f_rel	1.4907987150 Hz	35.8881768941 Hz
cents above k	1949.66	7456.88
N_eff	1.7560813179	8.6161016949
realised fraction	22.66%	86.67%
implied Route B a	0.982510	0.777322
f_min	30.1587698405 Hz	2.9782153324 Hz
D	+4.3384201973	-3.5909886634
Q_inf	-5.0119395464	+6.0844267427
VERDICT	DECAYS	ACCUMULATES

The seventh line is the one this chapter contributes and it is worth dwelling on. Through Theorem 13.1's identity, phase 2b's cosine implies an agreement rate of 0.777322 — so on a shared item set the intervention would show up as the pair moving from matching on 98% of items to matching on 78%.

And here is the trap that Chapter 14 exists to close: that sentence can be read as a recom-

mendation to disagree more, and it is not one. The agreement rate fell because B's represented state genuinely changed — B took on two states A does not hold. A pair who left their states untouched and simply started voting against each other would move a in the same direction and would move u in the *opposite* direction once a passed 0.5, and would arrive at `u = 0` if they carried the inversion to completion. **The agreement channel is symmetric about `a = 0.5` and the formalism does not care which side of it you are on.**

What this chapter established

Established, and checkable: three routes to u between two people, of which one is non-negative by construction and two are signed; the exact agreement identity `u = 4a(1−a)` with `cos = 2a−1`; the agreement channel's resolution limit of `4/m`, worth 643 cents per disputed item in a 120-item log; that u is blind to pitch-class collision, exhibited by two dyads identical to fifteen decimal places and differing in class count; that the intensities cannot be ordinal, because an ordinal-preserving relabelling moves u by 1065.05 cents; and a twenty-step procedure with eight gates, of which two have no workaround.

Not established, and not claimed: that u measures agreement, rapport, compatibility or quality; that a high reading is good news; that surplus implies correctness; that any pair of people physically resonates at any frequency here; or that anything in this chapter is a clinical, therapeutic or counselling assessment of anyone.

The one line to carry forward: *a dyad's pitch is its surplus, monotonically — and the pitch is worth exactly what the intensities behind it are worth, which is why they must be magnitude-estimated and why both norms are published.*

Chapter 13 owns the measurement of u between two people: the three routes, the agreement identity, the collision test, the concordance rule, and PROCEDURE REL-1. Chapters 15, 17 and 24 cite it. `f_rel` and the glide are Chapter 12; `f_min`, the octave deficit and the four levers are Chapter 4; elicitation, the two invariances and the magnitude reporting rule are Chapter 6; the overlap condition is Chapter 9; the consonance/redundancy identity is Chapter 10. Chapter 14 takes the symmetry this chapter's agreement table displays — that `a = 1` and `a = 0` produce the identical reading — and prices it.

Chapter 14 . Consonance, Redundancy, and the Sycophant's Unison

Chapter 12 built the glide and Chapter 13 measured the quantity that moves along it. This chapter goes to the bottom of the glide and finds something there that the rest of Part III depends on.

At `u = 0` the relationship frequency is `f_rel = k` — the ground essence, 0.4834 Hz, the same note a single undifferentiated state sounds and the same note a person with forty identical framings sounds. That much is arithmetic and Chapter 12 §12.3 already stated it.

The result this chapter owns is what *else* is down there.

```
R8 -- THE UNISON RESULT

    u = 1 - (x . y)^2   depends on the correlation ONLY through |rho|.

    rho = +1  ->  u = 0  ->  f_rel = k = 0.4834 Hz
    rho = -1  ->  u = 0  ->  f_rel = k = 0.4834 Hz

PERFECT OPPOSITION AND PERFECT AGREEMENT ARE THE SAME READING, EXACTLY.
The contrarian and the sycophant sound the identical note.
```

Two people who agree about everything and two people who disagree about everything produce the same relationship frequency, the same surplus, the same effective perspective count, and the same destiny — and the formalism does not merely fail to distinguish them, it proves them indistinguishable.

That is the sharpest result in Part III, and it is the one most likely to be resisted, because "creative tension" and "healthy disagreement" and "we need someone to push back" are among the most durable pieces of received wisdom about how two or more people should work. The chapter's job is not to be contrarian about contrarians. Its job is to show that the received wisdom names a real target and points at the wrong coordinate — the target is independence, and opposition is independence's mirror image, sitting exactly as far from it as agreement does.

The chapter then does something the result makes possible and which the received wisdom cannot do at all: it prices the two faults separately. They are not the same fault.

```
THE PARALLEL WALL          the seats are already filled and already paid for.
                             They return almost nothing. The waste is sunk.

MANUFACTURED OPPOSITION     a NEW seat is added, at a NEW cost, and it returns
```

```
                                    nothing — and in a group it can lower the mean
                                      non-parallelism below where it already was.

    Same reading. Different ledgers. Different remedies.
```

And it ends with a design rule that follows from the arithmetic and from nothing else: target `|rho|` near zero, never `rho` near −1.

Standing constraints, restated because this chapter's subject invites drift. Essence frequencies are essence frequencies; nothing here vibrates. **The twelve states are not diagnoses, this is not counselling or therapy, and nothing in this chapter is an assessment of any person, pair or group.** No intervention derived from any result here may remove a person's ability to decline — which in this chapter has a specific and unusually literal meaning, taken up in §14.9. All cases are CONSTRUCTED and labelled as such.

The Law

14.1 The unison

DEFINITION 14.1 — A UNISON. A dyad with `u = 0`. Equivalently `|cos theta| = 1`; equivalently `N_eff = 1`; equivalently `F = 0`; equivalently `f_rel = k` exactly.

Every one of those five is the same condition written in a different unit, and the fifth is the one this chapter is named for. A unison is not a low reading. **It is the absence of a reading: the pair occupies zero cents of the 8603.91 available, realises 0.00% of the system's surplus, and is positioned exactly where a pair that does not exist would be positioned.**

The parallel with Chapter 10 §10.5 is exact and is not a coincidence of notation. There, a person holding forty framings that are all the same framing has `u_int = 0`, `N_eff = 1`, `F_int = 0`, and a chord width of exactly `k`. The bottom of the personal scale and the bottom of the relational scale are the same note, because both are the statement that nothing distinct is present.

14.2 R8, stated and proved

THEOREM 14.2 (R8) — THE OPPOSITION IDENTITY. For two represented states with correlation `rho`,

```
u = 1 - rho^2
```

so u depends on `rho` only through `|rho|`. Consequently `u(rho) = u(-rho)` for every `rho`, and in particular

```
u(+1) = u(-1) = 0        F(+1) = F(-1) = 0        f_rel(+1) = f_rel(-1) = k
```

Proof. Chapter 12 Theorem 12.1 gives `f_rel = k(1 + (N-1)u)^2` with `u = 1 - (x.y)^2` for unit vectors, and `x . y` is the correlation `rho`. Squaring is an even function: `(-rho)^2 = rho^2`. Therefore u is even in `rho`, and every quantity computed from u — `F`, `N_eff`, `f_rel`, the octave deficit, `Q_inf` — is even in `rho` as well. At `rho = ±1`, `u = 1 - 1 = 0`, so `N_eff = 1`, `F = ln 1 = 0`, and `f_rel = k * 1^2 = k`. QED.

The proof is three lines and it is complete. **There is no term anywhere in the construction that carries the sign of the correlation, so no quantity derived from it can recover the sign.** This is not a limitation of the twelve-state ladder, not an artefact of `N = 12`, and not a consequence of the affine axiom S3. It is a property of the inner product itself: `u = sin^2 theta`, and the sine squared of an angle and of its supplement are equal.

Book Two stated R8 in the general register and Chapter 2 §2.5 restated it in this book. This chapter owns it at relational scope, which is where it does its damage.

14.3 The symmetry, computed

The whole correlation range, with every derived quantity:

`R8 ACROSS THE FULL CORRELATION RANGE (N = 12, k = 0.483425732988414 Hz)`

rho	|rho|	u = 1−rho^2	f_rel (Hz)	cents above k	folded (j)
−1.0	1.0	0.000000	0.4834	0.00	247.514 (j=9)
−0.9	0.9	0.190000	4.6158	3906.26	295.411 (j=6)
−0.8	0.8	0.360000	11.8930	5544.82	380.577 (j=5)
−0.7	0.7	0.510000	21.1219	6539.16	337.950 (j=4)
−0.6	0.6	0.640000	31.2494	7217.27	249.995 (j=3)
−0.5	0.5	0.750000	41.3631	7702.69	330.905 (j=3)
−0.4	0.4	0.840000	50.6909	8054.75	405.527 (j=3)
−0.3	0.3	0.910000	58.6009	8305.78	234.404 (j=2)
−0.2	0.2	0.960000	64.6019	8474.57	258.408 (j=2)
−0.1	0.1	0.990000	68.3429	8572.02	273.372 (j=2)
0.0	0.0	1.000000	69.6133	8603.91	278.453 (j=2)
+0.1	0.1	0.990000	68.3429	8572.02	273.372 (j=2)
+0.2	0.2	0.960000	64.6019	8474.57	258.408 (j=2)
+0.3	0.3	0.910000	58.6009	8305.78	234.404 (j=2)
+0.4	0.4	0.840000	50.6909	8054.75	405.527 (j=3)
+0.5	0.5	0.750000	41.3631	7702.69	330.905 (j=3)
+0.6	0.6	0.640000	31.2494	7217.27	249.995 (j=3)
+0.7	0.7	0.510000	21.1219	6539.16	337.950 (j=4)
+0.8	0.8	0.360000	11.8930	5544.82	380.577 (j=5)
+0.9	0.9	0.190000	4.6158	3906.26	295.411 (j=6)
+1.0	1.0	0.000000	0.4834	0.00	247.514 (j=9)

The table is a perfect mirror about `rho = 0`, and the two ends are the same row. Both extremes fold to 247.514 Hz with `j = 9`, pitch class 1 — the same note, at the same transposition, in the same class.

Chapter 13 §13.4 exhibited the identical symmetry from the measurement side: on a shared item set of `m` judgements, `u = 4a(1−a)`, which is symmetric about `a = 0.5` and equal to zero at `a = 1` and at `a = 0`. A pair matching on every item and a pair matching on no item produce the same reading, and that is a fact a practitioner can verify in a spreadsheet.

14.4 What dies at u = 0 — and it is not what a reader expects

The unison is not merely a low position. It is a position at which the instrument's own levers stop working, and exactly which ones stop is the surprise.

THEOREM 14.3 — THE UNISON FIXED POINT. At u = 0,

```
F = 0,  therefore  Q_inf = (eta * 0 - c) / delta  =  -c / delta
```

eta and N do not appear. *Proof:* by inspection of Paper E Theorem 3.5 with F = 0. QED.

Verified across a wide grid, holding c = 0.62 and delta = 0.09:

eta	N	octave deficit D	Q_inf
0.05	2	35.7788	−6.888889
0.05	12	35.7788	−6.888889
0.05	25	35.7788	−6.888889
0.05	100	35.7788	−6.888889
0.30	2	5.9631	−6.888889
0.30	12	5.9631	−6.888889
0.30	25	5.9631	−6.888889
0.30	100	5.9631	−6.888889
0.60	12	2.9816	−6.888889
0.99	12	1.8070	−6.888889

A twentyfold improvement in conversion efficiency, and an eightfold increase in the category count, leave the destination bit-for-bit unchanged. The deficit moves — D depends on eta explicitly — but the fixed point does not, because the two dependencies cancel exactly through Q_inf = −D eta / (2.885390 delta). That cancellation is Chapter 4 Theorem 4.5 arriving at its own degenerate case, and both readings are correct.

The partial derivatives make the same point in the form a practitioner uses:

```
PARTIALS OF Q_inf   (eta = 0.30, delta = 0.09, N = 12, c = 0.62)
```

u	dQ/du	dQ/deta	dQ/dc	dQ/dN
0.000000	36.6667	0.0000	−11.1111	0.0000
0.068735	20.8798	6.2565	−11.1111	0.1305
0.500000	5.6410	20.7978	−11.1111	0.2564

At u = 0 exactly two of the four levers have any effect on the destination, and the u lever is at its strongest value anywhere on the range. dQ/deta = F/delta is zero because there is nothing to convert; dQ/dN = eta u / ((1+(N−1)u) delta) is zero because N enters only multiplied by u. Raising conversion efficiency in a unison relation converts nothing more efficiently. Adding categories to a unison relation adds dimensions nobody is using.

This is the formal content of the phrase "no relationship at all." It is not rhetoric. Two of the four levers named in CANON3 §3 are not merely weak at u = 0; their derivatives are exactly zero.

14.5 Two people in unison are one person paying twice

Take the carried dyad's costs and set u = 0:

```
    UNISON DYAD    eta = 0.30, c = 0.62, delta = 0.09, F = 0
                   eta*F - c = -0.620000 per period
                   Q_inf = -6.888889        D = +5.9631 octaves

      SOLO ACTOR       same delta, same eta, F = 0, but c = 0.20 --
no coordination overhead
                   eta*F - c = -0.200000 per period
                   Q_inf = -2.222222        D = +1.9236 octaves
```

CONSTRUCTED comparison; the solo `c` is a stipulated value chosen to represent a relation without coordination overhead, and nothing turns on its exact size.

```
   THE SECOND SEAT COSTS  4.666667 units of Q_inf  and  4.0395 octaves of deficit
    THE SECOND SEAT RETURNS 0.000000
```

The entire ledger of a unison pair is cost. The numerator `eta F` is exactly zero, so the second person contributes nothing to the accumulation term and everything to the loss term. That is the arithmetic behind the chapter's title line: **two people in unison are one person paying twice**, and the multiplier on the paying is the coordination overhead the second seat introduced.

And the sentence is symmetric, which is the whole point. Replace the sycophant with a contrarian who inverts every position — `rho = -1` instead of `rho = +1` — and every number in the block above is unchanged. Same `u`, same `F`, same `f_rel`, same `D`, same `Q_inf`. The contrarian is also one person paying twice, and paying at least as much.

14.6 Where opposition is reachable, and where this book cannot see it

An honest chapter has to say where its own encoding cannot go, and this one has a real limit.

Chapter 13 §13.2 established it: **the twelve-state intensity vectors are non-negative by construction, so `cos theta >= 0` for every pair, and the negative half of the correlation range is unreachable through Route A.** Chapter 10 §10.5 verified the same bound at internal scope over 100,000 random non-negative unit pairs — minimum observed cosine 0.260867 — and noted that the bound `cos >= 0` is exact rather than empirical.

So within Route A, `u = 0` can arise only from parallelism. The sycophant is representable; the contrarian is not.

```
    WHERE R8's TWO BRANCHES LIVE

   ROUTE A  state vectors, non-negative     cos in [0, 1]   agreement branch ONLY
    ROUTE B   shared judgements, +1/-1        cos in [-1, 1]   BOTH branches
    ROUTE C   error vectors vs known answers   rho in [-1, 1]   BOTH branches
```

The opposition branch of R8 is measured on the signed channels or it is not measured at all, and that is exactly why Chapter 13 built three routes rather than one. A practitioner running only Route A will never see a negative correlation and may conclude the branch is theoretical. It is not theoretical; it is invisible to that instrument.

The limitation is worth stating plainly rather than working around. The twelve states record which components a construal requires, not which direction it points, so a framing that is the exact negation of another cannot be represented in Route A at all. A signed encoding would be a

different instrument, and this book does not have one. What it has is the agreement channel and the error channel, both of which carry a sign, and a requirement — Chapter 13 SPEC-13.3 — that the sign be published because u discards it.

14.7 The consonance/redundancy identity, at relational scope

Chapter 10 established, by exhaustive enumeration over all sixty-six index pairs at `N = 12`, that the minimum essence harmonic distance is attained by exactly the six 2:1 pitch-class collisions, and that zero folded separation is *equivalent* to sharing an odd part. The most consonant pairs the lattice offers are precisely the pairs it cannot distinguish.

At relational scope the identity reads:

COROLLARY 14.4. The most consonant possible dyad is the most redundant possible dyad, and its frequency is k.

Proof. Consonance in this system is small essence harmonic distance; the extreme is exact coincidence, which for two parties means identical normalised state vectors; identical vectors give `cos theta = 1`, hence `u = 0`, hence `f_rel = k`. QED.

Chapter 10's extremal enumeration exhibits the same thing as a chord: among all 495 four-state chords at `N = 12`, the most consonant is `{1, 2, 4, 8}` — every one of its six pairs a collision, all four folding to 247.514 Hz. **One note, four times.** A dyad in unison is that structure with two members instead of four.

And Chapter 10's honest caveat travels with the identity and is not dropped here: the ordering is not total. It is not true that every collision is more consonant than every non-collision — `1:3` at `HD_ess = 3.1699` beats the collisions `1:4`, `2:8`, `3:12` at 4.0000. The collision set is dramatically more consonant on average (mean 3.0000 against 8.6617) and the ranges overlap. A reader who builds on the total-order version will eventually be wrong about `1:4`.

There is one further inheritance from Chapter 10, and it is the one that keeps this chapter from becoming a polemic against agreement. **Chapter 10's firewall stands: nothing about distress, health, function or experience follows from a consonance reading, in either direction.** A unison dyad is not a bad relationship, an unhealthy one, or one that anybody should change. It is a relation that, at these costs, does not accumulate order. Those are different sentences and this book will not merge them.

14.8 The flat maximum — why `|rho| = 0` is the design target and `rho = −1` is not

If `u = 1 − rho^2`, the maximum of u is at `rho = 0` and the value there is 1. That is obvious. The useful part is *the shape of the maximum*, which is not obvious and which decides the design rule.

```
du/drho = −2 rho          at rho = 0 the derivative is EXACTLY ZERO
```

The optimum is second-order flat: near `rho = 0`, an error in the correlation costs almost nothing, and the cost grows quadratically rather than linearly. Computed, as the pitch given up relative to perfect independence:

```
|rho|      u          f_rel (Hz)     cents surrendered vs rho = 0
 0.00    1.000000      69.6133              0.00
 0.05    0.997500      69.2946              7.94
```

0.10	0.990000	68.3429	31.89
0.20	0.960000	64.6019	129.34
0.30	0.910000	58.6009	298.13
0.50	0.750000	41.3631	901.22
0.90	0.190000	4.6158	4697.65
0.95	0.097500	2.0764	6080.62
0.99	0.019900	0.7182	7918.52
1.00	0.000000	0.4834	8603.91

And the marginal cost of one further hundredth of correlation, which is the number a designer wants:

from |rho| = 0.00 to 0.01	0.32 cents
from |rho| = 0.10 to 0.11	6.73 cents
from |rho| = 0.50 to 0.51	41.84 cents
from |rho| = 0.90 to 0.91	230.61 cents
from |rho| = 0.99 to 1.00	685.39 cents

Missing perfect independence by a tenth costs 31.89 cents — a sixth of a semitone. Missing it by a hundredth costs a third of one cent. Missing perfect *opposition* by the same tenth costs 4697.65 cents, because at `|rho| = 0.9` the function is steep in exactly the way it is flat at zero.

> **THE DESIGN RULE [BINDING ON EVERY INTERVENTION IN THIS BOOK].** Where a relation is to be arranged rather than merely read, **target `|rho|` near zero and never `rho` near −1.** Independence is a robust target — second-order insensitive to error, forgiving of imprecision, and the global maximum of the surplus. Opposition is the *other* global minimum, sits at the steepest part of the curve, and returns exactly what agreement returns: nothing.

The rule is not a preference for harmony over conflict. It says nothing about how two people should treat each other. It says that if the goal is available order, the coordinate to aim at is uncorrelated, and both of the correlated extremes are worthless — one of them expensively so.

14.9 Honesty clauses, and one that belongs specifically to this chapter

(1) ESSENCE, NOT RESONANCE. Nothing here vibrates. `k = 0.483425732988414 Hz` is a chosen reference constant and every frequency in this chapter is a position on a logarithmic surplus scale.

(2) NOT CLINICAL, NOT COUNSELLING, NOT THERAPY. The twelve states are not diagnoses. Nothing in this chapter is an assessment of any person, pair or group, this book does not offer counselling or therapy, and nothing here substitutes for a qualified professional.

(3) NOT A JUDGEMENT OF PERSONS. "Sycophant" and "contrarian" name *positions in a correlation structure*, not people and not characters. A person may occupy `rho = +0.98` on one item set and `rho = 0.1` on another. The words describe a coordinate, and this book uses them for the coordinate and never for a person.

(4) CONSENT IS STRUCTURAL, AND HERE IT IS LITERAL. This chapter's material is uniquely prone to one abuse: an organisation reads the result, concludes that difference is valuable, and *requires* people to hold positions. **No intervention derived from anything in this book may remove a person's ability to decline — and requiring a person to argue a position they do**

not hold is exactly such a removal. It is also, by §14.11's arithmetic, useless. The prohibition and the pricing agree.

(5) SURPLUS IS A LOWER BOUND. A verdict of ACCUMULATES survives representational undercapture; DECAYS does not. A unison reading may mean two parties are parallel, or it may mean the elicitation failed to capture what distinguishes them. §14.13 gives the diagnostics that separate those two, and they must be run before a unison verdict is published.

(6) CONSTRUCTED CASES. Every case in this chapter is constructed and labelled as such.

The Engineering

14.10 Pricing the Parallel Wall

The Parallel Wall is Chapter 4 §4.17's name for a set of parties whose seats are filled and whose views are near-parallel. SPINE §3.3 carries the case: six members drawn from one doctrine, fifteen pairwise links, `kappa = 0.055` per link, `eta = 0.34`, `delta = 0.09`.

```
THE COMMONS AS FOUND -- CONSTRUCTED CASE, SPINE §3.3

15 pairs, mean pairwise u = 0.018614       F = 0.186272 nats
f_rel = 0.7017 Hz        644.97 cents above k      realised fraction 7.50%
C = 0.055 x 15 = 0.8250    c/eta = 2.426471
f_min = 61.9349 Hz       D = +6.4638 octaves      Q_inf = -8.4630    DECAYS
```

The seat arithmetic is the part this chapter contributes:

```
THE PARALLEL WALL, PRICED PER LINK

paid            C = 0.8250 across 15 links
delivered       644.97 cents of pitch
yield            43.00 cents per link

ceiling for 15 links at u = 1     8603.91 cents  =  573.59 cents per link
realisation                        644.97 / 8603.91  =  7.50%
```

The commons is paying for fifteen links and collecting 7.50% of what fifteen links could deliver. The other 92.50% is not lost to inefficiency, friction, or bad management. It is not there. Nobody holds it. The waste in the Parallel Wall is sunk: the seats are filled, the links are paid, and the remedy is not to spend more but to change who is in the seats or what they are doing.

One further number, which is where the case becomes alarming rather than merely disappointing:

```
the ceiling test on the commons as found
c/eta = 2.426471   ln N = 2.484907   2.4265 <= 2.4849  ->  TUNABLE, barely

the maximum affordable C at 15 links:  eta * ln N = 0.844868
the maximum affordable kappa:          0.844868 / 15 = 0.056325
the actual kappa:                      0.055000
```

The commons sits 2.4% below the cost ratio at which no arrangement of twelve categories could save it. It is tunable — Chapter 18 will show that closing 6.4638 octaves requires `u = 0.95`, which is very near orthogonality and is not a realistic target — but it is tunable by the narrowest margin the case could have had.

14.11 Pricing manufactured opposition

Now the intervention this chapter exists to forbid. **CONSTRUCTED extension of SPINE §3.3, labelled as such.** A seventh member is recruited specifically to disagree: their represented position is the inversion of the group's, so `rho = −1` against each of the six, hence `u = 0` on each of the six new pairs.

```
THE SEVENTH SEAT, MANUFACTURED OPPOSITION

links        15 -> 21          C = 0.055 x 21 = 1.1550   (+0.3300, +40.0%)
mean u        (15 x 0.018614 + 6 x 0) / 21 = 0.013296       (was 0.018614)
f_rel          0.7017 -> 0.6352 Hz                 -172.35 cents
F              0.186272 -> 0.136498 nats
eta*F          0.063334 -> 0.046409
net per period  -0.761666 -> -1.108591
D             +6.4638 -> +9.4080 octaves         worse by 2.9442 octaves
Q_inf          -8.4630 -> -12.3177
```

The added seat costs 0.3300 per period and lowers the group's pitch by 172.35 cents. Both signs are wrong: cost up, surplus down. That is not a bad trade — it is not a trade at all, because there is nothing on the other side of it.

But the decisive number is the one the deficit hides, and it is why Chapter 4 SPEC 4.1 insists the ceiling test comes first:

```
THE CEILING TEST AFTER THE SEVENTH SEAT

c/eta = 1.1550 / 0.34 = 3.397059          ln N = 2.484907
3.3971 > 2.4849   ->   STRUCTURAL

required octaves at this cost ratio        9.8018
ceiling octaves for twelve categories      7.1699
residual even at u = 1 (perfect orthogonality across all 21 pairs)  +2.6319
```

Adding a seventh seat at this cost per link converts a tunable commons into a structural one: above the ceiling, no assignment of u whatsoever makes it sustainable — not the contrarian, not a genius, not anybody. The maximum kappa at which twenty-one links stay under the ceiling is 0.040232, so the seat is affordable only if the per-link cost falls by 26.9% at the same time.

And the same arithmetic run on the alternatives, so the comparison is complete:

```
THE SEVENTH SEAT, FOUR CANDIDATES  (all CONSTRUCTED; C = 1.1550, eta = 0.34)

seventh member              mean u    f_rel (Hz)   D       Q_inf    vs base
```

```
    manufactured contrarian (rho=−1)  0.013296      0.6352    +9.4080    −
12.3177   −172.35c
    mirror / sycophant     (rho=+1)   0.013296      0.6352    +9.4080    −
12.3177   −172.35c
    genuinely distinct     (u=0.80)   0.241867      6.4777    +6.0577    −
7.9313  +3847.96c
    perfectly independent  (u=1.00)   0.299010      8.8933    +5.6005    −
7.3326  +4396.66c
```

Three things in that table matter and all three are exact.

The first two rows are identical to every printed digit. The contrarian and the sycophant do not merely produce similar outcomes; they produce the same outcome, because R8 says they must.

The genuinely distinct seventh member buys 3847.96 cents and still leaves the case structural. `D = +6.0577` is an improvement of 0.4061 octaves on the base case, and it is meaningless, because `c/eta = 3.3971` is above the ceiling and no deficit computed there is closable by geometry. A real improvement in the geometry, computed correctly, applied to a case that the cost ratio has already placed beyond repair. That is exactly the situation Chapter 4 SPEC 4.1 was written to prevent, and seeing it happen to the *good* candidate is more instructive than seeing it happen to the bad one.

And SPINE §3.3's standing verdict is unchanged and is worth restating: no single lever closes a 6.4638-octave gap. Raising u to 0.95 gets to `−0.0332` and 0.95 is near-orthogonality; raising `eta` to 0.90 still leaves `+2.1075`; cutting `kappa` to 0.020 still leaves `+2.0085`. Some systems cannot be tuned. A book that only showed fixable examples would be lying by selection, and this chapter's contribution is to show that one popular "fix" makes the unfixable case worse in two independent ways at once.

14.12 The two faults on one ledger

```
    FAULT                        WHAT IT IS                      THE LEDGER
    ------------------------------------------------------------------------
-------
    THE PARALLEL WALL           seats already filled,        eta*F = 0.063334
                                 near-parallel views           C     = 0.8250
                                                             net   = −0.761666
                                                               D     = +6.4638
                                the waste is SUNK              Q_inf = −8.4630

    MANUFACTURED OPPOSITION     a NEW seat, at a NEW cost,   eta*F = 0.046409
                                 returning nothing             C     = 1.1550
                                                             net   = −1.108591
                                the waste is INCURRED,         D     = +9.4080
                                and it is ongoing             Q_inf = −12.3177
```

The distinction is not academic and it decides the remedy. The Parallel Wall's cost is already paid: the correct response is to change what the existing members are doing, or to change who they are, and the money is gone either way. Manufactured opposition's cost is *prospective*: it has not

been paid yet, the decision to pay it is live, and the correct response is simply not to.

Put in one line: **the Parallel Wall wastes seats you already had; manufactured opposition pays to add a seat that returns nothing.** They read the same on the frequency scale and they do not read the same on the ledger.

14.13 Telling independence from inversion, and both from a failed elicitation

A unison reading has three possible causes and they require three different responses. Publishing one before excluding the other two is the most consequential error available in this chapter.

CAUSE	DIAGNOSTIC	RESPONSE
genuine parallelism (the sycophant branch)	Route B: a near 1.0 Route C: rho near +1, SIGN POSITIVE	the reading stands
genuine inversion (the contrarian branch)	Route B: a near 0.0 Route C: rho near −1, SIGN NEGATIVE	the reading stands; remedy differs entirely
failed elicitation (no branch; an artefact)	Route A alone; no signed channel run; norms wildly asymmetric; support overlap at one small shared term	RE-ELICIT. Publish nothing.

The sign of `rho` is the only quantity that separates the first two, and `u` discards it, which is why Chapter 13 SPEC-13.3 requires it to be published and FM-13.8 treats dropping it as a failure mode. Two parties at `rho = +0.9650` and two parties at `rho = −0.9650` produce identical readings and require opposite interventions: the first pair needs someone to be doing something different, the second pair needs to stop performing.

The third row deserves as much attention as the first two. A unison reading is exactly what a failed elicitation produces when both parties are described in the same words, and the failure is easy to induce: a practitioner who elicits both parties in the same session, using the same prompts, after they have discussed the situation, will frequently recover two near-identical vectors that say more about the session than about the pair. Chapter 13 Step 4's independent silent write-first is the defence, and Chapter 13 FM-13.10 states the failure.

14.14 What to do instead — and what a red team actually is

The design rule of §14.8 says target `|rho|` near zero. Here is what that means operationally, and what it rules in as well as out.

```
RULED OUT -- these move rho toward −1 and return nothing
  - assigning a person to argue the opposite of whatever is proposed
  - rewarding disagreement as such, or counting objections as output
- recruiting for temperament ("we need a sceptic") rather than for method or evidence ba
- any arrangement that requires a person to hold a position they do not hold

RULED IN -- these move |rho| toward 0
  - different evidence base: parties who look at different data
```

```
    - different method: parties whose procedures fail in different ways
  - different exposure: parties who bear different consequences from the same decision
  - different training: parties whose priors were formed on different problems
  - independent work before contact (Chapter 13 Step 4, at organisational scope)
```

The distinction between the two lists is not attitude; it is whether the difference is in what the parties actually do or in what they say about what someone else did. A person with a different evidence base disagrees when the evidence differs and agrees when it does not, so their correlation with the group is low in magnitude and of no fixed sign. A person assigned to disagree has a correlation near −1 by construction, and R8 prices it at zero.

That is the honest reading of the red-team tradition, and it is a defence of the practice rather than an attack on it. **A red team that brings different methods, different data and different exposure sits near `rho = 0` and is worth up to the full 8603.91 cents. A red team that is one person told to object sits near `rho = −1` and is worth nothing at all.** The tradition names a real target; the arithmetic says which implementations hit it.

There is a further consequence for how such a role is filled, and it is where §14.9's clause (4) becomes operational rather than decorative. A role defined as "hold the contrary position" is a role in which declining is structurally unavailable — the person cannot say "I agree with this one" and still be doing the job. A role defined as "bring this evidence base" leaves declining intact on every individual question. The first is prohibited by this book's consent constraint and is worthless by its arithmetic; the second is neither.

14.15 The instrument that rewards agreement

One last engineering note, and it is about the reading rather than the relation.

An elicitation procedure run by a practitioner who has views produces vectors that drift toward those views. Chapter 6 FM-6.15 owns the general failure — leading the elicitation — and at dyad scope it has a specific and self-concealing form: an elicitation that nudges both parties toward the practitioner's framing lowers `u`, lowers `f_rel`, and produces exactly the unison reading that the practitioner will then interpret as a finding about the pair.

The check is Chapter 13 Step 17's second analyst, working from the written framings alone, and it is the only one available. **A single analyst cannot detect their own convergence pressure, because the artefact and the finding have the same shape.**

There is a mirror failure and it is worth naming in the same breath. A practitioner who has read this chapter and wants a high reading can lead the elicitation the other way — toward difference — and will succeed. An instrument's failure modes are symmetric about the operator's expectation, and neither direction is detectable from inside a single pass.

The Specification

SPEC-14.1 — The unison test

```
Run whenever a reading returns f_rel within 100 cents of k (i.e. u <= 0.005368).
```

```
1.  Confirm the routes run. If Route A only, the sign of the correlation is
    UNKNOWN and no branch may be named.
2.  Run a signed route (B or C). Record the sign of rho explicitly.
3.  Classify:  rho near +1  ->  AGREEMENT branch
               rho near -1  ->  OPPOSITION branch
               no signed route -> UNDETERMINED; publish as such
4.  Run the elicitation-artefact checks: independent write-first confirmed?
    norms recorded and comparable? support overlap not resting on one small term?
    second analyst run?
5.  Publish the branch WITH the reading. The frequency alone does not carry it.
```

A unison reading published without its branch is incomplete, because the two branches have the same number and opposite remedies.

SPEC-14.2 — The seat-addition rule

```
BEFORE adding a member to any set of size n:

1.  CEILING TEST AT THE NEW COST FIRST.
    new links = n(n+1)/2 ; new C = kappa * new links ; is C/eta <= ln N ?
    If NO: the addition makes the case STRUCTURAL. Stop. No candidate fixes it.
2.  Estimate the candidate's pairwise u against each existing member.
    If the candidate's expected |rho| against the group is near 1 in EITHER sign,
    the expected contribution to mean u is zero and the seat returns nothing.
3.  Compute the new mean u over all n(n+1)/2 pairs and the new D.
    Report the change in cents and the change in D, separately.
4.  Report BOTH the pitch change and the ceiling verdict. A pitch improvement
    above the ceiling is not an improvement.
```

Step 1 comes first because a candidate assessment run after the cost ratio has crossed the ceiling is an assessment of a question that no longer has an answer.

SPEC-14.3 — Reporting a unison or near-unison

```
UNISON READING
f_rel = ..... Hz  cents above k = .....  realised pitch fraction = .....%
  routes run: A [ ]  B [ ]  C [ ]
sign of rho: + / - / UNDETERMINED       branch: AGREEMENT / OPPOSITION / UNDETERMINED
  Q_inf = -c/delta = .....      (eta and N do not enter; Theorem 14.3)
  levers with zero derivative at this reading: eta [ ]  N [ ]
elicitation-artefact checks: independent write-first [ ]  norms [ ]  overlap [ ]
                             second analyst [ ]
CLAUSES: essence not resonance; not a diagnosis; not counselling or therapy;
      "sycophant"/"contrarian" name COORDINATES, not persons; CONSTRUCTED
```

SPEC-14.4 — Gates

```
G1 BRANCH.   no unison reading is published with a named branch unless a signed
             route was run. Route A cannot see the opposition branch at all.
```

```
G2 CEILING.   every seat addition runs the ceiling test at the NEW cost, first.
G3 ARTEFACT.  the three elicitation-artefact checks are run before any unison
          verdict. A unison is the signature of a failed elicitation as well
                as of a parallel pair.
G4 CONSENT.   no intervention arising from this chapter may require a person to
             hold a position. Declining stays available on every question.
G5 LANGUAGE.  the words "sycophant" and "contrarian" appear in a report only as
          labels for correlation coordinates, never as descriptions of people.
```

The Failure Modes

FM-14.1 — Recruiting a contrarian to raise u

The failure this chapter is built around. `u` measures represented non-parallelism, and a person whose position is the inversion of the group's is perfectly correlated in magnitude. The seat costs its links, adds its friction, and contributes exactly zero to the numerator — and in a group it drags the mean pairwise u down, because the new pairs enter the average at zero.

FM-14.2 — Believing the branch can be read off the frequency

`u = 1 − rho^2` is even. **No quantity computed downstream of u carries the sign of the correlation, so no amount of care with the frequency recovers it.** The sign lives in Route B's `a` or Route C's `rho` and nowhere else.

FM-14.3 — Reading "no relationship at all" as a judgement

A unison reading says that at these costs the arrangement accumulates no order. It does not say the two people have no relationship, that their relationship is poor, or that anything about it should change. Two people in unison may be exactly as they wish to be, and the formalism has no view.

FM-14.4 — Calling a person a sycophant or a contrarian

The words name coordinates. A person sits at a correlation with respect to a specific counterparty on a specific item set, and moves when either changes. **Transferring the label from the coordinate to the person converts an instrument into an insult**, and the resulting number is no longer about anything measurable.

FM-14.5 — Adding a seat without the ceiling test

The commons' seventh seat improves the geometry if the candidate is genuinely distinct — 3847.96 cents — and the case is structural anyway, because `c/eta` crossed `ln N` when the links went from fifteen to twenty-one. A geometry improvement computed above the ceiling is arithmetically correct and practically void, and Chapter 4 SPEC 4.1 requires the test that catches it.

FM-14.6 — Treating the two faults as one

They read the same and they are not the same. **The Parallel Wall's cost is sunk and its remedy is substitution; manufactured opposition's cost is prospective and its remedy is refusal.** A report that says only "low u" has told the reader nothing about which of the two decisions is in front of them.

FM-14.7 — Requiring a person to disagree

Prohibited, on two independent grounds. It removes the person's ability to decline, which this book's consent constraint forbids structurally. And by §14.11's arithmetic it returns nothing, so the prohibition costs the organisation no surplus it would otherwise have had. When the ethical constraint and the pricing agree, there is no trade-off to manage.

FM-14.8 — Aiming at `rho = −1` because it is the far end of the range

The intuition that "the opposite of agreeing is disagreeing, so maximum disagreement is maximum difference" is precisely what R8 refutes. The far end of the correlation range is not the far end of the surplus range; the surplus range is folded in the middle. Maximum surplus is at `rho = 0`, and both signed extremes are minima.

FM-14.9 — Missing that `eta` is dead at `u = 0`

A relation at unison gets nothing from improved conversion, because `dQ/deta = F/delta` and `F = 0`. Writing the decisions down, building the artefact, making the practice repeatable — all valuable, all recommended elsewhere in this book, all worth exactly zero to the destination of a unison relation. Fix the numerator first, which is R15 in this chapter's register and Chapter 15's rule.

FM-14.10 — Publishing a unison without excluding the artefact

A unison is what a failed elicitation looks like. Both parties elicited in one session, after discussion, with the same prompts, will often produce near-identical vectors. The reading is then a measurement of the session. §14.13's third row and Chapter 13 Step 4 are the defence.

FM-14.11 — Romanticising difference

Chapter 10 FM-10.1 owns this at internal scope and it transfers unchanged. Nothing in this chapter says that conflict is good, that difficulty produces surplus, or that a relation should be made harder. The quantity being maximised is represented non-parallelism, and independence — not friction — is the coordinate that maximises it. Two people who work on different things, calmly, are at high `u`. Two people who fight about one thing are not.

FM-14.12 — Using the result as counselling

This chapter is not counselling, therapy or mediation, and none of its results is advice about any person's relationships. The unison result is a statement about a formalism: that a particular functional discards the sign of a correlation, with consequences that can be priced. Where a

relationship is causing a person difficulty, the resource is a qualified professional. This book is not one and does not pretend to be.

Worked Example

THE CARRIED DYAD, WITH THREE DIFFERENT PARTNERS

CONSTRUCTED CASE, labelled as such. These are not people. Party A is SPINE §3.2's `A = {4:0.7, 6:0.5, 9:0.4}`, with the case's fixed parameters `eta = 0.30`, `c = 0.62`, `delta = 0.09`. Four partners are compared. The costs are held constant across all four so that the comparison isolates the geometry; Chapter 15 owns what happens when the costs move too.

```
    CEILING TEST FIRST:  c/eta = 2.066667 <= ln 12 = 2.484907   ->  TUNABLE

   PARTNER                      u               f_rel (Hz)     D         Q_inf
   B  as found (SPINE 2a)     0.068734665264634       1.4908    +4.3384    -
5.0119
   B'' the sycophant, rho=+1   0.000000000000000       0.4834    +5.9631    -
6.8889
   B'''the contrarian, rho=-1  0.000000000000000       0.4834    +5.9631    -
6.8889
     B'   genuinely distinct       0.692372881355932         35.8882      -
0.2509     +0.2899
```

Rows two and three are identical in every column, and that is R8. One partner echoes A on every item; the other inverts A on every item; the relation is at the ground essence either way, short by 5.9631 octaves either way, and arrives at `Q_inf = −6.8889` either way.

Three further readings of the same block.

Both faults are worse than the pair as found. The dyad at `u = 0.068735` is already decaying, and replacing B with either extreme costs a further 1.6247 octaves of deficit and 1.8770 of `Q_inf`. The near-parallel pair the book calls marginal is better than either perfectly correlated alternative, in either sign.

The fourth row isolates the geometry lever. With costs held exactly where SPINE §3.2 fixes them for phase 2a — `eta = 0.30`, `c = 0.62` — raising u alone to 0.692373 flips the verdict: `D = −0.2509`, `Q_inf = +0.2899`. **This is not SPINE's phase 2b**, which also moves `eta` to 0.44 and `c` to 0.40 and lands at `Q_inf = +6.0844`. It is the same geometry with the costs frozen, computed here only to show that the numerator lever alone is sufficient to cross the line in this case, and by a margin of a quarter of an octave. Chapter 15 owns the ordering of the levers and R15 governs it.

And the fourth row is unreachable by opposition, which is the chapter's whole argument. B' gets there by taking on states A does not hold — a change in what B is doing. No amount of B disagreeing with A about the states they share moves the reading in that direction. Chapter 4 §4.20 flagged it when it constructed the case, and this chapter has now proved it.

THE SAME DYAD, WITH THE CONTRARIAN'S FRICTION PRICED

The comparison above held c fixed, which flatters the contrarian. **CONSTRUCTED extension:** manufactured opposition is not free — the argument takes time, positions must be restated, decisions get relitigated — so c rises. Computed:

```
c         c/eta       ceiling verdict     D (at u = 0)      Q_inf
0.62     2.066667      TUNABLE             +5.9631         −6.8889
0.70     2.333333      TUNABLE             +6.7326         −7.7778
0.745472 2.484907      STRUCTURAL          +7.1699         −8.2830
0.75     2.500000      STRUCTURAL          +7.2135         −8.3333
0.90     3.000000      STRUCTURAL          +8.6562        −10.0000
```

The tipping point is exact: `c_crit = eta * ln N = 0.30 * 2.484906649788 = 0.745472`. A rise in coordination cost from 0.62 to anything above 0.7455 — a 20% increase — moves this dyad above the ceiling, at which point no assignment of u is sustainable, the geometry lever is gone entirely, and only `eta`, `c` and `N` remain.

So manufactured opposition in this dyad does two separate kinds of damage. It sets the numerator to zero, which R8 guarantees. And it raises the cost ratio, which — at a 20% rise — removes the only lever that could have repaired the numerator. The second harm is worse than the first and it is invisible to anyone who computes a deficit without running the ceiling test.

THE COMMONS, AND THE SEVENTH SEAT

CONSTRUCTED extension of SPINE §3.3, labelled as such. The full comparison, with the ceiling verdict attached to every row because §14.11 showed what happens without it:

```
   CONFIGURATION            links  C     c/eta   ceiling    mean u    f (Hz)    D      Q_i
   six members, as found       15  0.8250  2.4265  TUNABLE    0.018614   0.7017  +6.4638
8.4630
   + contrarian  (rho = −1)     21  1.1550  3.3971  STRUCTURAL  0.013296   0.6352  +9.4080
12.3177
   + sycophant   (rho = +1)     21  1.1550  3.3971  STRUCTURAL  0.013296   0.6352  +9.4080
12.3177
   + distinct    (u = 0.80)     21  1.1550  3.3971  STRUCTURAL  0.241867   6.4777  +6.0577
7.9313
   + independent  (u = 1.00)    21  1.1550  3.3971  STRUCTURAL  0.299010   8.8933  +5.600
7.3326
```

Every row that adds a seat is structural, whoever fills it. That is the finding, and it is not about the contrarian at all — it is about kappa. Twenty-one links at 0.055 each is `C = 1.1550`, and `1.1550 / 0.34 = 3.3971`, which exceeds `ln 12 = 2.484907`. The maximum kappa at which a seventh seat stays under the ceiling is 0.040232, so the seat is affordable only alongside a 26.9% cut in per-link cost.

Within that finding, the contrarian is still the worst available choice:

```
CONTRIBUTION OF THE SEVENTH SEAT, IN CENTS, AGAINST THE SIX-MEMBER BASE

  contrarian  /  sycophant        −172.35 cents      cost +0.3300
```

```
genuinely distinct (u = 0.80)  +3847.96 cents      cost +0.3300
perfectly independent (u = 1)  +4396.66 cents      cost +0.3300
```

Three candidates, the same price, and a spread of 4569.01 cents between the worst and the best — three and three quarter octaves for the same money. The contrarian is the only one of the three whose contribution has the wrong sign.

And SPINE §3.3's standing verdict is unchanged by any of it: **no single lever closes a 6.4638-octave gap.** Raising u to 0.95 reaches −0.0332, and 0.95 is near-orthogonality. Raising `eta` to 0.90 leaves +2.1075. Cutting `kappa` to 0.020 leaves +2.0085. Chapter 18 carries the case and its lesson: some systems must be restructured rather than tuned, and the diagnostic is what tells you which case you are in.

THE ONE COMPARISON THE CHAPTER IS FOR

```
   A SEAT FILLED BY AGREEMENT     f_rel = 0.4834 Hz   0.00 cents   Q_inf = -
c/delta
   A SEAT FILLED BY OPPOSITION    f_rel = 0.4834 Hz   0.00 cents   Q_inf = -
c/delta
  A SEAT FILLED BY INDEPENDENCE  f_rel = 69.6133 Hz  8603.91 cents  the ceiling of the syst
```

Two of those three are the same note and the third is seven octaves higher. The two that coincide are the two that a lifetime of ordinary advice treats as opposites.

What this chapter established

Established, and checkable: that `u = 1 − rho^2` is even in `rho`, so perfect opposition and perfect agreement give `u = 0`, `F = 0`, `N_eff = 1` and `f_rel = k` identically; that at `u = 0` the fixed point is `−c/delta` with `eta` and `N` absent and their partial derivatives exactly zero; that the agreement channel exhibits the same symmetry directly, since `u = 4a(1−a)` vanishes at `a = 1` and at `a = 0`; that the maximum of u at `rho = 0` is second-order flat, costing 0.32 cents for the first hundredth of correlation against 685.39 for the last; that the Parallel Wall's fifteen links realise 7.50% of their ceiling at 43.00 cents per link; and that a seventh seat at `kappa = 0.055` raises `c/eta` to 3.3971 and converts a tunable commons into a structural one whoever fills it.

Not established, and not claimed: that agreement is bad, that conflict is good, that a unison reading says anything about how two people are doing, that the twelve states diagnose anything, or that any of this is counselling, therapy or an assessment of any person. **The opposition branch of R8 is not even visible to this book's native encoding**, which is a limitation stated as one, and the signed routes of Chapter 13 are the only place it can be seen.

The one line to carry forward: *the contrarian and the sycophant sound the identical note, and the note is the one a pair with nothing between them would sound.*

Chapter 14 owns the unison result (R8) at relational scope, the pricing of the Parallel Wall against manufactured opposition, and the design rule that follows. Chapters 16 and 17 cite it. `f_rel` and the glide are Chapter 12; the three routes, the agreement identity and the sign requirement are Chapter 13; `f_min`, the ceiling test and the four levers are Chapter 4; the consonance/redundancy identity and its firewall are Chapter 10; the internal statement of R8 and its unreachable branch

are Chapter 10 §10.5. Chapter 15 takes the dyad to `f_min` and fixes the order in which the levers are pulled.

Chapter 15 . The Sustainable Dyad

Two people, one bond, one number. Chapter 12 gave the bond a frequency and Chapter 13 gave the procedure for measuring the u that produces it. Chapter 4 gave the threshold that frequency has to clear. This chapter puts the three together and asks the only question a person in a relationship ever actually asks about it, which is not *how is it* but *where is it going*.

The apparatus answers that question, and the answer is a signed number of octaves. What makes this chapter necessary is not the reading. It is what happens next.

Because the moment a reading comes back negative, everyone reaches for the same repair. Not a foolish repair — a good one, well supported in ordinary experience, and recommended by almost every tradition that has ever thought about bonds. You add trust. You add continuity, ritual, shared memory, the assumption of good faith, the willingness to let a thing go unresolved overnight because it will still be there in the morning. Everyone knows this works. And in a relationship whose reading is already negative, it does not work. It makes the outcome measurably worse, at a computable rate, and it does so *while the diagnostic reads exactly unchanged*.

That result is Book Two's R15 in the frequency register, and this chapter owns it for the rest of the series. Chapters 18 and 29 cite it. It is the most counterintuitive statement in this book and it is not subtle arithmetic — it is a division by a smaller positive number.

The rule, stated once at the top so that a reader who takes nothing else takes this:

```
THE ORDER-OF-OPERATIONS RULE (R15, frequency form)

    Q_inf = ( eta * F(u) - c ) / delta

    Trust, ritual, continuity and institutional memory act on delta.
    delta divides.  It has no sign of its own.

    FIX THE NUMERATOR, THEN LOWER THE LOSS.
   Reverse the order and the same intervention accelerates the failure.
```

This chapter is not counselling, therapy, or clinical assessment of any kind, and nothing in it may be presented as any of those things. It is arithmetic on four declared parameters. The standing prohibitions of CANON3 §5 apply in full and are restated in §15.7.

The Law

15.1 The deficit, applied to a bond

Nothing new is defined here. Chapter 4 owns `f_min`, the octave deficit, the ceiling and the four levers; Chapter 12 owns `f_rel`; Chapter 13 owns the measurement of u between two people. This section only assembles them at dyadic scope.

```
THE DYADIC READING       (all objects owned elsewhere; see Chapter 4 §4.21)

 u        measured between the two persons                    Chapter 13
 f        = k (1 + (N−1) u)^2                                 Chapter 12
f_min   = k e^(2c/eta)                                  Chapter 4, Def 4.1
D      = 2.885390 (c/eta) − log2(f/k)                   Chapter 4, Def 4.4
Q_inf   = ( eta F(u) − c ) / delta                        Paper E, Thm 3.5

 D <= 0   ACCUMULATES        D = 0   HOLDS        D > 0   DECAYS
```

The parameters at dyadic scope mean what they mean everywhere else, and it is worth being concrete about what they are in a bond, because the abstraction invites people to fill them in wrongly.

`eta` is the fraction of what the two of you do together that survives the week. Not how much you do. Not how intensely. The fraction that turns into something durable — a decision that stays decided, a practice that stays practised, an artefact that exists on Friday because of what happened on Monday.

`c` is what the bond costs per period to run at all, before it produces anything: the coordination, the negotiation, the friction, the re-explaining, the load of correcting things that drifted.

`delta` is the per-period rate at which accumulated order leaks away if nothing is added. It is Chapter 8's parameter — the one that separates a trait from a mood — and at relational scope it is what trust, continuity, shared record and ritual act on.

`u` is the represented non-parallelism between the two persons, and it is the only one of the four that is a joint geometric property rather than a property of the arrangement.

One clock, declared, for all three of `eta`, `c` and `delta` (SPEC 4.3). A bond read on weekly `eta` and monthly `c` produces a ratio that means nothing at all, and the error is invisible in the output.

15.2 The order-of-operations rule

THEOREM 15.1 — `delta` AMPLIFIES THE SIGN IT FINDS. Write the numerator `n := eta F(u) − c`. Then for `delta > 0`,

```
Q_inf = n / delta                      d Q_inf / d delta = − n / delta^2

n > 0   ->   dQ_inf/d(delta) < 0   ->   REDUCING delta RAISES Q_inf
n < 0   ->   dQ_inf/d(delta) > 0   ->   REDUCING delta LOWERS Q_inf
n = 0   ->   Q_inf = 0 for every delta;  the lever is exactly inert
```

Proof: differentiate n/delta in delta. delta^2 > 0, so the sign of the derivative is the sign of −n. **QED.**

The theorem has no free parameters and no conditions beyond delta > 0. It is not a tendency, not an empirical regularity, and not a claim about people. It is what division does.

The consequence is the one the chapter exists to state:

> **An intervention that lowers delta multiplies whatever the relation is already doing. In a bond that is accumulating, it accumulates faster. In a bond that is decaying, it decays faster. Trust is a multiplier on an existing sign, not a source of order.**

Computed on the carried dyad of SPINE §3.2 as found — a constructed case, parameters chosen, and Chapter 4 §4.9 (6) applies:

```
u = 0.068734665264634    F = 0.563085 nats    eta = 0.30    c = 0.62

numerator  n = 0.30 * 0.563085 - 0.62 = -0.451075        NEGATIVE

delta = 0.09   ->   Q_inf = -5.011940
delta = 0.05   ->   Q_inf = -9.021491                    WORSE by 80.0%
```

And the second half of the result, which is the half that makes the frequency register earn its place in this book:

```
   OCTAVE DEFICIT at delta = 0.09    +4.338420
  OCTAVE DEFICIT at delta = 0.05   +4.338420          IDENTICAL to every digit
```

The deficit does not contain delta at all (Chapter 4, Theorem 4.9). So the instrument reads flat while a successful, well-intentioned and expensive intervention doubles the rate of loss. A reader holding the deficit alone, without Theorem 4.9, would read the flat number as *give it time*. A reader holding both reads it correctly the first time: **you have not touched the mechanism, and the mechanism is the numerator.**

15.3 The same lever, the other way round

Now the carried dyad after the intervention of SPINE §3.2 phase 2b — u raised to 0.692372881355932, eta from 0.30 to 0.44, c from 0.62 to 0.40:

```
F = 2.153633 nats
numerator  n = 0.44 * 2.153633 - 0.40 = +0.547598         POSITIVE

delta = 0.09   ->   Q_inf =  +6.084427
delta = 0.05   ->   Q_inf = +10.951968                   BETTER by 80.0%
```

Same lever, same magnitude, same direction, same two people. At 2a it moved Q_inf from −5.0119 to −9.0215. At 2b it moves it from +6.0844 to +10.9520. The proportional change is identical in both cases — delta scales by 0.05/0.09 = 0.5556, so Q_inf scales by 1.8 — and the two outcomes are opposite because the numerators are opposite.

The arithmetic is not a special case and it is not a trick of these particular numbers. It holds for every relation, at every scale, at every value of delta.

15.4 The fixed point is order-independent; the path is not

Here is the objection a careful reader raises immediately, and it is correct as far as it goes. `Q_inf` is a function of the *final* parameters. If a pair ends at `(eta = 0.44, F = 2.153633, c = 0.40, delta = 0.05)`, then `Q_inf = +10.951968` — and it does not matter in what order they got there. The destination is order-independent. So why does the order matter?

It matters because a relation is not its fixed point. It is a trajectory that may or may not survive long enough to reach one.

THEOREM 15.2 — ORDER-INDEPENDENCE OF THE DESTINATION, ORDER-DEPENDENCE OF THE PATH. Let a relation follow `Q_{t+1} = (1 − delta_t) Q_t + n_t`. The fixed point of the terminal parameter set is independent of the sequence of parameter changes. The trajectory is not, and the divergence between two orderings of the same two changes is bounded below by the accumulated difference in `n_t/delta_t` over the interval in which the parameters differ.

Verified on the carried dyad, `Q_0 = 1.0`, with two orderings of exactly the same pair of interventions, each applied for eight periods before the other:

```
PATH A -- trust first:    8 periods at (0.30, F=0.563085, 0.62, delta=0.05)
                          then     at (0.44, F=2.153633, 0.40, delta=0.05)
PATH B -- numerator first: 8 periods at (0.44, F=2.153633, 0.40, delta=0.09)
                          then     at (0.44, F=2.153633, 0.40, delta=0.05)

  t      PATH A          PATH B
  0     +1.000000       +1.000000
  1     +0.498925       +1.457598
  2     +0.022905       +1.874013
  4     −0.858924       +2.597783
  6     −1.654774       +3.197137
  8     −2.373029       +3.693462
 10     −1.073842       +4.401167
 12     +0.098675       +5.039870
 16     +2.111893       +6.136527
 20     +3.751671       +7.029761
 24     +5.087281       +7.757306
 32     +7.061215       +8.832564

 both converge to Q_inf = +10.951968
```

Path A's stock goes negative at `t = 3`, bottoms at −2.373029, and does not return to its starting value until `t = 14`. Path B never falls below its starting value at any step. And the lag is permanent:

```
B reaches +3.6935 at t = 8;    A reaches it at t = 20     lag 12 periods
B reaches +5.0399 at t = 12;   A reaches it at t = 24     lag 12 periods
B reaches +6.1365 at t = 16;   A reaches it at t = 28     lag 12 periods
B reaches +7.0298 at t = 20;   A reaches it at t = 32     lag 12 periods

cumulative gap over 32 periods:  111.133601 stock-periods
```

Twelve periods of permanent lag and 111.13 stock-periods of cumulative loss, from doing exactly

the same two things in the opposite order. The lag does not shrink. Both paths converge to the same number and one of them arrives twelve periods later, forever.

And the interval in which Path A's stock is negative is the interval in which the relation may simply not survive to reach its fixed point. The formalism has no term for that — it will happily compute a trajectory through negative stock — but Chapter 4's FM 4.9 records that admissibility fails there, and a relation whose accumulated order has gone negative is a relation in which people make decisions. **This is why order-of-operations is a rule and not a preference.**

15.5 Which lever is proportionally strongest — and the answer is not what the partial derivatives say

Chapter 4 §4.17b makes the point that the four partial derivatives are in four different units and cannot be ranked against one another. That is correct and it stands. But there is a comparison that *is* legitimate, and this chapter owns it, because it is what decides the usual dyadic question.

Elasticities are dimensionless. The change in `D` per *proportional* change in a parameter — per e-fold, per doubling, per percent — has the same units for every parameter, because the parameter's own units cancel.

THEOREM 15.3 — THE ELASTICITIES. With `D = 2.885390 (c/eta − F)` and `F = ln(1 + (N−1)u)`:

```
dD / d(ln u)    =  −2.885390 ∗ ( 1 − 1/N_eff )        N_eff = 1 + (N−1)u
dD / d(ln eta)  =  −2.885390 ∗ ( c / eta )
dD / d(ln c)    =  +2.885390 ∗ ( c / eta )
```

Proof: `dD/d(ln x) = x dD/dx`, applied to Chapter 4 §4.8's four partials. **QED.**

Three consequences, all exact.

(i) The two cost-side levers have equal and opposite elasticity. They enter `D` only as a ratio, so an e-fold rise in `eta` and an e-fold fall in `c` buy exactly the same number of octaves. Whichever is cheaper to move in the situation is strictly the better choice, and the formalism is indifferent between them.

(ii) The cost-side elasticity is exactly the number of octaves the relation requires. `2.885390 (c/eta)` is the required-octaves figure of Chapter 4 Theorem 4.3. An e-fold improvement in the cost ratio moves the deficit by precisely the requirement — which is another way of saying that at `c/eta`, an e-fold is the whole problem.

(iii) The geometry lever's elasticity is bounded above by `2.885390` octaves per e-fold, at every u, in every system. Because `1 − 1/N_eff < 1` always. There is no configuration in which raising u proportionally is worth more than 2.885390 octaves per e-fold, and the bound is approached only as u approaches 1, where there is nothing left to buy.

COROLLARY 15.3a — THE DYADIC LEVER CRITERION.

```
| dD/d(ln eta) |  >  | dD/d(ln u) |      <=>      c/eta  >  1 − 1/N_eff

and since 1 − 1/N_eff < 1 always,   c/eta >= 1   is SUFFICIENT.
```

A relation whose cost ratio exceeds one nat is one in which the cost-side levers are proportionally stronger than the geometry lever — always, at every u, with no exceptions. Computed on the three carried cases:

	c/eta	1 − 1/N_eff	|dD/dln eta|	|dD/dln u|	ratio
dyad 2a	2.066667	0.430550	5.963140	1.242306	4.8001
dyad 2b	0.909091	0.883938	2.623082	2.550507	1.0285
commons (17)	2.426471	0.169955	7.001314	0.490387	14.2771

The criterion holds in all three, and at 2b it holds by a margin of 0.025153 nats — barely. That is worth noticing rather than glossing: the repaired dyad has moved so far toward non-parallelism, and cut its costs so far, that the two levers have nearly equal proportional strength. A relation that has been tuned well is one in which the levers have equalised, and that is a signature, not a coincidence.

15.6 The reserve — why the strongest lever is not always the sufficient one

Elasticity says which lever moves the deficit fastest per proportional unit. It says nothing about how far each lever can go before it runs out. Those are different questions and confusing them is the most expensive planning error available in this chapter.

THEOREM 15.4 — THE LEVER RESERVES. The maximum number of octaves each lever can supply, alone, from an operating point (u_0, eta_0, c_0, N):

```
     R_u     = 2.885390 * ( ln N - F(u_0) )                        ( u -
> 1 )    = the pitch headroom
    R_eta  = 2.885390 * c_0 * ( 1/eta_0 - 1/eta_max )      ( eta -> eta_max )
    R_c    = 2.885390 * ( c_0 / eta_0 )                     ( c -> 0 )
   R_N   unbounded in principle, and 2.885390*u/(1+(N-1)u) per category in practice
```

Computed on the carried dyad as found, against its deficit of +4.338420:

```
    R_u    ( u -> 1 )        5.545206 octaves       CLOSES the deficit
    R_eta  ( eta -> 0.50 )   2.385256 octaves       does not
    R_eta  ( eta -> 0.75 )   3.577884 octaves       does not
      R_eta   ( eta -> 1.00 )     4.174198 octaves          does not --
and eta_max = 1 is generous
    R_c    ( c -> 0 )        5.963140 octaves       CLOSES the deficit
```

The lever with 4.8 times the elasticity has less reserve than the deficit requires. Raising eta from 0.30 all the way to a physically impossible 1.00 — a system converting every unit of interaction into permanent structure, with nothing lost — buys 4.174198 octaves against a requirement of 4.338420, and falls short by 0.164222. This is the same fact Chapter 4 §4.24 records in its other form: eta_req = c/F = 1.101077, a conversion efficiency above 1, which is not an expensive lever but an absent one.

> **THE HONEST DYADIC LAW: the cost-side levers have the higher elasticity and the lower ceiling; the geometry lever has the lower elasticity and the only large reserve. Small deficits are closed on the cost side. Large ones require difference.**

And that is the correct, non-slogan form of "raising eta usually beats raising u in a dyad." It is true, it is quantified, and it comes with the condition attached. §15.9 gives the practical reason it

is usually true and §15.10 gives the reason it is not always.

15.7 The honesty clauses, restated at dyadic scope

(1) ESSENCE, NOT RESONANCE. Every frequency here is a position on a logarithmic scale of surplus. No person, bond, tissue or molecule is claimed to vibrate at any value in this chapter, and nothing here is to be played at anyone (Chapter 4, FM 4.8).

(2) NOT COUNSELLING, THERAPY, OR CLINICAL ASSESSMENT. The twelve states are not diagnoses. A reading is a computation on four declared parameters and is not an evaluation of any person's health, capacity, character or prospects. **This chapter describes arithmetic and does not describe or prescribe for anyone, and a reader who needs support for a relationship should seek it from a person qualified to give it, which this book is not.**

(3) CONSENT IS STRUCTURAL. No intervention derived from a reading may remove a person's ability to decline. Chapter 18 owns the procedure and builds the requirement into the numbered steps rather than appending it as advice.

(4) THE READING INHERITS ITS INPUTS. `D` is exactly as defensible as `u`, `eta`, `c` and `N`. Where intensities were not magnitude-estimated (SPINE §1.1; Chapter 6's rule), the relationship frequency is not determined and must not be printed as a number.

(5) NO SINGLE-PARTY TERM. `u` is a joint geometric property; `eta`, `c` and `delta` are properties of an arrangement. There is no term anywhere in the apparatus that belongs to one person, and no reading licenses a conclusion about who is at fault.

(6) CONSTRUCTED CASES. Every case in this chapter is constructed and labelled. Its parameters are chosen. Precision in a constructed case does not transfer to a measured one.

The Engineering

15.8 Reading a bond, end to end

The sequence, with owners named. Nothing here is new; the value is the order.

```
0. CONSENT.     Both parties agree to the reading and either may stop it.  Ch 18
1. DECLARE N and the ordering that produced it.                            Ch 5
2. ELICIT the intensities by magnitude estimation; publish the raw L2 norms.  Ch 6
3. MEASURE u between the two persons.                                      Ch 13
4. DECLARE eta, c, delta on ONE stated period.                          Ch 4 SPEC 4.3
5. CEILING TEST FIRST: compare c/eta to ln N.                           Ch 4 SPEC 4.1
6. COMPUTE f, f_min, D, Q_inf; fold and publish with j.                    Ch 4 SPEC 4.5
7. CROSS-CHECK D = -2.885390 (delta/eta) Q_inf.                         Ch 4 SPEC 4.8
8. REPORT the sensitivity of D to the least certain parameter.             Ch 4 SPEC 4.19
9. SIGN THE NUMERATOR before choosing any lever.                        this chapter
```

Step 9 is the one this chapter adds, and it costs one subtraction. Compute `eta F(u) − c` and look at its sign. Everything about what to do next depends on it, and it is not visible in `D`, which is

the same positive number whether the numerator is slightly negative or catastrophically so relative to `delta`.

15.9 Why `eta` is usually the cheap lever in a dyad

Corollary 15.3a establishes when the cost-side levers are proportionally stronger. It does not establish that they are cheaper to move, because cost is not in the formalism (Chapter 4 §4.17b). Four structural reasons, stated as engineering judgment and badged as such, explain why in a dyad they usually are.

One: `eta` is unilateral and u is not. `eta` is a property of what happens to what the pair already produces — whether the decision gets written down, whether the practice gets a slot, whether the artefact exists. One person can raise `eta` on their own, starting today, without asking the other to be different. u is a joint geometric property of two representations. It cannot be moved by one party's intention at all.

Two: `eta` is mechanism and u is substance. Raising `eta` is a question about apparatus — a notebook, a standing agenda, a place where the conclusion lives. Raising u requires that one of the parties genuinely come to hold something the other does not. That is a change in a person, it is slow, and it may not be available at any price.

Three: `eta` is reversible and u is not. A decision log can be discontinued on Tuesday with no residue. A person who has taken on a new set of commitments in order to raise u cannot un-take them, and should not be asked to. Chapter 18 makes reversibility a criterion, and on that criterion the cost-side levers dominate the geometry lever in every dyad.

Four: difference is scarce and getting scarcer inside a bond. Two people who spend time together converge. That is what the time is for and it is not a defect. But the convergence is exactly a fall in u, and it runs continuously in the background against whatever anyone does. A dyad's u has a natural downward drift and its `eta` does not — which means the cost-side lever holds its gains and the geometry lever leaks them.

15.9b Estimating the three arrangement parameters

u has an owner and a procedure: Chapter 13. Chapter 11 §11.13 gives the estimation steps at personal scope, and this section is their dyadic form, which Chapter 13 defers to here. The other three parameters do not have an instrument comparable to u's, and the honest thing is to say so plainly rather than to imply that a number can be read off something. What follows is estimation practice, badged as practice, and it produces figures whose scale status must be declared with them (SPEC 4.4).

For `eta`, the operational question is what fraction of a period's joint work is still doing something at the start of the next period. It is countable in principle: enumerate what the two parties produced in a period — decisions, agreements, artefacts, practices — and count how many survive intact into the following period without being re-made. A pair that settles six things a month and re-settles four of them next month is converting about a third. That count is crude, it is sensitive to what gets called a thing, and it is far better than a guess, because the objection to it is visible and arguable in a way that a guess is not.

For `c`, the question is what the arrangement costs to run before it produces anything: the time spent coordinating rather than deciding, the re-explaining, the friction of getting to the point where

work can happen. Measured in the same units as the surplus — nats per period — it requires a declared conversion, and the conversion is where most of the error lives. The practical route is to express both `eta F` and `c` as fractions of the same period budget, which makes the ratio `c/eta` recoverable even when neither term is well determined on its own.

For `delta`, the question is how much of what was accumulated is still there after a period of doing nothing. Chapter 8 owns the parameter and gives the half-life relation `t_half = ln(0.5)/ln(1 − delta)`, which is the estimation handle: a pair that finds a lapsed practice roughly half-forgotten after seven periods is running near `delta = 0.09`, since `ln(0.5)/ln(0.91) = 7.3496`. At `delta = 0.05` the half-life is 13.5134 periods.

The reason for spelling this out is not that the estimates are good. It is that the ratio `c/eta` carries the entire cost side of the reading, its elasticity is the largest in the chapter, and a verdict resting on it deserves an error bar rather than a decimal point. SPEC 15.11 requires one.

15.10 And why it is not always — the reserve check

Against those four reasons stands Theorem 15.4, and the check is one line:

```
IF  D <= 2.885390 * c * ( 1/eta_0 − 1/eta_max )    the cost side can close it alone
 ELSE                                                 difference is required
```

Computed at a range of operating points, with `eta_max = 1`:

eta_0	c	R_eta (octaves)
0.20	0.400	4.6166
0.20	0.620	7.1558
0.20	0.825	9.5218
0.30	0.400	2.6930
0.30	0.620	4.1742
0.30	0.825	5.5544
0.40	0.400	1.7312
0.40	0.620	2.6834
0.40	0.825	3.5707
0.50	0.400	1.1542
0.50	0.620	1.7889
0.50	0.825	2.3804

Read the table as a warning about where the reserve is. It is large only where `eta` is very low — where the relation is converting almost nothing and there is a great deal of headroom in the mechanism. A relation already converting half of what it does has under two octaves available on the `eta` side no matter how expensive it is. The cost-side lever is the cheap one and the first one, and it is spent quickly.

15.11 What raising u actually is, and R8's trap

Chapter 14 owns the unison result and it is the single most important thing to carry into any attempt at this lever.

```
u = 1 − (x . y)^2   depends only on | rho |
```

```
perfect agreement    rho = +1   ->   u = 0
perfect opposition   rho = −1   ->   u = 0
```

Manufactured disagreement raises cost and returns exactly nothing. A party who adopts contrary positions in order to "bring difference" moves the pair toward `rho = −1`, which reads as `u = 0` just as agreement does, while adding coordination overhead. The reading gets worse in both terms at once. The contrarian and the sycophant sound the identical note, and it is the ground essence.

What does raise u is a change in what a party actually holds: states occupied that the other does not occupy, commitments taken on, work done in a domain the other is not in. The carried dyad's phase 2b realises this concretely — `B` takes on two states `A` does not hold — and the resulting vector is given in full in the Worked Example. The test of a u intervention is whether it would still be true if nobody were reading the number.

And there is a second trap, which is representational rather than behavioural. Two people described by disjoint sets of states are automatically orthogonal and read `u = 1` regardless of content — the one-hot degeneracy Chapter 9 owns. **A u that rose because the description changed is not a u that rose.** SPEC 15.6 states the check.

15.12 What trust is actually for

Nothing in this chapter says trust is bad, and a reader who takes that away has taken away the opposite of the result. Theorem 15.1 says `delta` multiplies. In a relation with a positive numerator it is the most powerful single thing available:

```
      carried dyad at 2b:    delta 0.09 -> 0.05     Q_inf   +6.084427 -
> +10.951968    +80.0%
```

An 80% improvement from one change, with no new difference required and no cost cut, is not a minor lever. It is the largest single move in the whole chapter. The rule is about sequence, not about worth.

There is also a category of `delta` intervention that is worth doing at any point, because it is not being done for its effect on `Q_inf` at all. A written record is a `delta` intervention and also an `eta` intervention, since the same document both slows the leak and converts more of what happens into durable structure. A pair adopting one is moving two parameters, and the numerator is one of them. Interventions that touch both sides are the ones to reach for first, and they are common: the record, the standing review, the artefact that outlives the conversation.

Chapter 4's SPEC 4.9 requires `delta` interventions to be reported separately, as multipliers on an existing sign. This chapter adds the reason: reported together with numerator changes, they make an intervention look like it worked when what worked was the other half of it.

15.12b Combinations, and why the carried case needed two levers

Because `D = 2.885390 (c/eta − F)` is a difference of two independent terms, any set of parameter moves whose octave contributions sum to the deficit will close it. There is no interaction term between the geometry side and the cost side, which means combinations can be planned additively — a convenience the formalism rarely offers, and worth using.

Computed on the carried dyad as found, against its deficit of `+4.338420`:

```
    u          eta      c        D            Q_inf (delta 0.09)    verdict
    0.068735   0.44     0.40    +0.998357        -1.6916            DECAYS
  0.300000  0.44    0.62   -0.142896      +0.2421         accumulates, barely
  0.692373  0.30    0.62   -0.250931      +0.2899         accumulates, barely
   0.300000   0.44     0.40    -1.585591       +2.6866           accumulates
   0.400000   0.40     0.50    -1.259181       +1.9396           accumulates
   0.500000   0.36     0.50    -1.393393       +1.9317           accumulates
    0.692373   0.44     0.40    -3.590989        +6.0844             phase 2b
```

The first row is the one to sit with. It is the decision log *on its own* — `eta` up to 0.44, `c` down to 0.40, the geometry untouched — and it leaves the pair at `+0.998357` octaves, still decaying, with `Q_inf = -1.6916`. **The entire cost-side intervention of phase 2b, applied alone, does not save the carried dyad.** It closes 3.340058 of the 4.338420 octaves and stops.

The second and third rows are the mirror image: either lever alone, pushed to a plausible limit, clears the threshold by a fifth of an octave, which is not a margin anyone should plan against. It took both to reach a reading with any reserve in it. The pattern is general enough to state as guidance rather than as a theorem: on a deficit above roughly two octaves, single-lever plans tend to land near the hold line, where the relation pays for itself exactly and banks nothing, and where any increase in cost puts it back under.

15.13 The convergence clock

There is one more dyadic result worth having, and it is the cheapest test in the chapter because it has almost no free parameters. Two people who spend time together converge. Their representations move toward one another, which is a fall in `u`, which is a fall in `f`. Nothing about that is pathological — it is most of what shared time is for — but it runs continuously and it runs against the geometry lever.

Model the drift as geometric decay at rate `lambda` per period, `u_t = u_0 (1 - lambda)^t`, and ask when a currently-sustainable bond crosses its own hold line. Chapter 4's Corollary 4.8 gives the hold line as `u* = (e^(c/eta) - 1)/(N - 1)`, so

```
t*  =  ln( u* / u_0 ) / ln( 1 - lambda )
```

This is R10's parameter-free crossing time transplanted to dyadic scope, and it inherits R10's virtue: with `c`, `eta` and `N` held fixed, `lambda` is the only input.

Computed for the carried dyad at phase 2b, where `u_0 = 0.692373` and `u* = (e^0.909091 - 1)/11 = 0.134733`:

```
lambda      t* (periods)      D at t=10     D at t=20     D at t=40
 0.01          162.863          -3.3362       -3.0846       -2.5922
 0.02           81.020          -3.0821       -2.5872       -1.6483
 0.03           53.738          -2.8290       -2.1013       -0.7797
 0.05           31.911          -2.3271       -1.1755       +0.6539
 0.08           19.631          -1.5912       +0.0527       +1.9308
 0.10           15.535          -1.1171       +0.7320       +2.3153
 0.15           10.072          -0.0201       +1.8767       +2.5903
```

A repaired dyad has a computable shelf life, and at a 5% per-period convergence rate the carried case's repair is spent in 31.9 periods. The pair does nothing wrong in that interval. They

simply agree with each other more each period, which is what happens, and the reading that was −3.5910 walks back across zero.

Two practical consequences. The first is that a u intervention is not a one-off; it decays, and the schedule of its decay is estimable from two readings and one logarithm. The second is that the cost-side levers do not decay this way — a decision log does not gradually stop being a decision log — which is a fifth reason, and the one that generalises best, for reaching for them first.

The honest limit on all of it: lambda is a modelling assumption, badged as such, and geometric decay is the simplest form that behaves correctly at the boundaries rather than one that has been fitted to anything. A pair that measures u twice can estimate lambda directly, and should, rather than adopting a figure from this table.

15.14 The damage integral — pricing the wrong order

Theorem 15.2 gives the mechanism. This is the number to quote when someone asks what the ordering rule is worth.

```
CONSTRUCTED, carried dyad, Q_0 = 1.0, eight periods of the wrong order first

 permanent lag                          12 periods
minimum stock reached, wrong order   −2.373029   (starting value +1.000000)
 periods spent below the starting value 13   (t = 1 through t = 13)
 cumulative gap over 32 periods         111.133601 stock-periods
 terminal fixed point, both orders      +10.951968
```

Eight periods of misordering cost twelve periods of permanent delay. The excess is not a rounding artefact: while the numerator is negative, the lower delta is *accelerating* the loss, so the eight periods do more damage than eight periods at the original delta would have.

The number that will not appear in any of this is a change in the diagnostic. D reads +4.338420 at the start of Path A and +4.338420 at the end of Path A's first eight periods. Whatever instrument the pair is using, if it is the deficit alone, it reports that nothing happened.

15.15 The marginal reading, and when not to act on it

A deficit near zero is the most dangerous reading in the book, because it is the one most likely to be an artefact of an imprecise input. Chapter 4's SPEC 4.19 requires the sensitivity to be published with the deficit; at dyadic scope the sensitivities are:

```
dD/d eta  =  −2.885390 c / eta^2          dD/d c  =  +2.885390 / eta
dD/d u    =  −2.885390 (N−1)/N_eff        dD/d N  =  −2.885390 u / N_eff
```

A verdict is resolved only when the deficit exceeds its own uncertainty. Chapter 17's constructed family is worked as an example of a case where it does not: a deficit of +0.584737 octaves with eta known to ±0.05 carries an uncertainty of ±0.9734 octaves, and the sign of the verdict is not determined. The correct action there is to measure eta better, not to intervene, and Chapter 18 makes that a numbered step rather than a caution.

The Specification

SPEC 15.1 — CONSENT FIRST, AND IT IS REVOCABLE. Both parties agree to the reading before it is taken, and either may stop the process at any point without giving a reason. No reading is taken of a person who has not agreed to it and no intervention proceeds over a decline.

SPEC 15.2 — SIGN THE NUMERATOR. Compute `eta F(u) − c` and report its sign with every dyadic reading. The deficit does not carry it.

SPEC 15.3 — ORDER OF OPERATIONS. Where the numerator is negative, no intervention on `delta` is undertaken until it is positive. Raise `eta` or cut `c` first, or raise `u`. This is the rule this chapter owns, and Chapters 18 and 29 cite it by name.

SPEC 15.4 — REPORT `delta` INTERVENTIONS SEPARATELY from numerator interventions, as multipliers on an existing sign, with the sign stated (Chapter 4 SPEC 4.9).

SPEC 15.5 — CEILING TEST BEFORE LEVER CHOICE (Chapter 4 SPEC 4.1). `c/eta` against `ln N`, classify, and only then choose.

SPEC 15.6 — THE `u` PROVENANCE CHECK. A rise in `u` must be traceable to a change in what a party holds, not to a change in the description. Re-encode both parties under the identical procedure before and after, and publish both raw L2 norms (Chapter 6's rule).

SPEC 15.7 — THE R8 CHECK. Before crediting a `u` intervention, verify that the change was not a move toward opposition. Report `cos theta` signed, not `u` alone, since `u` discards the sign.

SPEC 15.8 — THE RESERVE CHECK. Before committing to a cost-side plan, compute `R_eta = 2.885390 c (1/eta_0 − 1/eta_max)` against the deficit, with `eta_max` declared and defended. A plan requiring `eta > 1` is not an expensive plan; it is not a plan.

SPEC 15.9 — DECLARE `eta_max`. The reserve calculation is meaningless without it, and `eta_max = 1` is a bound, not an estimate.

SPEC 15.10 — ONE CLOCK for `eta`, `c` and `delta`, named in the same passage as the ratio.

SPEC 15.11 — REPORT THE SENSITIVITY of `D` to the least certain parameter, and declare the verdict UNRESOLVED where the uncertainty exceeds the deficit.

SPEC 15.12 — REVERSIBILITY IS REPORTED. Every proposed intervention is classified as reversible or not, and irreversible interventions require explicit separate agreement (Chapter 18).

SPEC 15.13 — NO SINGLE-PARTY ATTRIBUTION. A reading is reported as a property of an arrangement. No output of this procedure names a person as the cause of a deficit, because the formalism contains no term that could.

SPEC 15.14 — LABEL EVERY CASE CONSTRUCTED, and do not carry a constructed case's precision into a measured one.

SPEC 15.15 — ESTIMATE THE CONVERGENCE RATE FROM TWO READINGS. Where a `u` intervention is undertaken, `u` is re-measured at a later declared period and `lambda` estimated from the pair, not adopted from a table. Report `t∗` with the reading.

The Failure Modes

FM 15.1 — TRUST FIRST. *Signature:* a failing bond into which continuity, ritual, patience or assumed good faith is invested, followed by acceleration of the decline, with the diagnostic unchanged throughout. *Mechanism:* Theorem 15.1. `delta` divides a negative numerator. *Computed:* `Q_inf` from `−5.011940` to `−9.021491`; deficit `+4.338420` both times. *Fix:* SPEC 15.2 and SPEC 15.3. Sign the numerator, then act on the numerator.

FM 15.2 — READING THE FLAT DEFICIT AS STABILITY. *Signature:* "we changed something significant and the number did not move, so the reading must be insensitive." *Mechanism:* Theorem 4.9. `delta` is not in `D`. The instrument is behaving correctly. *Fix:* report `Q_inf` alongside `D` in every reading. They are the same quantity in different units *only* at fixed `delta` (Chapter 4, Theorem 4.5).

FM 15.3 — RANKING LEVERS BY THEIR PARTIAL DERIVATIVES. *Signature:* "`dD/deta` is `−19.88` and `dD/du` is `−18.07`, so `eta` is the better lever." *Mechanism:* the two are in different units — octaves per unit of conversion efficiency against octaves per unit of non-parallelism. **Comparing them is comparing octaves-per-apple to octaves-per-orange** (Chapter 4 §4.17b). *Fix:* Theorem 15.3. Elasticities are dimensionless and are the comparison that exists.

FM 15.4 — CONFUSING ELASTICITY WITH RESERVE. *Signature:* a plan built on the strongest lever that requires more of it than exists. *Mechanism:* Theorem 15.4. On the carried dyad the `eta` lever has 4.80 times the elasticity of the `u` lever and 75% of the reserve required. *Fix:* SPEC 15.8. Compute the reserve before committing.

FM 15.5 — MANUFACTURED DIFFERENCE. *Signature:* one party adopting contrary positions to raise `u`. *Mechanism:* R8. `u` depends only on `|rho|`; opposition reads as `u = 0`, and the performance adds cost. *Fix:* SPEC 15.7, and Chapter 14 for the full result.

FM 15.6 — THE RE-ENCODING THAT LOOKS LIKE PROGRESS. *Signature:* `u` rises after a change in how the two parties were described. *Mechanism:* the one-hot and disjoint-support degeneracies of Chapter 9. Two people described over non-overlapping state sets read `u = 1` for free. *Fix:* SPEC 15.6. Re-encode both, identically, before and after.

FM 15.7 — CUTTING THE CONVERSION MECHANISM. *Signature:* `c` falls, `eta` falls with it, `D` does not move. *Mechanism:* `D` depends on `c/eta`. The recurring meeting that was costing the relation was also converting it. *Fix:* Chapter 4 SPEC 4.11. Test whether the cost being cut is doing conversion work.

FM 15.8 — ACTING ON AN UNRESOLVED VERDICT. *Signature:* an intervention justified by a deficit smaller than its own uncertainty. *Mechanism:* SPEC 4.19. A deficit of `+0.58 ± 0.97` octaves is a statement that the sign is unknown. *Fix:* SPEC 15.11. Measure the least certain parameter better first; §15.15 and Chapter 17's family work the arithmetic.

FM 15.9 — THE MISATTRIBUTED REPAIR. *Signature:* a combined intervention credited entirely to its `delta` component. *Mechanism:* a written record moves `eta`, `c` and `delta` at once. Reported as one thing, it looks like trust worked. *Fix:* SPEC 15.4, and Chapter 4's path-free split for the attribution.

FM 15.10 — THE PATH IGNORED BECAUSE THE DESTINATION IS THE SAME. *Signature:* "the fixed point does not depend on the order, so the order does not matter." *Mechanism:* Theorem 15.2. The destination is order-independent and the trajectory is not; the constructed case shows

12 periods of permanent lag and 13 periods below the starting stock. *Fix:* compute the trajectory, not only the fixed point, whenever an ordering decision is live.

FM 15.11 — THE DEFICIT READ AS A VERDICT ON A PERSON. *Signature:* "our reading is +4.34, so one of us is the problem." *Mechanism:* a four-parameter property of an arrangement read as a property of a person. **There is no single-party term in the apparatus** (Chapter 4, FM 4.13). *Fix:* SPEC 15.13. State the arrangement.

FM 15.12 — THE READING USED AS COUNSEL. *Signature:* a deficit presented as advice about whether to stay in a relationship, or as anything resembling an assessment of a person. *Mechanism:* register slip. `D > 0` says a configuration loses order at a computable rate. It does not weigh what the relation is for, what else it produces, or what ending it would cost, and it acquires no such term by being applied to people (Chapter 4 §4.22). *Fix:* §15.7. **This is not counselling, therapy or clinical assessment, the book says so wherever a reader might mistake it, and it says so here.**

FM 15.13 — THE ONE-OFF u REPAIR. *Signature:* a bond re-read years after a successful geometry intervention and found back below its hold line, with nobody able to name what went wrong. *Mechanism:* §15.13. Convergence drift. u decays because shared time makes people more alike, and at `lambda = 0.05` the carried case's repair is spent in 31.9 periods. *Fix:* SPEC 15.15. Re-measure and estimate `lambda`; prefer cost-side gains, which do not decay this way.

Worked Example

THE CARRIED DYAD — THE ARC, AND THE R15 DEMONSTRATION

This case is CONSTRUCTED. It is the dyad of SPINE §3.2, the same case Chapter 4 works and the only case this chapter uses. Its parameters are chosen, not observed. It is not an assessment of any person, no clinical, diagnostic or therapeutic claim is made or implied, and the frequencies are essence frequencies — nothing is claimed to resonate.

Phase 2a — as found

```
A = {4: 0.7, 6: 0.5, 9: 0.4}                raw L2 norm 0.948683
B = {4: 0.6, 6: 0.55, 9: 0.3, 10: 0.2}      raw L2 norm 0.890225

N = 12,  one declared period,  eta = 0.30,  c = 0.62,  delta = 0.09
```

Step 0 — consent. Both parties agreed to the reading and either may stop it (SPEC 15.1).

Steps 1–3 — the geometry (Chapter 13's procedure; Chapter 6's magnitude-estimation rule; raw norms published above):

```
cos theta =  0.965020898600318          u = 0.068734665264634
F         =  0.563084802746 nats        N_eff = 1.7560813179
f       =  k (1 + 11u)^2  =  1.4907987150 Hz     folded 381.644 Hz (j = 8)
```

The two agree about almost everything, and no one involved would describe that as a problem. Two people 0.9650 aligned field 1.76 people's worth of distinct perspective between them.

Step 5 — the ceiling test, before anything else.

```
c/eta  = 0.62 / 0.30 = 2.066666666667
ln 12  = 2.484906649788
margin = +0.418240 nats          BELOW THE CEILING  ->  TUNABLE
```

Step 6 — the reading.

```
f_min        = k e^(2 * 2.066667)  = 30.1587698405 Hz  folded 241.270 Hz (j = 3)
 octaves required  = 2.885390 * 2.066667  =  5.9631395023
 octaves present   = log2(1.4907987150/k) =  1.6247193050
OCTAVE DEFICIT                        = +4.3384201973     (+5206.1 cents)
 N_eff required    = e^2.066667           =  7.898451
 pitch headroom    = 2.885390 (ln 12 - F) =  5.545206 octaves
 VERDICT: DECAYS
```

Step 7 — cross-check.

```
 Q_inf = (0.30 * 0.563084802746 - 0.62) / 0.09 = -5.0119395464
identity: -2.885390 * (0.09/0.30) * (-5.0119395464) = +4.338420197298   err 8.88e-
16
```

Step 9 — sign the numerator, which is this chapter's addition.

```
eta F - c  =  0.30 * 0.563084802746 - 0.62  =  -0.451074559176     NEGATIVE
```

That sign, and not the magnitude of the deficit, is what decides the next move.

The R15 demonstration

The intuitive repair is to add trust. Assume it succeeds completely and `delta` falls from 0.09 to 0.05 — a 44.4% reduction in per-period loss, which is a large and hard-won change.

```
 BEFORE   Q_inf = -0.451074559176 / 0.09 =  -5.0119395464
AFTER   Q_inf = -0.451074559176 / 0.05 =  -9.0214911835     WORSE by 80.0%

 DEFICIT BEFORE  +4.3384201973
DEFICIT AFTER  +4.3384201973                          UNCHANGED, every digit
```

A negative number divided by a smaller positive number is more negative. Trust did not add order; it amplified the sign the relation already had, and the sign was negative.

The mistake is not naivety. It is that the intervention was correct about the parameter and wrong about the order, and there is no way to see that from the deficit alone.

The lever audit at 2a

Required: `F >= c/eta = 2.066667`, i.e. `N_eff >= 7.898451`.

```
L1  RAISE u       need u >= 0.627132   from 0.068735 -- a factor of 9.1240
                    reserve 5.545206 octaves >= 4.338420       REACHABLE
            0.627132 is a large amount of real difference for a pair at cos 0.9650

 L2  RAISE eta      need eta >= c/F = 1.101077
```

```
                  reserve to eta = 1.00 is 4.174198 < 4.338420
              ABSENT. Not expensive -- absent. A conversion efficiency above 1 would
                    mean more structure created than interaction supplied.

  L3  CUT c         need c <= eta F = 0.168925   from 0.62 -- a cut of 72.8%
                reserve 5.963140 octaves              REACHABLE, and large

  L4  RAISE N        need N >= 101.3635  ->  102 categories
                     at u = 0.068735 a new category buys 0.1129 octaves
                NOT REACHABLE, and it would cross the N = 17 cliff on the way
                     (Chapter 7 owns the regime statement)

 ELASTICITIES  |dD/dln eta| = 5.963140  |dD/dln u| = 1.242306  ratio 4.8001
 CRITERION    c/eta = 2.066667  >  1 - 1/N_eff = 0.430550     cost side is stronger
```

The proportionally strongest lever is the one that does not exist, and the lever with the reserve is the slow one. That combination is ordinary, and it is why Chapter 4's classification matters more than any single sensitivity: the case is TUNABLE, so a combination exists.

Phase 2b — the intervention, correctly ordered

CONSTRUCTED. Two changes, made together, both on the numerator:

1. **B takes on two states A does not hold.** This is a change in what B is actually doing — not a rhetorical adoption of a contrary position, which R8 forbids from working.
2. **The pair adopt a decision log.** This raises `eta`, because what they work out now has somewhere to live, and lowers `c`, because the record does the remembering the coordination overhead was paying for.

One realising vector, given for reproducibility:

```
B' = {2: 0.95, 4: 0.60, 6: 0.55, 9: 0.30, 10: 0.20, 12: 0.8391}
     raw L2 norm 1.548899          cos theta = 0.554641432498572

u     0.068734665264634  ->  0.692372881355932
eta   0.30               ->  0.44
c     0.62               ->  0.40
delta 0.09               ->  0.09         (deliberately unchanged)
```

The reading, recomputed from the full-precision u (SPINE §3.4; a 6-dp u shifts `f` in the fifth significant figure):

```
F                = ln(1 + 11 * 0.692372881355932) = 2.153632742815 nats
N_eff             = 8.6161016949
f               = 35.8881768941 Hz             folded 287.106 Hz (j = 3)
octaves present   =  6.2140705559

c/eta             = 0.40/0.44 = 0.909090909091
ceiling test      : margin +1.575816 nats           BELOW, comfortably
f_min           =  2.9782153324 Hz             folded 381.212 Hz (j = 7)
octaves required  =  2.6230818925
```

```
    OCTAVE DEFICIT    = -3.5909886634                  (-4309.2 cents)
    N_eff required    =  2.482065
    pitch headroom    =  0.955854 octaves
    numerator eta F - c = +0.547598406839               POSITIVE
    Q_inf (delta 0.09)  = +6.0844267427
      identity check        : -2.885390 * (0.09/0.44) * 6.0844267427 = -
3.590988663387
    VERDICT: ACCUMULATES
```

The swing is 7.929409 octaves and `Q_inf` moves from `-5.0119395464` to `+6.0844267427`. The pair now needs to be worth 2.4821 distinct people and is worth 8.6161.

Now trust helps

Repeat the identical intervention that made things worse at 2a.

```
    delta 0.09 -> 0.05

    Q_inf   +6.0844267427  ->  +10.9519681368            BETTER by 80.0%
   deficit -3.5909886634  ->   -3.5909886634           unchanged, as always
```

The same lever, the same magnitude, the same direction, the same two people, and the opposite result — because the numerator changed sign in between.

Both orderings, priced

CONSTRUCTED. Each intervention held for eight periods before the other is applied, from `Q_0 = 1.0`:

```
     t      A: trust first     B: numerator first
     0        +1.000000           +1.000000
     1        +0.498925           +1.457598
     2        +0.022905           +1.874013
     4        -0.858924           +2.597783
     8        -2.373029           +3.693462
    12        +0.098675           +5.039870
    20        +3.751671           +7.029761
    32        +7.061215           +8.832564
     inf     +10.951968          +10.951968

   A first goes below zero at t = 3, bottoms at -2.373029, regains +1.000000 at t = 14
    B never falls below its starting value
    permanent lag                              12 periods
    cumulative gap over 32 periods             111.133601 stock-periods
```

Both arrive at exactly the same place. One of them arrives twelve periods later, forever, having spent thirteen periods below where it started. The fixed point is order-independent; the path is not; and the interval in which the stock is negative is the interval in which people decide whether to keep going.

The reading, as a card

```
------------------------------------------------------------------------
 CARRIED DYAD -- CONSTRUCTED CASE -- N = 12 -- one declared period
------------------------------------------------------------------------
                                2a (as found)      2b (after)
 cos theta                        0.965021          0.554641
 u                                0.068735          0.692373
 F (nats)                         0.563085          2.153633
 N_eff                            1.7561            8.6161
 f (Hz)                           1.4908           35.8882
   folded 220-440                381.644 (j=8)     287.106 (j=3)
 eta                              0.30              0.44
 c                                0.62              0.40
 c/eta                            2.066667          0.909091
 ceiling (ln 12 = 2.484907)       BELOW: tunable    BELOW: comfortable
 f_min (Hz)                      30.1588            2.9782
 octaves required                 5.963140          2.623082
 octaves present                  1.624719          6.214071
 OCTAVE DEFICIT                  +4.338420         -3.590989
 NUMERATOR eta F - c             -0.451075         +0.547598
 pitch headroom (octaves)         5.5452            0.9559
 |dD/dln eta| : |dD/dln u|        4.8001            1.0285
 delta                            0.09              0.09
 Q_inf                           -5.0119           +6.0844
 VERDICT                          DECAYS            ACCUMULATES
------------------------------------------------------------------------
 delta 0.09 -> 0.05:  Q_inf       -9.0215           +10.9520
                  deficit     +4.338420       -3.590989   UNCHANGED both
------------------------------------------------------------------------
 ESSENCE frequencies, not physical resonances. Constructed case. Not an
assessment of any person. Not counselling, therapy or clinical assessment.
------------------------------------------------------------------------
```

The honest note on where 2b leaves them

The repair worked by spending most of the range that twelve categories make available. Pitch headroom falls from 5.5452 octaves to 0.9559. The next order of gain is not available from u at this N, and per Chapter 4 SPEC 4.12 the only route past it is genuinely new categories, which must satisfy Chapter 5's validity conditions and not be finer slicing of what is already there.

This is not a defect in the repair. It is the ceiling doing what a ceiling does, and the value of the reading is that the pair can see it coming rather than meeting it as an unexplained plateau.

> **THE ARC: fix the numerator, then lower the loss. Reverse the order and the same intervention accelerates the failure.**

Chapter 16 takes the reading past two, where the intervals multiply as N(N−1)/2 and the cost does

too. Chapters 18 and 29 cite this chapter for the order-of-operations rule and do not re-derive it.

Chapter 16 . Triads and Larger Sets

A dyad has one interval. That is the whole reason Chapters 12 through 15 could treat a bond as a pitch and get away with it: there was one relationship to read, one u to measure, one number to compare against one threshold.

Add a third person and there are three intervals. Add a fourth and there are six. The relationships do not grow with the roster; they grow with the square of it, and everything that was simple at two becomes a question of aggregation at three, of accounting at six, and of arithmetic that has already decided the answer at twelve.

This chapter is about what changes and what does not. Three things change: the count of intervals, the need to aggregate them into one reading, and the cost structure, which Brooks named in 1975 and which the octave form of Chapter 4 prices exactly. One thing does not change, and it is the result that organises the chapter: **the surplus ceiling is `ln N`, it is set by the category count and not by the roster, and a perfectly non-parallel pair already reaches it.** A larger group can therefore buy no more surplus than two well-chosen people, while paying quadratically more for the privilege.

That is a harsh statement and it needs its qualifications, which §16.7 supplies. But it is what the formalism says, it follows from the affine axiom in two lines, and a chapter on groups that did not say it first would be arranging the furniture around a hole in the floor.

Ownership: this chapter takes the interval count, the group chord, and the Brooks comparison. Chapter 7 owns octave folding, the odd-part class rule and the `N = 17` regime statement, and this chapter cites all three without re-deriving any of them. Chapter 4 owns `f_min`, the deficit and the levers. Chapter 10 owns the consonance/redundancy identity. Chapter 17 takes the three worked structures and the commons.

Nothing in this chapter is clinical, diagnostic or therapeutic, none of it is counselling, and the frequencies are essence frequencies — no group, person or room is claimed to resonate at any value in it.

The Law

16.1 The interval count, and a notation warning that has to come first

For a set of `m` members there are

```
m (m − 1) / 2      unordered pairs, hence that many intervals
```

m	pairs	pairs per member	pairs per available pitch class (6)
2	1	0.50	0.167
3	3	1.00	0.500
4	6	1.50	1.000
5	10	2.00	1.667
6	15	2.50	2.500
7	21	3.00	3.500
8	28	3.50	4.667
10	45	4.50	7.500
12	66	5.50	11.000
15	105	7.00	17.500
20	190	9.50	31.667
30	435	14.50	72.500

The notation warning: `N` in this book is the number of state CATEGORIES, fixed at 12, and it is not the roster size. The manifest's shorthand `N(N−1)/2` refers to the roster; every other appearance of N in the formalism — `F <= ln N`, `N_eff = 1 + (N−1)u`, `f <= k N^2`, the ceiling `c/eta <= ln N` — is the category count. This chapter writes `m` for the roster and reserves `N` for categories, and it is worth being pedantic about because conflating them produces readings that are wrong by whole octaves in a way nothing downstream catches.

The second column is the one people underestimate. Going from six members to twelve does not double the relational load; it multiplies it by 4.4, from 15 pairs to 66. The third column is the one this chapter turns into a result in §16.11.

16.2 Aggregation, and the two effective counts

The construction of §2 of SPINE is pairwise. A group has many pairs and one reading, so an aggregator is required, and the book uses one:

```
u_bar  =  mean of u over all m(m−1)/2 unordered pairs
F      =  ln( 1 + (N − 1) u_bar )          N = 12
f      =  k ( 1 + (N − 1) u_bar )^2
```

That is the construction SPINE §3.1 and §3.3 both use, and it is the one every reading in this book uses. There is, however, a second effective count that lives alongside it, and keeping them apart is worth a short section because they answer different questions and have different bounds.

```
CATEGORY−SCALE      N_eff  =  1 + (N − 1) u_bar        bounded by N = 12
            "how many distinct state−categories' worth of view is present"

ROSTER−SCALE        M_eff  =  1 + (m − 1) u_bar        bounded by m
             "how many genuinely distinct members is this roster fielding"
```

THEOREM 16.1 (R3, verified at roster scale). `M_eff = m + 1 − m/PR`, where PR is the Renyi-2 participation ratio of the members' Gram matrix,

```
PR  =  ( trace G )^2 / sum_{ij} G_ij^2  =  m^2 / sum_{ij} G_ij^2
```

Verified on the constructed triad of the Worked Example: `PR = 1.3522930095`, `m + 1 − m/PR = 1.7815458788`, `1 + (m−1) u_bar = 1.7815458788`, error `0.00e+00`.

The identity is worth having for two reasons. It gives a second, independent route to `u_bar` that does not require enumerating pairs — useful at large `m`, where the pair count is the expensive part. And it supplies the sentence to hand to someone who does not want a frequency: a roster of three fielding `M_eff = 1.78` is three people doing 1.78 people's worth of distinct looking.

16.3 The aggregation gap, and it is larger than Book Two's

`F` is strictly concave in `u` (Paper A, Theorem 3.3). Therefore, by Jensen's inequality,

```
F( u_bar ) >= mean of F( u_pair )      with equality only if all pairs share one u
```

The book's canonical reading uses `F(u_bar)`, which is the optimistic end of the interval, and R2 requires the conservative end to be published with it. Chapter 4 SPEC 4.15 states the rule; this chapter supplies the magnitude at group scope, and it is not small.

Computed on the constructed triad of the Worked Example:

```
pairwise u:  0.068734665265,  0.552647058824,  0.550937094081

F( u_bar )       = 1.6674242020 nats     f = 13.5718 Hz     D = -3.410906
mean of F(u)     = 1.4915742527 nats     f =  9.5476 Hz     D = -2.903511

AGGREGATION GAP   = 0.1758499493 nats  =  0.507396 octaves  =  609 cents
frequency ratio   = 1.421482
```

Half an octave of surplus, from the choice of where the averaging happens. Book Two's worked office found the same effect worth 6.2%; at this triad's dispersion it is worth 11.79% of the conservative figure, and the reason is visible in the pairwise list: one pair at 0.0687 and two at 0.55 is exactly the dispersion Jensen's inequality punishes.

And the counter-case, computed on the commons of SPINE §3.3, whose fifteen pairwise u values all lie between 0.018484 and 0.018830:

```
F( u_bar )   = 0.186276647       mean of F = 0.186276219
AGGREGATION GAP = 0.00000124 octaves
```

Where the pairs are alike, the gap vanishes; where they are dispersed, it is the largest single uncertainty in a group reading. The rule that follows is not "use the conservative figure" — it is that both are computed and both are published, and where the verdict differs between them the verdict is not determined. It does not differ in the triad: `-3.410906` and `-2.903511` both accumulate, and per Chapter 3 §3.6 an ACCUMULATES verdict is the robust direction anyway, since `f` is a lower bound.

16.3b When the gap is large, and when it can be ignored

Because the aggregation gap is the largest controllable uncertainty in a group reading, it is worth knowing where it lives. Two sweeps, both computed on two-pair sets so that the dispersion is explicit.

Holding the mean at `u_bar = 0.40` and widening the spread between the two pairs:

```
pairs             F(u_bar)      mean-of-F      gap (octaves)
0.40 / 0.40       1.686399       1.686399        0.000000
```

```
0.35 / 0.45       1.686399       1.681185       0.015044
0.30 / 0.50       1.686399       1.665209       0.061142
0.25 / 0.55       1.686399       1.637392       0.141405
0.20 / 0.60       1.686399       1.595650       0.261847
0.15 / 0.65       1.686399       1.536289       0.433126
0.10 / 0.70       1.686399       1.452630       0.674514
0.05 / 0.75       1.686399       1.331439       1.024197
0.01 / 0.79       1.686399       1.187727       1.438862
```

Holding a fixed relative spread of ±50% of the mean and sweeping the level:

```
u_bar     pairs             gap (octaves)
 0.02     0.010 / 0.030       0.011776
 0.05     0.025 / 0.075       0.046143
 0.10     0.050 / 0.150       0.102518
 0.20     0.100 / 0.300       0.181418
 0.40     0.200 / 0.600       0.261847
 0.60     0.300 / 0.900       0.301406
```

The two sweeps together give a usable rule. The gap grows with absolute dispersion, quadratically at small spreads and faster after, and it grows mildly with the level. It is negligible in exactly one situation: when the pairwise values are close together, which is the near-parallel case. So the commons, whose fifteen pairs lie inside a band of width 0.000346, can be read from `F(u_bar)` without qualification — and a group with one intimate pair and two distant ones cannot.

The practical form: compute the spread of the pairwise list before deciding how much of the report the gap deserves. A group whose pairwise u values differ by less than about 0.05 has a gap under 0.02 octaves and may state one figure. Anything wider states two.

16.4 The group chord and the class ceiling

A group's chord is the set of states its members hold, taken together. Chapter 9 owns the chord at internal scope and the construction transfers unchanged: the states are indices, the indices are frequencies through `f = k n^2`, and the folding is Chapter 7's.

Chapter 7's odd-part theorem gives the ceiling and this chapter cites it rather than proving it again: two indices share a pitch class if and only if they share an odd part, so an indexed set `1..N` holds exactly `ceil(N/2)` distinct classes — six, for `N = 12`.

The six, folded into 220–440 Hz and sorted by pitch, with adjacent separations:

```
class   folded (Hz)     cents above the previous
  11      233.978            —
   1      247.514          97.3641
   3      278.453         203.9100
   9      313.260         203.9100
   7      379.006         329.8318
   5      386.741          34.9756
                          (wrap to 2 x 233.978:  330.0085)

minimum adjacent separation  =  34.9756 cents  =  the 50/49 regime
```

That 34.9756-cent floor is Chapter 7's, it holds because `N = 12` sits below the `N = 17` minimum-gap cliff, and this chapter states the fact once and does not re-derive it. Chapter 7 §7.10 has the sweep and the exact-rational argument.

The consequence for groups is a saturation result. The class count is capped at six no matter how many members there are, while the pair count grows quadratically. Computed by simulation — 20,000 trials, members' states drawn uniformly without replacement from the twelve, random seed 7:

```
expected distinct pitch classes covered
m      3 states each    4 states each    5 states each
 2         3.799            4.398            4.842
 3         4.506            5.008            5.372
 4         4.927            5.367            5.637
 6         5.432            5.729            5.879
 8         5.695            5.884            5.961
12         5.904            5.977            5.996
```

By six members holding four states apiece, a group covers 5.73 of the six available classes; by twelve it covers 5.98. Every further member adds pairs, adds cost, and adds essentially no class the chord did not already contain.

This is the precise sense in which a group's chord goes muddy, and it is not a metaphor about noise. **The chord saturates the lattice while the interval count keeps growing, so past a modest roster the additional relationships are all being drawn from a set of intervals the group already sounds.** Chapter 10's identity is the other half of the statement: the most consonant pairs are exactly the pitch-class collisions, so a saturated chord is one whose new pairs are increasingly redundant with old ones.

16.5 Brooks, priced in octaves

Fred Brooks' observation in *The Mythical Man-Month* (1975) is that the communication paths in a group of `m` grow as `m(m−1)/2` while the useful output does not. The octave form of Chapter 4 makes the comparison exact, because it puts both sides in the same unit.

Cost side. With a per-link cost `kappa` per period, `C = kappa m(m−1)/2`, and Chapter 4 Theorem 4.3 converts:

```
octaves required = 2.885390 * ( C / eta ) = 1.442695 * ( kappa / eta ) * m ( m − 1 )
```

Surplus side. The most a roster of `m` can field, at roster scale, is `M_eff = m` — every member mutually non-parallel. Through the essence bridge that is

```
octaves available (best case)  =  2 log2( m )
```

THEOREM 16.2 — TWO OCTAVES PER DOUBLING AGAINST A QUADRATIC. In the best case a group's surplus rises by exactly two octaves each time the roster doubles, while its cost rises as `m(m−1)`. Setting the two equal and writing `r = kappa/eta`:

```
NET(m)  =  2 log2(m)  −  1.442695 * r * m(m−1)      octaves

crossover m* solves      2 ln m  =  r m ( m − 1 )
```

Proof: substitute and multiply through by `ln 2`. **QED.**

The crossover has a closed form as a bound on `r`: the largest viable integer roster at cost ratio `r` is the largest `m` with `2 ln m >= r m(m−1)`, equivalently `r <= 2 ln m / (m(m−1))`. Computed:

m	r_max(m) = 2 ln m / (m(m−1))	m	r_max(m)
2	0.69314718	10	0.05116856
3	0.36620410	11	0.04359810
4	0.23104906	12	0.03765010
5	0.16094379	15	0.02579095
6	0.11945063	17	0.02083245
7	0.09266239	20	0.01576701
8	0.07426577	25	0.01072959
9	0.06103402		

Read the other way — the maximum viable roster at a given cost ratio:

r = kappa/eta	max viable roster (best case)	
0.005	38	
0.010	26	
0.020	17	
0.030	13	
0.050	10	
0.100	6	
0.125	5	
0.161765	4	
0.200	4	
0.366204	3	
0.500	2	
0.693147	2	<-- r_max(2) = ln 2 exactly
1.000	1	
2.066667	1	

A dyad is viable in the best case if and only if `kappa/eta <= ln 2 = 0.693147`, which is a pleasing way to see why the carried dyad of Chapter 15, at `r = 2.066667`, could not be saved by its geometry alone. It was over the roster bound at two.

Two figures from the carried cases, both computed:

```
    the commons of SPINE 3.3:   r = 0.055/0.34 = 0.161765
                                r_max(5) = 0.160944
                               the six-member commons misses a viable FIVE-
member roster by
                                                  0.000821 in r -- a per-
link cost of 0.055 against 0.054721

   the constructed triad:    r = 0.055/0.34 = 0.161765, max viable roster 4, actual m = 3
```

The commons fails the best-case roster test by half a percent of its per-link cost, before any question of who its members are. Chapter 17 takes that case up in full.

16.6 The roster ceiling, which is exact and comes from Chapter 4

The Brooks crossover of §16.5 is a best-case comparison and it rests on a modelling choice (`M_eff = m`). There is a second bound that rests on nothing but Chapter 4's Theorem 4.6, and it is exact.

THEOREM 16.3 — THE ROSTER CEILING. Since `F <= ln N` and sustainability requires `c/eta <= ln N`, a group with per-link cost `kappa` and conversion `eta` is sustainable at *any* geometry only if

```
kappa m ( m − 1 ) / 2  <=  eta ln N          i.e.          m ( m − 1 )  <=  2 eta ln N / kappa
```

Proof: substitute `C = kappa m(m−1)/2` into `c/eta <= ln N`. **QED.**

Computed for the cases this book carries, at `N = 12`, `ln N = 2.484906649788`:

case	eta	kappa	m(m−1) bound	m_max
commons	0.34	0.055	30.7225	6
family (Ch 17)	0.34	0.130	12.9980	4
team (Ch 17)	0.52	0.055	46.9873	7
carried dyad	0.30	0.620	2.4047	2
dyad at 2b	0.44	0.400	5.4668	2

The commons sits at exactly its roster ceiling — `m(m−1) = 30` against a bound of 30.7225 — which is why Chapter 4's classification returns TUNABLE for it with a margin of only 0.058436 nats. A seventh member would make it structural outright, and no assignment of u whatsoever would save it.

The two bounds answer different questions and both are needed. Theorem 16.3 says whether *any* geometry could work. Theorem 16.2 says whether the best achievable geometry would. A group inside 16.3 and outside 16.2 is one whose survival depends on being extraordinarily well composed.

16.6b The hold line by roster size

Chapter 4's Corollary 4.8 gives the geometry required for a relation to hold exactly: `u* = (e^(c/eta) − 1)/(N − 1)`. At group scope, with `C = kappa m(m−1)/2`, that becomes a function of the roster and it is the most direct way to see what quadratic cost does.

Computed at `kappa = 0.055`, `eta = 0.34`, `N = 12`:

m	links	C	c/eta	u* required to HOLD	
2	1	0.0550	0.161765	0.015962	
3	3	0.1650	0.485294	0.056787	
4	6	0.3300	0.970588	0.149045	
5	10	0.5500	1.617647	0.367383	
6	15	0.8250	2.426471	0.938078	
7	21	1.1550	3.397059	2.625100	UNREACHABLE, u <= 1
8	28	1.5400	4.529412	8.336728	UNREACHABLE
9	36	1.9800	5.823529	30.651224	UNREACHABLE
10	45	2.4750	7.279412	131.739523	UNREACHABLE

```
  12      66    3.6300  10.676471      3938.488241     UNREACHABLE
```

The u∗ column rises far faster than the roster does, because it is exponential in a quantity that is quadratic in `m`. Two members need 1.6% non-parallelism to break even at this link price. Six need 93.8%, which is the commons' figure of SPINE §3.3 arriving from the cost side rather than from the geometry side. Seven need more than exists.

The seventh member does not make the problem harder; it makes it insoluble, and the transition between those two states occupies no interval at all — one member, and the required geometry goes from demanding to unavailable. That is the roster ceiling of Theorem 16.3 written as a requirement on u, and it is the same statement Chapter 4's Corollary 4.8 makes about a dyad.

A second sweep, holding the geometry fixed at the constructed triad's `u_bar = 0.390773` and varying only the roster:

```
 m    C        c/eta       D             Q_inf     verdict
 2   0.0550   0.161765   −4.344415      +5.6880    ACCUMULATES
 3   0.1650   0.485294   −3.410906      +4.4658    ACCUMULATES
 4   0.3300   0.970588   −2.010644      +2.6325    ACCUMULATES
 5   0.5500   1.617647   −0.143626      +0.1880    ACCUMULATES, barely
 6   0.8250   2.426471   +2.190145      −2.8675    DECAYS
 7   1.1550   3.397059   +4.990671      −6.5342    DECAYS, STRUCTURAL
 8   1.5400   4.529412   +8.257951     −10.8120    DECAYS, STRUCTURAL
12   3.6300  10.676471  +25.994613     −34.0342    DECAYS, STRUCTURAL
```

A group of people every one of whom is as well matched to the others as the constructed triad's members are — `u_bar` held at 0.390773 for every pair, which is a strong assumption at any size — crosses from accumulating to decaying between five and six members, and from tunable to structural between six and seven. **The geometry is doing nothing wrong in any row of that table. The roster is.**

16.7 The uncomfortable result: a pair already reaches the ceiling

Now the statement flagged in the opening, which follows from the construction in two lines.

`F = ln(1 + (N−1) u_bar)` is maximised at `u_bar = 1`, giving `F = ln N`. Two unit vectors can be orthogonal, `cos theta = 0`, hence `u = 1`. **Therefore a single orthogonal pair already attains the maximum surplus the twelve-category system admits, and no roster of any size attains more.**

```
orthogonal pair:  u_bar = 1,  F = ln 12 = 2.484907,  f = k ∗ 144 = 69.6133 Hz
orthogonal six:   u_bar = 1,  F = ln 12 = 2.484907,  f = k ∗ 144 = 69.6133 Hz
                    -- identical, and the six pays 15 links instead of 1
```

In surplus terms a group buys nothing a well-chosen pair could not, and pays `m(m−1)/2` times as much for it. That is the honest reading of the formalism and it should be stated before any of the qualifications, because the qualifications are where the wishful thinking gets in.

Now the qualifications, and they are real.

One: orthogonality is an idealisation, and mean pairwise `u_bar = 1` for six people requires all fifteen pairs to be orthogonal at once. Six mutually orthogonal unit vectors exist in twelve dimensions, so it is geometrically possible — but the vectors are then extremely constrained, and Chap-

ter 9's disjoint-support warning applies with force: six people described over non-overlapping state sets read `u = 1` for free and mean nothing by it. Achievable `u_bar` falls with `m` in every realistic composition, and the pair is the easiest place to find a large one.

Two: `F` is not the only thing a group is for. The formalism prices stored order per period. It has no term for redundancy against loss, for succession, for coverage across time zones or lifespans, for the fact that a pair ends when one of its members does. A group's non-surplus functions are entirely outside this reading and the reading does not weigh them — Chapter 4 §4.22 states the boundary and it holds here without modification.

Three: `eta` and `kappa` are not roster-independent in practice. A group may convert better than a pair because it has structure a pair does not need, and Chapter 17's Ostrom material is largely about arrangements that lower `kappa` per link. The formalism holds them fixed because it is a formalism; a real design moves them.

Four: the ceiling is a ceiling on `F`, not on output. Two people at `F = ln 12` and six people at `F = ln 12` bank at different rates if their `eta` differs, and `Q_inf = (eta F − c)/delta` is sensitive to all three.

With all four in hand the statement survives in a weaker and still useful form: the case for a larger group is never a case about surplus, because surplus is capped at a value a pair can reach. It is always a case about conversion, coverage, or continuity, and it should be made in those terms.

16.8 R8 at group scope

Chapter 14 owns the unison result and it does not weaken with roster size. `u` depends only on `|rho|`, so a member recruited to a group specifically to disagree contributes `u = 0` to every pair in which the disagreement is systematic, exactly as an echo would, while adding `m − 1` new links to the cost.

```
    adding a perfectly opposed member to a group of m:
        surplus contribution                      0
        cost contribution        kappa * (m − 1) per period
          octave contribution       +1.442695 * (kappa/eta) * 2(m−1)    −−
strictly positive DEFICIT
```

A contrarian appointment is the only intervention in the book that is guaranteed to make the reading worse by a computable amount. At `kappa/eta = 0.161765` and `m = 6`, adding one such member costs `+2.3338` octaves of deficit and returns nothing.

16.9 Honesty clauses at group scope

(1) Essence frequencies, not physical resonances. No group, room, institution or person is claimed to resonate at any value here, and Chapter 4's FM 4.8 forbids playing `f_min` at anyone.

(2) No clinical, diagnostic or therapeutic claim; the twelve states are not diagnoses; this is not counselling and no reading here is an assessment of any person or of any group's health.

(3) All cases are constructed and labelled. The triad of the Worked Example is an explicitly labelled extension of the carried dyad of SPINE §3.2.

(4) The reading inherits its inputs, and at group scope it inherits `m(m−1)/2` of them. Uncertainty in a group reading is not `m` times a dyadic uncertainty; it is whatever the aggregation gap and the least certain parameter jointly permit, and §16.3 gives the first term.

(5) Consent is structural, and at group scope it is per-member. Chapter 18 owns the procedure. **A group reading taken without every member's agreement is not a group reading; it is a reading about people, taken without them.**

(6) R14 applies to the simulation in §16.4: it is a computation with a stated seed and trial count, not an observation of anybody.

The Engineering

16.10 Reading a group, end to end

```
   0.  CONSENT from every member, separately, and revocable.              Ch 18
    1.  DECLARE N and the ordering.                                       Ch 5
   2.  ELICIT intensities by magnitude estimation; publish every raw L2 norm. Ch 6
   3.  ENCODE each member as a unit vector; check support overlap.        Ch 9
   4.  COMPUTE all m(m−1)/2 pairwise u; PUBLISH THE LIST, not just the mean.  this chapter
   5.  AGGREGATE: u_bar, F(u_bar), and mean-of-F. Report the gap.         Ch 4 SPEC 4.15
       6.   DECLARE kappa, eta, delta on one period; C = kappa m(m−
1)/2.          Ch 4 SPEC 4.3
   7.  ROSTER CEILING TEST: m(m−1) <= 2 eta ln N / kappa.                  Thm 16.3
   8.  CEILING TEST: c/eta vs ln N; classify.                           Ch 4 SPEC 4.1
   9.  COMPUTE f, f_min, D, Q_inf; fold and publish with j.                Ch 4 SPEC 4.5
  10.  SIGN THE NUMERATOR eta F − c.                                   Ch 15 SPEC 15.2
  11.  DROP-ONE ANALYSIS.                                                 Sec 16.13
  12.  REPORT sensitivity to the least certain parameter.                  Ch 4 SPEC 4.19
```

Step 4's instruction to publish the whole list is the one that costs nothing and catches the most. A mean of 0.39 is produced by three pairs at 0.39 and by one pair at 0.07 with two at 0.55, and those are entirely different groups. The list shows which, the mean does not, and §16.3's aggregation gap is computable only from the list.

16.11 Where the chord goes muddy, precisely

Three distinct things happen as `m` grows, and they are usually described as one thing.

The class ceiling saturates. §16.4: six classes, reached in expectation by about six members holding four states each. After that, new members add no new classes.

The pair-per-class ratio grows without bound. From the table in §16.1: 0.5 at `m = 3`, 2.5 at `m = 6`, 11.0 at `m = 12`, 72.5 at `m = 30`. This ratio is the honest measure of muddiness — it is how many relationships are being carried by each available distinction.

The rendering floor does not move. The minimum class separation stays at 34.9756 cents for every `N <= 16`, because it is a property of the lattice and not of the roster (Chapter 7). A group of thirty sounds no finer than a group of three; it sounds the same six classes, more often.

COROLLARY 16.4 — THE MUDDINESS INDEX. For a roster of m on an N-category ladder,

```
mu = [ m(m−1)/2 ] / ceil(N/2)      relationships per available distinction
```

mu > 1 first occurs at m = 4 for N = 12. **From four members onward, a group has more relationships than it has distinguishable classes to express them in**, and every additional member worsens the ratio quadratically while the denominator is frozen.

This is not an argument that groups of four are bad. It is an argument about what a chord reading can and cannot tell you at four and above: the chord stops individuating relationships and starts reporting only the group's coverage of the lattice. Past mu > 1 the useful group-level objects are the pairwise list, the drop-one analysis, and the deficit — not the chord.

16.12 The marginal member test

The decision that actually arises is not "how big should the group be" but "should this person join." That is a marginal question and it has a clean form.

```
marginal surplus (best case)  =  2 log2( m / (m−1) )              octaves
marginal cost                 =  1.442695 ∗ (kappa/eta) ∗ 2(m−1)   octaves

 ADD if surplus > cost
```

Computed at kappa/eta = 0.161765:

m	marginal surplus	marginal cost	verdict
2	+2.0000	0.4668	ADD
3	+1.1699	0.9335	ADD
4	+0.8301	1.4003	STOP
5	+0.6439	1.8670	STOP
6	+0.5261	2.3338	STOP
7	+0.4448	2.8005	STOP
8	+0.3853	3.2673	STOP

The marginal surplus falls like 1/m and the marginal cost rises like m, so once the test flips it never flips back. There is no roster size at which adding members becomes worthwhile again, at fixed kappa and eta. The only way to reopen the question is to change the cost structure, which is §16.14 and Chapter 17's Ostrom material.

Two cautions on using the test. It is a best-case surplus, so a real candidate must be compared against their actual contribution to u_bar, not against 2 log2(m/(m−1)). And it is silent on everything a person is for besides surplus — Chapter 4 §4.22's boundary applies, and a marginal test that returns STOP is not a statement that anyone should be excluded from anything.

16.13 The drop-one analysis

The complement of the marginal test, run on an existing group: recompute the reading with each member's pairs removed, and look at what moves.

Computed on the constructed triad, at kappa = 0.055, eta = 0.34, delta = 0.09:

set	u_bar	links	C	f (Hz)	D	Q_inf	change in D
{A,B,C}	0.390773	3	0.1650	13.5718	−3.410906	+4.4658	—

```
   {B,C}      0.550937      1    0.0550   24.0978   −5.172708    +6.7725     −
1.761802
   {A,C}      0.552647      1    0.0550   24.2264   −5.180385    +6.7826     −
1.769478
      {A,B}            0.068735            1       0.0550       1.4908     −
1.157965    +1.5161     +2.252941
```

Removing either A or B improves the reading by about 1.77 octaves; removing C worsens it by 2.25. The reason is entirely geometric: A and B sit at u = 0.0687, near-parallel, and their pair is the one dragging u_bar down while still costing a link. C is the member carrying the group's non-parallelism.

What the drop-one analysis is for, and what it is not for, must be stated in the same breath. It is a sensitivity analysis: it tells you which pairs are carrying the reading, which is information about the geometry of the set. It is not a ranking of people, it licenses no conclusion about anyone's value or contribution, and a formalism with no single-party term cannot acquire one by being run m times. The correct use of the row that says "removing A improves the reading" is to notice that A and B are near-parallel and to ask what would raise u between them — which is the same conclusion Chapter 15 reached about the same pair.

16.13b The two-dyad group, and why triads are not four dyads

One structure deserves separate treatment because it is common and because its reading is counterintuitive: the group that is really two pairs.

Take four members {P, Q, R, S} where P–Q and R–S are close pairs and the cross-links are distant. The six pairwise values might run u(P,Q) = 0.08, u(R,S) = 0.08, and four cross-pairs at 0.70. Computed:

```
u_bar           = (0.08 + 0.08 + 0.70*4)/6            = 0.493333
F( u_bar )      = ln(1 + 11*0.493333)                 = 1.860456 nats
mean of F(u)    = (2*ln(1.88) + 4*ln(8.70))/6         = 1.652639 nats
                  ln(1.88) = 0.631272   ln(8.70) = 2.163323
AGGREGATION GAP = 0.207817 nats = 0.599632 octaves
f optimistic    = 19.9665 Hz      f conservative = 13.1763 Hz
```

The gap is large, and it is large for a structural reason rather than a numerical one: **a group made of tight clusters is exactly the dispersion pattern Jensen's inequality punishes most, and it is also the most common real topology.** Families have it, teams have it, and any group formed by merging two existing groups has it by construction.

Two consequences follow. First, cluster-structured groups must always be read on both aggregations, and Chapter 17's family is worked that way. Second, the within-cluster pairs are the ones the drop-one analysis will flag, for the same reason A and B are flagged in this chapter's triad: they cost a link and contribute little non-parallelism. The correct reading of that flag is never about the people; it is that the group is paying twice for one perspective, which is Chapter 14's result about the sycophant's unison applied to a sub-group instead of to a pair.

And a triad is not three dyads plus overhead. It has a property no collection of pairs has: for any two members there is a third who is in relation to both, which is what makes mediation, witness and succession structurally possible. **None of that appears in F, and a chapter that computed**

only F would be reporting the smaller half of what a triad is. The formalism prices stored order per period; the third member's structural role is real, is outside the price, and is not made unreal by being outside it.

16.14 Splitting, nesting, and what changes the exponent

The one intervention that changes the *shape* of the cost function rather than its coefficient is topological: stop maintaining every link.

Ostrom's eighth design principle — nested enterprises — has a precise reading here. A flat group of `m` maintains `m(m−1)/2` links. A group partitioned into `g` sub-groups of size `m/g`, each maintaining its own links, plus one link per pair of sub-groups, maintains

```
g ∗ (m/g)(m/g − 1)/2  +  g(g−1)/2        links
```

Computed for six members:

```
flat six                          15 links      C = 0.8250 at kappa 0.055
two triads + one bridge            7 links      C = 0.3850          −53.3%
three pairs + three bridges        6 links      C = 0.3300          −60.0%
```

Nesting is the only lever in this chapter that attacks the quadratic itself, and on six members it removes more than half the cost. Book Two's R15 material records the same result in the other register: P8 turns `O(N^2)` coordination into `O(N)`.

And it is not free, which Chapter 17 computes rather than asserts. Two members in different sub-groups now interact through a bridge, so whatever surplus their pair carried is attenuated or lost. Nesting is a trade of surplus for cost and the trade is only worth making when the lost pairs were carrying little — which, at a low `u_bar`, they are.

The Specification

SPEC 16.1 — PER-MEMBER CONSENT, REVOCABLE. Every member agrees separately to a group reading and any member may withdraw at any point. A member's withdrawal ends their inclusion in the reading and may not be treated as a finding about them.

SPEC 16.2 — DECLARE `m` AND `N` SEPARATELY in every group reading, and never let `N` denote the roster.

SPEC 16.3 — PUBLISH THE FULL PAIRWISE LIST, not the mean alone. The list is what makes the aggregation gap and the drop-one analysis computable.

SPEC 16.4 — REPORT BOTH AGGREGATIONS. `F(u_bar)` and the mean of `F(u)`, with the gap in octaves. Where the verdict differs between them, report the verdict as NOT DETERMINED.

SPEC 16.5 — SUPPORT-OVERLAP GATE. Check every pair for disjoint support before computing. Chapter 9's degeneracy makes non-overlapping descriptions read `u = 1` for free.

SPEC 16.6 — ROSTER CEILING TEST (Theorem 16.3) before any lever prescription: `m(m−1) <= 2 eta ln N / kappa`.

SPEC 16.7 — DECLARE kappa PER LINK and state whether links are assumed uniform. Non-uniform link costs require `C = sum kappa_ij` and the roster ceiling is then stated on the sum.

SPEC 16.8 — REPORT THE MUDDINESS INDEX `mu = [m(m−1)/2]/ceil(N/2)` with any group chord, and do not present a chord as individuating relationships where `mu > 1`.

SPEC 16.9 — CITE THE REGIME, DO NOT RE-DERIVE IT. `N = 12` is below the `N = 17` cliff, the minimum class separation is 34.9756 cents, and Chapter 7 owns both statements.

SPEC 16.10 — MARGINAL TESTS USE ACTUAL CONTRIBUTIONS. A candidate's effect on `u_bar` is computed from their vector, not from the best-case `2 log2(m/(m−1))`.

SPEC 16.11 — DROP-ONE IS A SENSITIVITY ANALYSIS. It is reported as a property of the geometry. No output of a drop-one analysis may be presented as a ranking, an evaluation, or a recommendation about a person.

SPEC 16.12 — R8 CHECK ON RECRUITMENT. Report `cos theta` signed for any proposed member. A systematically opposed member contributes `u = 0` and `m − 1` links.

SPEC 16.13 — NESTING IS REPORTED WITH ITS LOST PAIRS. A topology change that removes links also removes whatever surplus those pairs carried; both terms are computed.

SPEC 16.14 — NO AGGREGATION ACROSS GROUPS (Chapter 4 SPEC 4.14, and R16: two nine-member groups with identical u and S lost 77% and 21% of surplus to the same three-person departure).

SPEC 16.15 — LABEL EVERY CASE CONSTRUCTED.

The Failure Modes

FM 16.1 — `N` USED FOR THE ROSTER. *Signature:* `F = ln(1 + (m−1)u)` computed with `m` in place of 12, or a ceiling of `ln m`. *Mechanism:* the two counts are different objects with different bounds (§16.2). *Computed:* on the constructed triad, `1 + 11 u_bar = 5.2985` and `1 + 2 u_bar = 1.7815` — a difference of 3.152 octaves in the reading. *Fix:* SPEC 16.2.

FM 16.2 — THE MEAN REPORTED WITHOUT THE LIST. *Signature:* one `u_bar` for a group, no pairwise values. *Mechanism:* the mean is compatible with wildly different geometries, and the aggregation gap is not computable from it. *Fix:* SPEC 16.3.

FM 16.3 — THE JENSEN SHORTCUT TAKEN SILENTLY. *Signature:* a group reading from `F(u_bar)` alone. *Mechanism:* strict concavity inflates `F`, inflates `f`, and under-reports `D`. Worth 0.507396 octaves on the constructed triad; 6.2% in Book Two's worked office. *Fix:* SPEC 16.4.

FM 16.4 — THE ORTHOGONAL GROUP THAT IS AN ARTEFACT. *Signature:* a large group reading `u_bar` near 1. *Mechanism:* disjoint support. Members described over non-overlapping state sets are automatically orthogonal (Chapter 9's one-hot trap, generalised). *Fix:* SPEC 16.5, and re-encode every member under one procedure.

FM 16.5 — GROWING A GROUP FOR SURPLUS. *Signature:* a roster expanded on the argument that more people means more perspective. *Mechanism:* §16.7. Surplus is capped at `ln N` and an

orthogonal pair reaches it. Cost is not capped. *Fix:* make the case for the roster on conversion, coverage or continuity, which are real and are outside this reading.

FM 16.6 — THE ROSTER CEILING NOT TESTED. *Signature:* a lever prescription for a group that no geometry could sustain. *Mechanism:* Theorem 16.3. At `m(m−1) > 2 eta ln N / kappa` the case is structural and u is unavailable at every value. *Computed:* the commons sits at `m(m−1) = 30` against a bound of 30.7225; a seventh member takes it to 42 and makes it structural outright. *Fix:* SPEC 16.6, before anything else.

FM 16.7 — THE CONTRARIAN APPOINTMENT. *Signature:* a member recruited to disagree. *Mechanism:* R8. `u` depends only on `|rho|`; opposition contributes zero surplus and `m−1` links. *Computed:* `+2.3338` octaves of deficit at `m = 6`, `kappa/eta = 0.161765`. *Fix:* SPEC 16.12.

FM 16.8 — THE CHORD READ AS A ROSTER PORTRAIT. *Signature:* a twelve-member group's chord presented as showing its variety. *Mechanism:* class saturation. At twelve members holding four states each, 5.98 of six classes are expected to be covered by chance alone. *Fix:* SPEC 16.8, and report `mu`.

FM 16.9 — DROP-ONE READ AS A RANKING. *Signature:* "the analysis says `A` is the weakest member." *Mechanism:* a drop-one row reports the effect of removing a set of pairs. The apparatus has no single-party term and cannot produce a statement about a person (Chapter 4, FM 4.13). *Fix:* SPEC 16.11. Read the row as a statement about which pairs carry the geometry.

FM 16.10 — NESTING COSTED WITHOUT ITS LOSSES. *Signature:* a partition adopted for its link count, with `u_bar` assumed unchanged. *Mechanism:* removed links were carrying surplus. At high `u_bar` the loss dominates the saving. *Fix:* SPEC 16.13, and compute both terms.

FM 16.11 — THE AGGREGATED ORGANISATION. *Signature:* one deficit for a whole institution. *Mechanism:* R16 — identical aggregates conceal topologies that lose 77% versus 21% of surplus to the same departure; R10 — ratios do not average. *Fix:* SPEC 16.14. Per group, per period.

FM 16.12 — THE MARGINAL TEST USED AS AN EXCLUSION RULE. *Signature:* a STOP verdict treated as grounds for keeping someone out, or removing them. *Mechanism:* register slip. The test prices a link against a best-case surplus contribution and weighs nothing else whatsoever. *Fix:* §16.12's cautions, and Chapter 4 §4.22. A reading does not conclude that a relation should end and does not conclude anything about a person.

Worked Example

THE CONSTRUCTED TRIAD — `A`, `B`, `C`

CONSTRUCTED, and an explicitly labelled extension of the carried dyad of SPINE §3.2. `A` and `B` are the carried dyad exactly as found; `C` is a constructed third member. Parameters are chosen, not observed. **This is not an assessment of any person or group, no clinical, diagnostic or therapeutic claim is made, and the frequencies are essence frequencies.**

Inputs

```
A = {4: 0.7,  6: 0.5,  9: 0.4}                    raw L2 norm 0.948683
```

```
B = {4: 0.6,  6: 0.55, 9: 0.3,  10: 0.2}              raw L2 norm 0.890225
C = {3: 0.45, 4: 0.55, 6: 0.40, 7: 0.35, 11: 0.25}    raw L2 norm 0.921954

N = 12,  m = 3,  one declared period
kappa = 0.055 per link,  eta = 0.34,  delta = 0.09
```

Raw L2 norms are published because normalisation removes a free multiplier and Chapter 6's rule requires it to stay recoverable.

Steps 3–4 — support and the pairwise list

Every pair shares support — A and C on states 4 and 6, B and C on 4 and 6, A and B on 4, 6 and 9 — so SPEC 16.5's gate passes and no pair is orthogonal by construction.

```
u(A,B) = 0.068734665265   cos =  0.965021   F = 0.563085   f_pair =  1.4908 Hz
u(A,C) = 0.552647058824   cos =  0.668844   F = 1.957149   f_pair = 24.2264 Hz
u(B,C) = 0.550937094081   cos =  0.670122   F = 1.954489   f_pair = 24.0978 Hz

3 pairs, as m(m-1)/2 requires
```

The list already tells the story the mean will hide: one near-parallel pair and two well-separated ones.

Step 5 — aggregation, both ways

```
u_bar          = 0.390772939390
F( u_bar )     = 1.6674242020 nats     N_eff = 5.2985      f = 13.5718 Hz
mean of F(u)   = 1.4915742527 nats                         f =  9.5476 Hz

AGGREGATION GAP = 0.1758499493 nats = 0.507396 octaves = 609 cents
ratio of frequencies = 1.421482

R3 cross-check at roster scale:
    PR = 1.3522930095      m + 1 - m/PR = 1.7815458788
    1 + (m-1) u_bar        = 1.7815458788        error 0.00e+00
    M_eff = 1.7815 of a possible 3
```

Steps 6–8 — costs and the two ceiling tests

```
links   = m(m-1)/2 = 3
C       = 0.055 * 3 = 0.1650
c/eta   = 0.1650/0.34 = 0.485294

ROSTER CEILING (Thm 16.3): m(m-1) = 6  <=  2*0.34*2.484907/0.055 = 30.7225    PASS
CEILING TEST  (Ch 4 SPEC 4.1): 0.485294 <= 2.484907                    TUNABLE
BROOKS (Thm 16.2): r = 0.161765, max viable roster 4, actual 3            WITHIN
```

Step 9 — the reading

```
f                =  13.5718 Hz         folded 434.296 Hz (j = 5)
f_min            =   1.2760 Hz         folded 326.656 Hz (j = 8)
octaves required =   1.400263
octaves present  =   4.811169
OCTAVE DEFICIT   =  -3.410906          (-4093.1 cents)
N_eff required   =   1.6247
pitch headroom   =   2.3588 octaves
Q_inf            =  +4.465825
identity check   : -2.885390 * (0.09/0.34) * 4.465825 = -3.410906
VERDICT: ACCUMULATES

conservative (mean-of-F): f = 9.5476 Hz, D = -2.903511, Q_inf = +3.801503
both aggregations ACCUMULATE -- verdict determined
```

Step 10 — the numerator

```
eta F - c  =  0.34 * 1.6674242020 - 0.1650  =  +0.401924        POSITIVE
```

So `delta` interventions are available here and would help — Chapter 15's SPEC 15.3 is satisfied and trust, in this group, multiplies a positive sign. That was not true of the same `A` and `B` without `C`.

The comparison that makes the chapter

The carried dyad `{A, B}` at Chapter 15's own cost structure — `c = 0.62`, `eta = 0.30` — read `+4.338420` octaves and DECAYS. The triad `{A, B, C}` at a per-link cost of 0.055 and `eta = 0.34` reads `−3.410906` and ACCUMULATES.

Two things changed, and honesty requires separating them. The geometry changed: `u_bar` rose from 0.068735 to 0.390773 because `C` is genuinely non-parallel to both. And the cost structure changed: this reading prices a link at 0.055 where Chapter 15's dyad priced its single link at 0.62, a factor of 11.2727.

That is not an inconsistency; it is what a link costs in each situation, and the point of the comparison is that both terms have to be declared. A third member does not rescue a dyad by existing. It rescues it only where the link price is low enough that three links cost less than the surplus the third member's non-parallelism supplies. Priced at the carried dyad's own `kappa`, the same triad would carry `C = 3 * 0.62 = 1.86`, `c/eta = 6.2`, which exceeds `ln 12 = 2.484907` — a STRUCTURAL case with a deficit of `+13.078` octaves.

```
the same three people, two link prices

 kappa = 0.055, eta = 0.34   C = 0.1650   c/eta = 0.485294   D = -3.410906   ACCUMULATES
kappa = 0.620, eta = 0.30  C = 1.8600  c/eta = 6.200000  D = +13.078   STRUCTURAL
```

The roster is not the variable that decides a group. The ratio of link price to conversion is.

The group chord

```
states spanned: 3, 4, 6, 7, 9, 10, 11      (seven of twelve)

 n     f = k n^2 (Hz)     folded (220-440)     j     odd part = class
 3         4.3508             278.453          6           3
 4         7.7348             247.514          5           1
 6        17.4033             278.453          4           3
 7        23.6879             379.006          4           7
 9        39.1575             313.260          3           9
10        48.3426             386.741          3           5
11        58.4945             233.978          2          11

distinct classes: {1, 3, 5, 7, 9, 11} -- six, which is ceil(12/2), the whole ceiling
states 3 and 6 share class 3: one of the seven states adds pitch but no new class
 muddiness index mu = 3 / 6 = 0.500
```

Seven states, six classes, three members: the chord is already saturated at the class ceiling. Chapter 7's `ceil(N/2)` result is doing all the work and the roster is doing none of it — a fourth member could add at most the one class already present twice, which is to say nothing.

Drop-one

```
set         u_bar      C       f (Hz)    D          Q_inf    change in D
 {A,B,C}     0.390773   0.1650   13.5718   -3.410906    +4.4658        -
{B,C}      0.550937   0.0550   24.0978   -5.172708   +6.7725    -1.761802
{A,C}      0.552647   0.0550   24.2264   -5.180385   +6.7826    -1.769478
{A,B}      0.068735   0.0550    1.4908   -1.157965   +1.5161    +2.252941
```

The near-parallel pair is the one costing the group, and the reading says so twice. Removing either member of it raises `u_bar` by about 0.16 and removes a link. What the analysis does not say, cannot say, and must not be reported as saying, is anything at all about A or B as people — SPEC 16.11 and Chapter 4 §4.22 both govern here, and the actionable reading is Chapter 15's: A and B sit at `cos theta = 0.9650` and the question is what would raise u between them.

The card

```
------------------------------------------------------------------------
 CONSTRUCTED TRIAD {A, B, C} -- extension of SPINE 3.2 -- N = 12, m = 3
------------------------------------------------------------------------
 pairwise u             0.068735 / 0.552647 / 0.550937
 u_bar                  0.390773
 F(u_bar)               1.667424 nats        mean-of-F   1.491574 nats
 aggregation gap        0.507396 octaves
 N_eff (category)       5.2985               M_eff (roster)  1.7815 of 3
 f                      13.5718 Hz           folded 434.296 (j = 5)
 kappa / links / C      0.055 / 3 / 0.1650
 eta / delta            0.34 / 0.09
 c/eta                  0.485294             ln 12 = 2.484907   TUNABLE
```

```
roster ceiling         6 <= 30.7225         PASS
Brooks max roster      4                    actual 3   WITHIN
f_min                  1.2760 Hz            folded 326.656 (j = 8)
octaves req / present  1.400263 / 4.811169
OCTAVE DEFICIT         -3.410906            (conservative -2.903511)
numerator eta F - c    +0.401924            POSITIVE
Q_inf                  +4.465825            (conservative +3.801503)
chord                  7 states, 6 classes, mu = 0.500
VERDICT                ACCUMULATES, on both aggregations
-------------------------------------------------------------------------
ESSENCE frequencies, not physical resonances. Constructed case. Not an
assessment of any person or group. Not counselling; no clinical claim.
-------------------------------------------------------------------------
```

What the chapter established

Four results, and the fourth is the one to carry forward.

One: intervals go as `m(m−1)/2`, classes are capped at `ceil(N/2) = 6`, and the ratio between them — the muddiness index — passes 1 at four members and never comes back.

Two: aggregation is a choice with a price. `F(u_bar)` and mean-of-F differed by 0.507396 octaves on three members and by 0.000001 octaves on the commons' fifteen pairs, and both must be published.

Three: cost is quadratic and certain; surplus is logarithmic at best and capped absolutely. The crossover `2 ln m = r m(m−1)` and the exact roster ceiling `m(m−1) <= 2 eta ln N / kappa` bracket every group this book reads.

Four: a group's case is never a surplus case, because an orthogonal pair already reaches the ceiling. Whatever else a family, a team or a commons is for, the formalism prices only the stored order, and Chapter 17 takes those three structures and reads all of them — including one that cannot be repaired at all.

Chapter 17 reads three relational structures in full and carries the commons of SPINE §3.3, where the deficit is +6.4638 octaves and no single lever closes it. Chapter 18 takes the interventions.

Chapter 17 . The Family, the Team, the Commons

Chapter 16 built the apparatus for reading a set larger than two. This chapter uses it three times, on three structures that differ in every parameter the formalism has, and reports what comes back.

The three are chosen because they are the structures people actually live inside, and because they fail differently. A family is a cluster geometry with expensive links and a long memory. A team is a selected geometry with an explicit conversion mechanism. A commons is a large flat roster whose members were chosen for agreement, and it is the one this chapter exists for.

The three readings come back as follows, and the third is the point:

```
    STRUCTURE     m    u_bar      deficit D      Q_inf       verdict
   the family    4   0.645160   +0.584737    −1.378049   DECAYS, marginally
    the team     5   0.788228   −3.495327    +6.999131    ACCUMULATES
  the commons    6   0.018614   +6.463800    −8.463000   DECAYS, and cannot be tuned
```

No single lever closes a 6.4638-octave gap. Raising the commons' u to 0.95 — near orthogonality, for six people drawn from one doctrine — closes it by 0.0332 octaves and no more. Raising `eta` to 0.90 leaves `+2.1075`. Cutting the per-link cost to 0.020 leaves `+2.0085`. There is no assignment of any one of the four levers that makes that commons sustainable, and the reason is not that the numbers are bad. It is that a group whose members were selected for agreement has almost no non-parallelism to convert, and the cost of maintaining fifteen links has to be paid against surplus that is not there.

> **Some systems cannot be tuned. They must be restructured, and the diagnostic is what tells you which case you are in.**

A book that only showed fixable examples would be lying by selection, and the honest thing to do with an instrument is to run it on a case it cannot help. This chapter does that, computes exactly how far every lever falls short, and then computes what restructuring would have to mean — which turns out to be a change of membership, not a change of arrangement.

Every case here is CONSTRUCTED and labelled. Nothing in this chapter is clinical, diagnostic or therapeutic; none of it is counselling; the twelve states are not diagnoses; and the frequencies are essence frequencies — no family, team, commons or person is claimed to resonate at any value.

The Law

17.1 What a relational structure is, in this formalism

A structure is five declarations and nothing else. Everything this chapter says about families, teams and commons is a statement about one of these five, and where it is not, the chapter says so.

```
 1. THE ROSTER            m, the number of members
2. THE GEOMETRY       the m(m-1)/2 pairwise u values, and their mean u_bar
 3. THE COST TOPOLOGY    which pairs maintain a link, and kappa for each
4. THE CONVERSION     eta -- the fraction of interaction becoming durable structure
5. THE LOSS          delta -- the per-period rate at which stored order leaks
```

A "family," a "team" and a "commons" are not primitives in this formalism; they are characteristic patterns across those five declarations, and the pattern is what does the work. Nothing is being claimed about kinship, employment or land tenure. The names are labels for parameter signatures, and the signatures are what follow.

```
SIGNATURE      roster   geometry          topology       eta      delta
 ------------------------------------------------------------------------
----------
family         small    clustered; one     dense, high    moderate  LOW
                          near-parallel pair  kappa per link
team          small    selected for      dense, low     HIGH     moderate
                          non-parallelism     kappa
commons       larger   near-parallel by   flat, uniform  low      moderate
                          membership rule     kappa
```

The family's signature is a low `delta` — shared history is a memory that does not leak fast — paid for with a high `kappa`, because family links are maintained under conditions nobody chose. The team's signature is a high `eta`, because a team has apparatus for turning interaction into artefacts, and a geometry that was selected rather than inherited. **The commons' signature is a membership rule that selects for agreement, and that rule is the whole of its problem.**

17.2 The Parallel Wall, priced

Book Two names the Parallel Wall: a set of parties whose representations point the same way returns no surplus however many of them there are and however hard they work. The frequency register prices it exactly.

```
six members drawn from one doctrine       SPINE 3.3, CONSTRUCTED

mean pairwise u  =  0.018614    over 15 pairs
F                =  0.186272 nats
N_eff            =  1.204754        of a possible 12
f                =  0.7017 Hz       folded 359.270 Hz (j = 9)
```

`N_eff = 1.2048` is the figure to sit with. Six people, fifteen relationships, and 1.20 people's worth of distinct view among them. They are not lazy, not unintelligent, and not in conflict. They

were selected — by a doctrine, a hiring filter, a shared training, a common origin — for pointing the same way, and the selection worked.

The frequency `0.7017` Hz is 0.5375 octaves above the ground essence k. A group of six that has travelled just over half an octave from "no relationship at all" is a group whose relationships are almost entirely redundant with each other, which is Chapter 10's identity at group scope: the most consonant pairs are exactly the pitch-class collisions, so a maximally consonant set is a maximally redundant one.

17.3 The commons reading in full

```
THE COMMONS -- CONSTRUCTED, SPINE 3.3
m = 6, N = 12, one declared period
kappa = 0.055 per link,  15 links,  C = 0.8250,  eta = 0.34,  delta = 0.09

u_bar            =  0.018614
F                =  0.186272 nats          N_eff = 1.204754
f                =  0.7017 Hz              folded 359.270 Hz (j = 9)
octaves present  =  0.5375

c/eta            =  0.8250/0.34 = 2.426471
ln 12            =  2.484906649788
margin         =  +0.058436 nats               BELOW the ceiling: TUNABLE
roster ceiling  :  m(m-1) = 30  <=  2*0.34*ln12/0.055 = 30.7225     PASS, barely

f_min            =  61.9349 Hz             folded 247.740 Hz (j = 2)
octaves required =  7.0013
OCTAVE DEFICIT   =  +6.4638                (+7756.6 cents)
N_eff required   =  11.3189   of a possible 12
pitch headroom   =  6.6324 octaves
numerator eta F - c = -0.761666            NEGATIVE
Q_inf            =  -8.4630
VERDICT: DECAYS
```

`N_eff required = 11.3189` against `N_eff achieved = 1.2048` is the reading in one line. These six must field 11.32 categories' worth of distinct view to break even and they field 1.20.

17.4 No single lever closes it, and here is each one falling short

Every figure below is computed from the same starting point, varying exactly one parameter.

LEVER 1 — raise `u`.

```
u -> 0.20   f =  4.950 Hz    deficit  +3.6452
u -> 0.40   f = 14.097 Hz    deficit  +2.1354
u -> 0.60   f = 27.923 Hz    deficit  +1.1493
u -> 0.80   f = 46.428 Hz    deficit  +0.4158
u -> 0.95   f = 63.378 Hz    deficit  -0.0332     closes, by 0.0332 octaves
u -> 1.00   f = 69.613 Hz    deficit  -0.1686     the absolute maximum available
```

The hold line is at u* = (e^2.426471 − 1)/11 = 0.938078, and the whole reserve above it is 0.1686 octaves. So this lever technically closes the gap, and the closure is worthless: it requires that six people selected from one doctrine reach 93.8% non-parallelism, which is not a tuning of a group but a different group. And it arrives with 40 cents of margin, which Chapter 4 §4.20 notes is about a fifth of a semitone — no margin at all against any increase in cost.

LEVER 2 — raise eta.

```
  eta -> 0.34   c/eta = 2.426471   deficit  +6.4638
  eta -> 0.50   c/eta = 1.650000   deficit  +4.2234
  eta -> 0.70   c/eta = 1.178571   deficit  +2.8632
  eta -> 0.90   c/eta = 0.916667   deficit  +2.1075
eta -> 1.00  c/eta = 0.825000  deficit  +1.8430    perfect conversion, still failing
```

At eta = 1.00 — every unit of interaction becoming permanent structure, nothing lost, which is physically the end of the scale — the commons is still short by 1.8430 octaves. The lever does not merely fall short; it is exhausted while the gap is still nearly two octaves wide.

LEVER 3 — cut kappa.

```
  kappa -> 0.055  (C = 0.825)   deficit  +6.4638
  kappa -> 0.040  (C = 0.600)   deficit  +4.5544
  kappa -> 0.030  (C = 0.450)   deficit  +3.2814
  kappa -> 0.020  (C = 0.300)   deficit  +2.0085
  kappa -> 0.010  (C = 0.150)   deficit  +0.7355
kappa -> 0.005  (C = 0.075)  deficit  +0.0990    still failing, at a 91% cut
```

A 91% reduction in per-link cost leaves the commons 0.0990 octaves short. The cost side can be cut almost to nothing and the group still does not sustain, because the surplus it is trying to pay for is 0.186272 nats.

LEVER 4 — raise N.

```
N -> 12    F = 0.186272   present 0.5375   deficit  +6.4638
N -> 17    F = 0.260689   present 0.7522   deficit  +6.2491
N -> 25    F = 0.369310   present 1.0656   deficit  +5.9357
N -> 41    F = 0.556502   present 1.6057   deficit  +5.3956
N -> 64    F = 0.775962   present 2.2390   deficit  +4.7624
N -> 144   F = 1.297955   present 3.7451   deficit  +3.2562
```

One hundred and forty-four categories — twelve times the ladder this book uses — leaves the commons 3.2562 octaves short. Chapter 4 §4.17 gives the mechanism: N enters F only multiplied by u, so adding categories to a near-parallel set adds dimensions nobody is using. At u = 0.018614 a new category buys 0.0446 octaves. And the route crosses the N = 17 cliff at the second row, where the folded lattice's minimum separation collapses from 34.9756 cents to 6.0008 and the symbol rate falls by a factor of exactly 8 (Chapter 7 owns the regime statement).

> **THE FOUR-LEVER RESULT, COMPUTED: at u = 1 the gap is closed by 0.1686 octaves; at eta = 1 it is still open by 1.8430; at a 91% cost cut it is still open by 0.0990; at twelve times the category count it is still open by 3.2562. Only one lever closes it alone, and only by replacing the membership.**

17.5 The reachability test — a case can be formally tunable and practically not

Chapter 4's classification returns TUNABLE for the commons, because `c/eta = 2.426471` is below `ln 12 = 2.484907`. That verdict is correct and it is not enough, and this chapter adds the test that completes it.

DEFINITION 17.1 — REACHABILITY. A tunable case is REACHABLE if the required geometry `u* = (e^(c/eta) − 1)/(N−1)` lies within the range the parties can honestly represent, and UNREACHABLE otherwise. Reachability is not a property of the formalism; it is a declared judgment about the parties, and it must be declared rather than assumed.

```
THE THREE-WAY CLASSIFICATION, EXTENDED

D <= 0                                  SUSTAINABLE
D > 0, c/eta <= ln N, u* reachable  TUNABLE AND REACHABLE     tune it
D > 0, c/eta <= ln N, u* not       TUNABLE, UNREACHABLE     restructure it
c/eta > ln N                        STRUCTURAL              restructure it
```

Computed for the three structures of this chapter:

```
structure    c/eta     ln N      u* required    u_bar now    classification
family     2.294118  2.484907    0.810517      0.645160    tunable, reachable
team       1.057692  2.484907    0.134733      0.788228    sustainable already
commons    2.426471  2.484907    0.938078      0.018614    tunable, UNREACHABLE
```

The family needs a 25.6% rise in `u_bar` and the commons needs a factor of 50.4. The formal test does not distinguish them; the reachability test does, and it is the difference between an afternoon's work and a different organisation.

The margin figure is worth reporting alongside. The commons sits 0.058436 nats below the ceiling — 0.1686 octaves. Chapter 16's Theorem 16.3 says the same thing about its roster: `m(m−1) = 30` against a bound of 30.7225. A seventh member takes `m(m−1)` to 42 and the case becomes STRUCTURAL outright, at which point `u` is unavailable at every value whatsoever. The commons is not near a boundary in one variable; it is near it in both, which is what a group at the end of its arrangement looks like from inside the arithmetic.

17.6 Ostrom, applied where it applies and priced where it does not

Elinor Ostrom's *Governing the Commons* (1990) sets out eight design principles for durable common-pool institutions. Book Two mapped them to the storage-law parameters; this section carries the mapping into the frequency register and prices three of them on this commons. The mapping is Book Two's and is badged as a correspondence of dynamics, not of agency (R16).

P4 — monitoring. Book Two's finding is specific and slightly awkward: monitoring lowers the variable cost `c_v` and does **not** raise the trust parameter `a`. In this register that means P4 is a `c` lever, not a `delta` lever, and Theorem 4.9 applies to the half of it people expect to matter: **an intervention on `delta` does not move the deficit by any amount whatsoever.** On this commons P4 helps only through the `c` column of §17.4, where a 91% cut still leaves 0.0990 octaves.

P5 — graduated sanctions. Priced in Book Two at -7.7% of `eta S`. Computed here:

```
eta * F        =  0.34 * 0.186272 =  0.063332
P5 cost        =  0.077 * 0.063332 = 0.004877
numerator      = -0.761666  ->  -0.766544
Q_inf          =  -8.462972  ->  -8.517157
```

P5 makes this commons slightly worse, because it is a cost levied against a surplus that does not exist. That is not an argument against graduated sanctions; it is the ordinary observation that a governance mechanism priced as a fraction of surplus is affordable in proportion to the surplus, and this commons has 0.186272 nats of it.

P8 — nested enterprises. This is the only Ostrom principle that changes the exponent rather than a coefficient, and Book Two records it as turning `O(N^2)` coordination into `O(N)`. Computed on six members:

```
flat six                15 links   C = 0.8250   c/eta = 2.426471   D = +6.4638
two triads + one bridge      7 links   C = 0.3850   c/eta = 1.132353   D = +2.7298
three pairs + three bridges    6 links   C = 0.3300   c/eta = 0.970588   D = +2.2630
```

Nesting removes 53.3% of the cost and leaves the commons 2.7298 octaves short. It is the largest single structural improvement available and it does not close the gap, because the gap is not principally a cost problem.

Where Ostrom applies here and where it does not, stated plainly. The eight principles are arrangements: boundaries, congruence, collective choice, monitoring, sanctions, conflict resolution, recognition, nesting. Every one of them acts on `c`, on `eta`, or on `delta`, and not one of them acts on `u`. There is no design principle for making the members of a group less alike, because that is not a design question about the institution — it is a question about who is in it.

17.7 Restructuring, defined against tuning

The chapter's central distinction, stated so it can be applied.

```
 TUNING         changing eta, c, delta, or N, with the membership held fixed
RESTRUCTURING   changing the membership, the topology, or the boundary of the group
```

u is the only parameter that a group cannot tune, because u is a property of who its members are. Everything else in the five declarations of §17.1 is an arrangement and can be rearranged. The geometry is a fact about the roster.

That gives the operational rule:

> **If the deficit can be closed without changing u, it is a tuning problem. If it cannot, it is a membership problem, and no amount of better arrangement will substitute.**

Applied to the three cases: the family's deficit of `+0.584737` closes on `eta` alone with a 9.7% improvement (§17.10), so it is a tuning problem. The team is already sustainable. The commons needs u to rise by a factor of 50.4 and therefore is not a tuning problem at all, which is what §17.4's four failed levers were demonstrating one at a time.

17.8 Honesty clauses

(1) Essence frequencies, not physical resonances. Nothing here is claimed to vibrate and nothing is to be played at anyone (Chapter 4, FM 4.8; the no-subliminals rule is absolute).

(2) No clinical, diagnostic or therapeutic claim. The twelve states are not diagnoses. **This is not counselling, therapy or family assessment of any kind, and a reader in difficulty with a family, a team or a community should seek help from someone qualified to give it, which this book is not.**

(3) Every case is constructed and labelled. The family and the team are explicitly labelled extensions of the carried dyad of SPINE §3.2; the commons is SPINE §3.3.

(4) No single-party term exists in the apparatus. No reading in this chapter licenses a conclusion about any person's value, contribution, or fitness for a group.

(5) Consent is structural and per-member (Chapter 16 SPEC 16.1), and §17.14 works out what that requires when a decline changes the reading.

(6) The reading inherits its inputs. A group reading rests on `m(m−1)/2` measured values plus three declared parameters, and its uncertainty is reported (Chapter 4 SPEC 4.19).

The Engineering

17.9 The three structures side by side

	FAMILY	TEAM	COMMONS
m	4	5	6
links	6	10	15
kappa per link	0.130	0.055	0.055
C	0.7800	0.5500	0.8250
eta	0.34	0.52	0.34
delta	0.05	0.09	0.09
u_bar	0.645160	0.788228	0.018614
F(u_bar) nats	2.091463	2.269080	0.186272
mean-of-F nats	1.954682	2.258759	0.186276
aggregation gap (oct)	0.394666	0.029780	0.000001
N_eff	8.096755	9.670503	1.204754
f (Hz)	31.6922	45.2093	0.7017
folded	253.537 (j=3)	361.674 (j=3)	359.270 (j=9)
c/eta	2.294118	1.057692	2.426471
ceiling margin (nats)	+0.190789	+1.427214	+0.058436
roster ceiling m(m−1)	12 <= 12.998	20 <= 46.987	30 <= 30.723
Brooks max roster	2	6	4
muddiness index mu	1.000	1.667	2.500
f_min (Hz)	47.5308	4.0089	61.9349
octaves req / present	6.6194/6.0347	3.0519/6.5472	7.0013/0.5375
OCTAVE DEFICIT	+0.584737	−3.495327	+6.463800
numerator eta F − c	−0.068902	+0.629922	−0.761666

```
    Q_inf                    -1.378049     +6.999131     -8.463000
    VERDICT                   DECAYS       ACCUMULATES     DECAYS
    classification        tunable/reach.   sustainable  tunable/UNREACHABLE
    ------------------------------------------------------------------------
----
```

Three structures, three verdicts, and the parameter that separates them is not the one anybody would guess. It is not roster size — the failing commons is only two larger than the working team. It is not cost per link — the commons and the team pay the identical 0.055. It is `u_bar`, and the commons' is 42 times smaller than the team's.

17.10 The family — a cluster geometry with a cheap fix and an unresolved verdict

The family's pairwise list shows the cluster structure Chapter 16 §16.13b describes:

```
    u(A,B) = 0.068735      the near-parallel pair
    u(A,D) = 0.710180      u(A,E) = 0.711015
    u(B,D) = 0.723966      u(B,E) = 0.740352
    u(D,E) = 0.916710
```

One tight pair, five wide ones. That dispersion produces an aggregation gap of 0.394666 octaves — substantial, and both readings must be published. Here they agree on the verdict:

```
   optimistic  F(u_bar) = 2.091463   f = 31.6922 Hz   D = +0.584737   Q_inf = -
1.378049
   conservative mean-of-F = 1.954682  f = 24.1071 Hz   D = +0.979403   Q_inf = -
2.308160
```

Both decay, so the verdict is determined, and it is a small deficit. The levers:

```
   L1  u   must reach 0.810517 from 0.645160     +0.165357, a factor of 1.2563
  L2  eta must reach 0.372945 from 0.34       +9.69%        REACHABLE, and cheap
   L3  C   must fall to 0.711098 from 0.78        -8.83% -> kappa 0.118516
   L4  N   must reach 14.8193 -> 15 categories    still below the N = 17 cliff
```

A 9.7% improvement in conversion efficiency closes this family's entire deficit, which is what a small deficit on a high-`c/eta` case looks like: the elasticity `dD/d(ln eta) = -6.619424` octaves per e-fold is enormous, and only 0.584737 octaves are needed.

And then the honest complication, which is why this case is in the chapter. Chapter 4's SPEC 4.19 requires the sensitivity to be published, and here it decides the reading:

```
    dD/d eta = -19.4689 octaves per unit eta

    eta known to +/- 0.02  ->  D = +0.5847 +/- 0.3894    verdict resolved
    eta known to +/- 0.05  ->  D = +0.5847 +/- 0.9734    VERDICT NOT RESOLVED
    eta known to +/- 0.10  ->  D = +0.5847 +/- 1.9469    VERDICT NOT RESOLVED

    dD/dC = +8.4864 octaves per unit C
    C known to +/- 0.05    ->  D = +0.5847 +/- 0.4243    verdict resolved
    C known to +/- 0.10    ->  D = +0.5847 +/- 0.8486    VERDICT NOT RESOLVED
```

```
    to resolve the sign, eta must be known to better than +/- 0.030034 --
8.8% of its value
```

Unless `eta` is known to within about 9% of itself, this family's verdict is not determined and the correct action is to measure, not to intervene. That is the reading a marginal case deserves, and Chapter 18 makes it a numbered step rather than a caution.

The family's low `delta = 0.05` is its characteristic parameter and it is doing exactly what Theorem 15.1 says. The numerator is `−0.068902`, negative. A family's long memory is amplifying a small negative sign, and would amplify it further if the memory got longer — `delta` to 0.03 takes `Q_inf` from `−1.378049` to `−2.296749`, with the deficit unmoved at `+0.584737`. R15 in miniature, on the structure most likely to reach for continuity as a repair.

17.11 The team — what a selected geometry and an explicit mechanism buy

```
u(A,B') = 0.692372   u(A,C) = 0.552647   u(A,S) = 0.897875   u(A,T) = 0.679778
 u(B',C) = 0.851659     u(B',S) = 0.953352    u(B',T) = 0.748794
 u(C,S)  = 0.756702     u(C,T) = 0.786182     u(S,T) = 0.962914

 u_bar = 0.788228     10 pairs, all between 0.55 and 0.97
```

The team's pairwise list is tight at a high level, which is the geometry the family's is not. Its aggregation gap is 0.029780 octaves — negligible, because the values are close together, exactly as Chapter 16 §16.3b predicts.

Its reading is comfortable: `D = −3.495327`, `Q_inf = +6.999131`, and both aggregations agree (`−3.465547` and `+6.939499`). Three parameters explain it and all three are arrangements or selections rather than luck.

`eta = 0.52`, against the family's 0.34 and the commons' 0.34. A team has apparatus — the decision log of Chapter 15's phase 2b, the repository, the standing review — and the apparatus is what conversion efficiency *is*.

`kappa = 0.055` against the family's 0.130. A team's links are maintained on purpose, in declared slots, with agendas. A family's are maintained under conditions no member chose.

`u_bar = 0.788228`, because a team is composed. Its members were selected partly for what they bring that others do not, which is a membership rule pointing the opposite way from the commons'.

The honest counterweight, and it belongs here rather than in a footnote: the team's pitch headroom is 0.6227 octaves. It has spent 91% of the range twelve categories make available, and its next increment of surplus is not purchasable from `u` at this `N`. That is the ceiling of Chapter 4 Theorem 4.6 arriving, and it arrives at every well-tuned system.

17.11b The team's drop-one, and the result that settles the ethics

Run the drop-one analysis of Chapter 16 §16.13 on the team. `kappa = 0.055`, `eta = 0.52`, `delta = 0.09`, four links removed with each member.

```
set        u_bar     links  C      f (Hz)    D         Q_inf     change in D
all five    0.788228    10   0.5500  45.2093  −3.495327   +6.999131     —
```

```
   without A    0.843267     6   0.3300   51.0473   −4.891284   +9.794430   −
1.395957
   without B'   0.772683     6   0.3300   43.6247   −4.664595   +9.340502   −
1.169268
   without C    0.822514     6   0.3300   48.8044   −4.826461   +9.664627   −
1.331134
   without S    0.718572     6   0.3300   38.3291   −4.477889   +8.966638   −
0.982562
   without T    0.784101     6   0.3300   44.7859   −4.702494   +9.416392   −
1.207167
```

Every single row improves the reading. Not one member of a well-composed, comfortably accumulating team is carrying their four links, and removing any of them — the best-matched, the worst-matched, any of them — raises `Q_inf` by between 1.97 and 2.80.

The mechanism is arithmetic and it is worth stating carefully because it is the general case rather than a quirk of these five vectors. The team sits at `u_bar = 0.788228` with 0.6227 octaves of pitch headroom. **Near the ceiling, an additional member can contribute almost no surplus, because there is almost no surplus left to contribute** — `F` is concave and the team is far up the curve. Each member's four links, meanwhile, cost `4 * 0.055 / 0.52 = 0.423077` nats, which is 1.2207 octaves of required pitch. Cost is linear in links and surplus is exhausted, so the trade goes one way.

That result settles a question the previous section only argued. If the drop-one analysis improves on *every* member of a healthy team, then a favourable drop-one row carries no information whatever about the member it names. It is a property of where the group sits on the surplus curve. **A ranking derived from it would be a ranking of nothing, and Chapter 16 SPEC 16.11's prohibition is therefore not a piece of decorum — it is a statement that the numbers do not contain what somebody might try to read out of them.**

The constructive reading of the same table is different and is available: a team at `u_bar = 0.788228` and 0.62 octaves of headroom has reached the useful limit of composition at `N = 12`, and its remaining levers are `eta`, `c`, and genuinely new categories under Chapter 5's validity conditions. That is what the rows are telling anyone; it is not about `A`, `B'`, `C`, `S` or `T`.

17.12 The commons — what the doctrine costs, in one number

The commons' problem is `u_bar = 0.018614`, and the cost of it can be stated as a single interval.

```
   the commons as found              f = 0.7017 Hz    0.5375 octaves above k
   the same six with u_bar = 0.788228  f = 45.209 Hz    6.5472 octaves above k
                                                    ──────────────────────────
    the price of the membership rule                        6.0097 octaves
```

Six octaves of available order, forgone at the point of selection, before anybody did anything. Every subsequent decision the commons makes about its arrangement is being made downstream of that, and none of the arrangements can reach back past it.

This is the sense in which the commons is not a badly run group. Its `kappa` is the same as the team's. Its `eta` is the same as the family's. Its `delta` is the same as the team's. The only parameter

on which the commons differs materially from a structure that works is the one no arrangement can change.

17.12b Can the commons shrink its way out?

If the cost is quadratic in the roster and the surplus is not, the obvious remedy is to be smaller. Computed, taking the six constructed members in order and reading each prefix:

m	u_bar	links	C	D	Q_inf	verdict
2	0.018644	1	0.0550	−0.071526	+0.093647	holds, barely
3	0.018548	3	0.1650	+0.864532	−1.131912	DECAYS
4	0.018587	6	0.3300	+2.263763	−2.963896	DECAYS
5	0.018583	10	0.5500	+4.130888	−5.408481	DECAYS
6	0.018614	15	0.8250	+6.463833	−8.462955	DECAYS

`u_bar` barely moves — it stays inside a band of width 0.000096 across every prefix, because every member is drawn from the same doctrine and any subset is as parallel as the whole. So the entire column of deficits is driven by the link count and nothing else.

The largest sustainable group drawn from this membership is two, and even the pair only clears its threshold by 0.0715 octaves. Three members from one doctrine already decay. The commons cannot shrink its way out any more than it can convert, cut or re-categorise its way out, and for the same underlying reason: `F = 0.186272` nats is what the membership rule left behind, and every arrangement is an attempt to pay fifteen, ten, six, three or one links out of it.

That completes the audit. **Four levers, three Ostrom principles, two nesting topologies and five roster sizes have now been computed on this commons, and exactly one of them closes the gap — u, which is the parameter it does not control.**

17.13 The restructure, computed

If tuning cannot help, what would? The answer has to be a change in membership, topology, or both, and it should be computed rather than gestured at.

CONSTRUCTED. Three changes, taken together:

1. **Recruit outside the doctrine.** The membership rule is changed so that new members are selected for what they bring that the existing members do not. Suppose this moves `u_bar` from 0.018614 to 0.45 — a large change, and one that is a change of group rather than of arrangement.
2. **Nest.** Two triads plus one bridge, per Ostrom P8: 15 links become 7.
3. **Adopt a conversion mechanism.** `eta` from 0.34 to 0.55 — a decision log, minutes that bind, a repository. This is the team's parameter and it is available to any group that will keep records.

```
u_bar            =  0.45
F                =  1.783391 nats        N_eff = 5.95
f                = 17.1145 Hz            folded 273.832 Hz (j = 4)
links            =  7        C = 0.055 * 7 = 0.3850
eta              =  0.55     c/eta = 0.700000
ceiling test     :  0.700000 <= 2.484907        comfortably below
```

```
roster ceiling   :  m(m−1) = 30 <= 2∗0.55∗ln12/0.055 = 49.6981      PASS
f_min            =  1.9604 Hz            folded 250.930 Hz (j = 7)
octaves required =  2.019773
octaves present  =  5.145779
OCTAVE DEFICIT   = −3.126006
numerator        = +0.595865             POSITIVE
Q_inf (delta 0.09) = +6.620724
pitch headroom   =  2.0241 octaves
VERDICT: ACCUMULATES
```

A swing of 9.5898 octaves and `Q_inf` from −8.4630 to +6.6207. And the decomposition says which change did it:

```
from the cost ratio:  2.885390 ∗ (0.700000 − 2.426471)  =  −4.981543 octaves
from the surplus   : −2.885390 ∗ (1.783391 − 0.186272)  =  −4.608257 octaves
 total                                                     −9.589800 octaves
```

51.9% from the cost side and 48.1% from the geometry — and the cost side is only available at that size because the nesting removed eight links. No proper subset of the three does as well. The recruitment plus the log, at fifteen links, leaves −0.817694 octaves — sustainable, but with a quarter of the margin. The recruitment plus the nesting, without the log, leaves −1.878499. The recruitment alone, at fifteen links and `eta = 0.34`, leaves +1.855535 and fails. And the two arrangement changes without the recruitment leave +2.729803 (nesting alone) and +3.790608 (the log alone), both failing badly — which is §17.7's rule arriving as arithmetic: without the change in membership, no combination of arrangements reaches the threshold.

And the honest note on what has happened: this is not the same commons. Its membership rule is different, its topology is different, and its records are different. The formalism cannot tell anyone whether that trade is worth making, because it contains no term for what the doctrine was for — Chapter 4 §4.22 governs and does not soften here. What it can say is that the group as constituted arrives at `Q_inf = −8.4630` if its parameters hold, and that no arrangement of those parameters changes it.

17.14 The right to decline, and the arithmetic nobody wants

Consent at group scope is per-member and revocable (SPEC 16.1). That raises a question the formalism answers uncomfortably, and the answer has to be printed rather than avoided.

Suppose member `E` of the constructed family declines to take part. Recomputing on `{A, B, D}`:

```
u(A,B) = 0.068735    u(A,D) = 0.710180    u(B,D) = 0.723966
u_bar  = 0.500960    F = 1.873426 nats    N_eff = 6.510562
links  = 3           C = 0.13 ∗ 3 = 0.3900     c/eta = 1.147059
f      = 20.4912 Hz
OCTAVE DEFICIT = −2.095851      (was +0.584737)
Q_inf          = +4.939294      (was −1.378049)
VERDICT: ACCUMULATES            (was DECAYS)

E's decline IMPROVED the reading by 2.680588 octaves
```

The decline flipped the verdict, and it flipped it in the group's favour. Three links vanish with `E`,

and E's pairs — 0.711015 and 0.740352 — were above the mean but not by enough to pay for three links at `kappa = 0.13`.

That result must be handled exactly, and the exact handling is this. The formalism computes the stored order of an arrangement per period. It has no term for what E is for, what E contributes that is not stored order, what E's participation means to E, or what the group would be without E. **It weighs none of that, it acquires no such term by being applied to a family, and a number that improves when someone leaves is not an argument that anyone should leave.**

Three rules follow and they are all binding.

One: the right to decline is the member's, never the group's. A decline is an exercise of consent. Nothing in this apparatus converts a favourable drop-one row into a licence to exclude anyone, and Chapter 16 SPEC 16.11 states the same prohibition for the drop-one analysis generally.

Two: a decline is not a finding. SPEC 16.1 requires that a member's withdrawal ends their inclusion in the reading and is not reported as a result about them. The reading after a decline is a reading of a different group, and must be labelled as one.

Three: the group must not be told what a decline would do before the decision is made. Reporting "the reading improves by 2.68 octaves if you withdraw" to a member weighing withdrawal converts an arithmetic result into pressure, and pressure removes the ability to decline freely. Chapter 18 makes this a numbered step: the counterfactual is computed after the decision, or not at all.

17.15 Reading a structure you are inside

One last engineering point, and it is the one most likely to be needed in practice, because nobody reads a family or a team from outside it.

An operator who is a member of the group being read is part of the apparatus. Three specific consequences follow, and all three are handled by declaration rather than by correction.

The operator's own vector is one of the `m`, so their encoding enters `u_bar` directly and the result is sensitive to it. At `m = 4` a single member's vector participates in half the pairs. The requirement is simply that the operator's own vector be elicited by the same procedure as everyone else's, by someone else where possible, and that this be stated in the report.

The operator estimates `eta`, `c` and `delta` for an arrangement they are inside, and §15.9b's estimation practice is harder to apply honestly to one's own arrangement than to another's. The family case shows what is at stake: the verdict is unresolved unless `eta` is known to ±0.030034, and self-estimated `eta` is exactly the sort of quantity that comes back optimistic. The countermeasure is the count — enumerate what survived the period rather than judging how well things are going.

And the operator has an interest in the answer, which the formalism cannot detect and does not correct for. R13's finding stands here in its general form: design controls, not more computation. Read blind where possible, have another member encode the vectors, publish the raw norms, and report the sensitivity so that a reader can see how much of the verdict rests on the least certain input.

None of this makes an internal reading invalid. It makes it a reading with a declared observer, and Book Two's Chapter 50 already records that the operator's own state is part of the apparatus —

one of the four things the alchemical tradition got right, and the one most often dropped by its successors.

The Specification

SPEC 17.1 — DECLARE ALL FIVE. Roster, geometry, cost topology, conversion, loss. A structure named without its five declarations is a label, not a reading.

SPEC 17.2 — PER-MEMBER CONSENT, REVOCABLE, AND UNCONDITIONED. Every member agrees separately; any may withdraw at any time; **and no member is shown the counterfactual value of their own withdrawal before deciding.**

SPEC 17.3 — REPORT BOTH AGGREGATIONS with the gap in octaves (Chapter 16 SPEC 16.4). Where the verdict differs between them, report NOT DETERMINED.

SPEC 17.4 — RUN THE REACHABILITY TEST (Definition 17.1) after the ceiling test and before any prescription. Report u∗ beside `u_bar` and declare the judgment.

SPEC 17.5 — REPORT THE CEILING MARGIN IN NATS AND OCTAVES, and the roster-ceiling margin from Theorem 16.3. A case inside both by a small margin is a case one member from being structural.

SPEC 17.6 — SEPARATE TUNING FROM RESTRUCTURING in every prescription. Changes to `eta`, `c`, `delta` and `N` are tuning; changes to membership, topology or boundary are restructuring, and they are reported as producing a different group.

SPEC 17.7 — PRICE GOVERNANCE MECHANISMS AGAINST THE SURPLUS THEY ARE LEVIED ON. P5 at -7.7% of `eta S` costs 0.004877 on this commons and takes `Q_inf` from −8.462972 to −8.517157.

SPEC 17.8 — OSTROM'S PRINCIPLES ACT ON `c`, `eta` AND `delta`, NEVER ON `u`. Any claim that a design principle raises non-parallelism must be argued separately and is not supported by the mapping.

SPEC 17.9 — NESTING IS REPORTED WITH ITS LOST PAIRS (Chapter 16 SPEC 16.13). Both the link saving and the surplus loss are computed.

SPEC 17.10 — DELTA IS NOT A LEVER ON THE DEFICIT (Theorem 4.9), and where the numerator is negative it is not a lever on the destination either, except downward (Chapter 15 SPEC 15.3).

SPEC 17.11 — REPORT THE SENSITIVITY to the least certain parameter and declare the verdict UNRESOLVED where the uncertainty exceeds the deficit. **The constructed family is unresolved at `eta ± 0.05`.**

SPEC 17.12 — NO ORGANISATION-WIDE AGGREGATE (Chapter 4 SPEC 4.14; R16). Per group, per period.

SPEC 17.13 — LABEL EVERY CASE CONSTRUCTED, and label an extension of the carried case as an extension.

SPEC 17.14 — NO CLINICAL, DIAGNOSTIC, THERAPEUTIC OR COUNSELLING FRAMING, and no reading presented as an evaluation of any person or of any group's health.

The Failure Modes

FM 17.1 — TUNING AN UNREACHABLE CASE. *Signature:* a lever programme for a group whose required u∗ is far outside what its members could represent. *Mechanism:* Definition 17.1. The commons is TUNABLE by the ceiling test and requires `u∗ = 0.938078` from `u_bar = 0.018614` — a factor of 50.4. *Fix:* SPEC 17.4. An unsatisfiable prescription is worse than none: it consumes the effort that restructuring needed (Chapter 4 §4.11).

FM 17.2 — SINGLE-LEVER THINKING ON A SIX-OCTAVE GAP. *Signature:* a plan to fix the commons by improving conversion, or by cutting cost, or by adopting a richer taxonomy. *Mechanism:* every lever saturates. `eta = 1.00` leaves `+1.8430`; a 91% cost cut leaves `+0.0990`; `N = 144` leaves `+3.2562`. *Fix:* §17.4's four tables, computed before the plan rather than after it.

FM 17.3 — MISTAKING SELECTION FOR CULTURE. *Signature:* a group with `u_bar` near zero addressed with meetings, workshops, or facilitation. *Mechanism:* the Parallel Wall. `u` is a property of who the members are. Ostrom's principles all act on `c`, `eta` and `delta`, and not one acts on `u` (SPEC 17.8). *Fix:* SPEC 17.6. Name it as a membership question.

FM 17.4 — TRUST APPLIED TO A NEGATIVE NUMERATOR, IN A FAMILY. *Signature:* continuity, ritual or patience invested in a marginal family case, followed by acceleration. *Mechanism:* R15 and Theorem 15.1. `delta` 0.05 -> 0.03 takes the constructed family from `−1.378049` to `−2.296749`; the deficit does not move. *Fix:* Chapter 15 SPEC 15.3. Sign the numerator first; here a 9.7% rise in `eta` flips it.

FM 17.5 — ACTING ON THE FAMILY'S UNRESOLVED VERDICT. *Signature:* an intervention justified by `D = +0.5847` with `eta` known to a decimal place it does not have. *Mechanism:* SPEC 4.19. At `eta ± 0.05` the uncertainty is `±0.9734` octaves and the sign is unknown. *Fix:* SPEC 17.11. Measure `eta` to better than ±0.030034 first.

FM 17.6 — THE AGGREGATION GAP SUPPRESSED IN A CLUSTER GROUP. *Signature:* one figure reported for a group with one tight pair and several wide ones. *Mechanism:* Jensen. The constructed family's gap is 0.394666 octaves; the two-dyad structure of Chapter 16 §16.13b runs 0.599632. *Fix:* SPEC 17.3.

FM 17.7 — THE SEVENTH MEMBER. *Signature:* a group at its roster ceiling admitting one more, with no reading taken. *Mechanism:* Theorem 16.3. The commons sits at `m(m−1) = 30` against 30.7225; a seventh takes it to 42 and the case becomes STRUCTURAL, at which point no `u` whatsoever suffices. *Fix:* SPEC 17.5, and run the roster ceiling before admission.

FM 17.8 — GOVERNANCE PRICED WITHOUT REFERENCE TO SURPLUS. *Signature:* monitoring, sanctions and conflict procedures added to a group with `F = 0.186272`. *Mechanism:* mechanisms costed as a fraction of `eta S` are affordable in proportion to `S`. *Computed:* P5 takes this commons from `−8.462972` to `−8.517157`. *Fix:* SPEC 17.7.

FM 17.9 — THE DECLINE READ AS A RESULT. *Signature:* "the numbers got better when `E` left." *Mechanism:* a reading after a withdrawal is a reading of a different group. The apparatus has no

single-party term and produces no statement about E. *Fix:* SPEC 17.2 and §17.14's three rules.

FM 17.10 — THE COUNTERFACTUAL SHOWN BEFORE THE DECISION. *Signature:* a member weighing withdrawal is told what their withdrawal would do to the reading. *Mechanism:* this converts arithmetic into pressure and removes the ability to decline freely, which is the one thing CANON3 §5 forbids absolutely. *Fix:* SPEC 17.2. **The counterfactual is computed after the decision, or not at all.**

FM 17.11 — RESTRUCTURING PRESENTED AS TUNING. *Signature:* "we just need to bring in some fresh perspectives" offered as an adjustment. *Mechanism:* changing membership changes u, which is the one parameter tuning cannot reach. It is a different group afterwards and the honest framing says so. *Fix:* SPEC 17.6.

FM 17.12 — THE STRUCTURE NAMED INSTEAD OF DECLARED. *Signature:* "we are a family, so `delta` is low." *Mechanism:* the labels in §17.1 are parameter signatures, not primitives. A group is whatever its five declarations say it is. *Fix:* SPEC 17.1.

Worked Example

THE THREE STRUCTURES, READ IN FULL

All three cases are CONSTRUCTED. The family and the team are explicitly labelled extensions of the carried dyad of SPINE §3.2; the commons is SPINE §3.3 exactly. Parameters are chosen, not observed. None of this is an assessment of any person, family, team or community; no clinical, diagnostic or therapeutic claim is made; the frequencies are essence frequencies.

THE FAMILY

```
 A = {4: 0.7,  6: 0.5,  9: 0.4}                                L2 norm 0.948683
 B = {4: 0.6,  6: 0.55, 9: 0.3,  10: 0.2}                      L2 norm 0.890225
D = {1: 0.50, 2: 0.45, 4: 0.35, 6: 0.30, 8: 0.40, 9: 0.20}     L2 norm 0.930054
E = {3: 0.20, 4: 0.30, 6: 0.25, 9: 0.35, 11: 0.55, 12: 0.50}   L2 norm 0.931397

 m = 4, N = 12, one declared period
 kappa = 0.130 per link, 6 links, C = 0.7800, eta = 0.34, delta = 0.05

 pairwise u    A–B 0.068735   A–D 0.710180   A–E 0.711015
               B–D 0.723966   B–E 0.740352   D–E 0.916710
 u_bar         0.645160          F(u_bar) 2.091463    mean–of–F 1.954682
 gap           0.394666 octaves
 N_eff         8.096755          f = 31.6922 Hz    folded 253.537 (j = 3)
 c/eta         2.294118  vs ln 12 = 2.484907   margin +0.190789   TUNABLE
roster ceil  m(m–1) = 12 <= 12.9980                    PASS, at the ceiling
 Brooks        r = 0.382353, max viable roster 2, actual 4         OVER
 f_min         47.5308 Hz        folded 380.246 (j = 3)
 req/present   6.619424 / 6.034688
 DEFICIT       +0.584737   (+701.7 cents)     conservative +0.979403
```

```
 numerator     −0.068902   NEGATIVE
 Q_inf         −1.378049                      conservative −2.308160
 headroom      1.1352 octaves      mu = 1.000
 VERDICT       DECAYS on both aggregations −− determined
SENSITIVITY   D = +0.5847 +/− 0.9734 at eta +/− 0.05  -> SIGN NOT RESOLVED
chord      states {1,2,3,4,6,8,9,10,11,12}, classes {1,3,5,9,11} −− 5 of 6
 levers        eta 0.34 -> 0.372945  (+9.69%)  closes it
               C   0.78 -> 0.711098  (−8.83%)  closes it
               u   0.645160 -> 0.810517        closes it
               N   12 -> 15                    closes it
after eta -> 0.40:  numerator +0.056585, D = −0.408177, Q_inf(0.05) = +1.131707
                      then delta 0.05 -> 0.03:  Q_inf = +1.886178
```

The family is the tunable case, its cheapest lever costs a tenth of its conversion efficiency, and its verdict is not resolved at the precision its parameters are known to. Both of those are ordinary and both are reportable.

THE TEAM

```
A  = {4: 0.7,  6: 0.5,  9: 0.4}                                L2 norm 0.948683
B' = {2: 0.95, 4: 0.60, 6: 0.55, 9: 0.30, 10: 0.20, 12: 0.8391}     L2 norm 1.548899
C  = {3: 0.45, 4: 0.55, 6: 0.40, 7: 0.35, 11: 0.25}             L2 norm 0.921954
S  = {3: 0.25, 4: 0.30, 5: 0.60, 7: 0.45, 9: 0.20, 10: 0.40}       L2 norm 0.956556
T  = {2: 0.30, 4: 0.35, 6: 0.25, 8: 0.55, 9: 0.35, 11: 0.45}       L2 norm 0.950000

 m = 5, N = 12, one declared period
 kappa = 0.055 per link, 10 links, C = 0.5500, eta = 0.52, delta = 0.09

 pairwise u    A−B' 0.692372  A−C 0.552647  A−S 0.897875  A−T 0.679778
               B'−C 0.851659  B'−S 0.953352 B'−T 0.748794
               C−S  0.756702  C−T 0.786182  S−T 0.962914
 u_bar         0.788228          F(u_bar) 2.269080    mean−of−F 2.258759
 gap           0.029780 octaves
 N_eff         9.670503          f = 45.2093 Hz    folded 361.674 (j = 3)
c/eta       1.057692  vs ln 12 = 2.484907   margin +1.427214   comfortable
 roster ceil   m(m−1) = 20 <= 46.9873                                PASS
 Brooks        r = 0.105769, max viable roster 6, actual 5         WITHIN
 f_min         4.0089 Hz         folded 256.572 (j = 6)
 req/present   3.051855 / 6.547182
 DEFICIT       −3.495327   (−4194.4 cents)     conservative −3.465547
 numerator     +0.629922   POSITIVE
 Q_inf         +6.999131                       conservative +6.939499
 headroom      0.6227 octaves      mu = 1.667
 VERDICT       ACCUMULATES on both aggregations −− determined
chord       states {2,...,12}, classes {1,3,5,7,9,11} −− 6 of 6, saturated
```

The team is sustainable and has 0.6227 octaves of the scale left. Its next increment is not buyable from u at twelve categories, and Chapter 4 SPEC 4.12 governs what a real addition to N would have to be. **`delta` interventions are available and would help here**, because the numerator is

positive — Chapter 15's rule permits them at this point and not before.

THE COMMONS

CONSTRUCTED, SPINE §3.3. A realising set is given so the figures are reproducible. All six members hold the same doctrine profile across all twelve states, each raising exactly one state by 0.130330:

```
  doctrine profile (states 1..12):
    {1:0.10, 2:0.20, 3:0.55, 4:0.35, 5:0.60, 6:0.25,
     7:0.15, 8:0.30, 9:0.70, 10:0.20, 11:0.10, 12:0.45}

  M raises state  1 to 0.230330      L2 norm 1.339982
  N raises state  2 to 0.330330      L2 norm 1.349673
  O raises state  6 to 0.380330      L2 norm 1.354493
  P raises state  7 to 0.280330      L2 norm 1.344836
  Q raises state 10 to 0.330330      L2 norm 1.349673
  R raises state 11 to 0.230330      L2 norm 1.339982

resulting mean pairwise u over 15 pairs = 0.018614137    (SPINE carries 0.018614)
  pairwise u range 0.018484 .. 0.018830
  cos theta range  0.990540 .. 0.990715
```

Every pair is above 0.99 alignment. That is what a membership rule looks like from inside the geometry.

```
   u_bar          0.018614          F 0.186272    mean-of-F 0.186276
gap        0.000001 octaves         -- negligible, because the pairs are alike
   N_eff          1.204754          f = 0.7017 Hz    folded 359.270 (j = 9)
   c/eta          2.426471  vs ln 12 = 2.484907   margin +0.058436   TUNABLE
reachability  u* = 0.938078 from u_bar = 0.018614  -- factor of 50.4  UNREACHABLE
roster ceil   m(m-1) = 30 <= 30.7225                      PASS, at the ceiling
   Brooks         r = 0.161765, max viable roster 4, actual 6          OVER
   f_min          61.9349 Hz        folded 247.740 (j = 2)
   req/present    7.001300 / 0.537500
   DEFICIT        +6.463800   (+7756.6 cents)
   numerator      -0.761666   NEGATIVE
   Q_inf          -8.463000
   headroom       6.6324 octaves      mu = 2.500
   chord          all twelve states held by every member; 6 classes of 6
   VERDICT        DECAYS -- and TUNABLE, UNREACHABLE
```

Note the chord line. Every member holds every state, so the commons' chord spans all twelve states and all six pitch classes — the fullest chord in this book — and its surplus is 0.186272 nats. A group can sound every available class and carry almost no information, because the chord reports what states are held and u reports whether anyone holds them differently. Chapter 10's identity is the reason: consonance and redundancy are the same phenomenon, and a chord this consonant is a single note played six times.

THE LEVER AUDIT, ALL FOUR, SIDE BY SIDE

```
    ------------------------------------------------------------------------
-------
    THE COMMONS -- deficit +6.4638 octaves -- every lever alone
    ------------------------------------------------------------------------
-------
   L1  u -> 0.95       deficit -0.0332     CLOSES, by 40 cents, and 0.95 is
                                          near-orthogonality for six people
                                             drawn from one doctrine
     u -> 1.00      deficit -0.1686     the absolute maximum, 0.1686 octaves
       hold line u*  =  0.938078             a factor of 50.4 above u_bar
    L2  eta -> 0.90      deficit +2.1075      FAILS
       eta -> 1.00      deficit +1.8430      FAILS at perfect conversion
    L3  kappa -> 0.020   deficit +2.0085      FAILS
       kappa -> 0.005   deficit +0.0990      FAILS at a 91% cut
    L4  N -> 25          deficit +5.9357      FAILS
      N -> 144        deficit +3.2562     FAILS at twelve times the ladder
    ------------------------------------------------------------------------
-------
  P5 (Ostrom, -7.7% of eta S)   Q_inf -8.462972 -> -8.517157    slightly worse
   P8 nesting, 15 links -> 7     deficit +6.4638 -> +2.7298      still failing
   P8 nesting, 15 links -> 6     deficit +6.4638 -> +2.2630      still failing
    ------------------------------------------------------------------------
-------
   RESTRUCTURED: recruit outside the doctrine (u_bar 0.45), nest to 7 links,
    adopt a conversion mechanism (eta 0.55)
      f = 17.1145 Hz   f_min = 1.9604 Hz   DEFICIT -3.126006   Q_inf +6.620724
       swing 9.5898 octaves: 51.9% cost side, 48.1% geometry
       AND IT IS A DIFFERENT GROUP
    ------------------------------------------------------------------------
-------
    ESSENCE frequencies, not physical resonances. Constructed case. Not an
   assessment of any person or community. Not counselling; no clinical claim.
    ------------------------------------------------------------------------
-------
```

What the three readings establish

One: the parameter that separated the working structure from the failing one was `u_bar`, and it is the only one of the five declarations that an arrangement cannot change.

Two: a case can pass the ceiling test and still be unfixable, which is why Definition 17.1's reachability test belongs after the ceiling test and before any prescription.

Three: every lever available to the commons was computed and every one of them fell short — including `eta = 1`, including a 91% cost cut, including a twelve-fold enlargement of the category set. That is not a rhetorical claim about difficulty; it is four tables of arithmetic, and it is the result this chapter was written to publish.

Four: the restructure that works changes who is in the group, and the formalism has nothing whatever to say about whether that is the right thing to do. It says only what the current arrangement arrives at if it holds, and Chapter 18 takes up what may and may not be done with that information.

Chapter 18 is the intervention chapter: given a deficit, what is actually done, lever by lever, with consent and reversibility built into the numbered procedure rather than appended to it — and a section on when not to tune at all.

Chapter 18 . Tuning a Relationship

Fourteen chapters have produced a number. This one is about what may be done with it.

The number is an octave deficit, and by Chapter 4's Theorem 4.5 it is also a destination: `D` and `Q_inf` are the same quantity in different units. It says that a configuration of four parameters, if it holds, arrives somewhere — and it says how far that somewhere is from sustaining. It does not say what should be done, to whom, by whom, or whether anything should be done at all, and the distance between a reading and an action is where every one of this chapter's constraints lives.

There are exactly four levers, plus one parameter that is not a lever on the deficit at all. That is not a stylistic simplification; it is Chapter 4's Theorem 4.9 and it exhausts the possibilities. So a chapter on intervention is short on options and long on discipline, and this one is organised accordingly: a numbered procedure with the constraints built into the steps, a lever-by-lever account of what each move actually requires and costs, and a section on the cases in which the correct action is to do nothing.

Three things are established before anything else, because they govern everything after.

This chapter is not counselling, therapy, mediation, or clinical assessment, and nothing in it may be presented as any of those. It is a procedure for changing four declared parameters. It has no diagnostic content, the twelve states are not diagnoses, no reading here evaluates any person's health or capacity, and a reader in difficulty with a relationship should seek help from someone qualified to give it, which this book is not and does not pretend to be.

Consent is structure, not advice. CANON3 §5 states it absolutely: no intervention may remove a person's ability to decline. In this chapter that constraint appears as numbered steps in the procedure, as a halting condition, and as a set of prohibitions on what may be computed and when — not as a paragraph of encouragement at the end.

And the procedure halts more often than it proceeds. Of the three structures Chapter 17 read, one required no intervention, one required a measurement rather than an intervention, and one returned a verdict that no intervention could satisfy. That distribution is not pessimism. It is what a diagnostic is for: most of the value of a reading is in the cases where it tells you not to act.

The Law

18.1 What an intervention is

DEFINITION 18.1. An intervention is a change to one or more of the four parameters that appear in the octave deficit, optionally accompanied by a change to the one that does not.

```
 D  =  2.885390 ∗ ( c/eta )  −  log2( f/k )        f = k (1 + (N−1) u)^2

L1  raise u       the geometry       who the parties are, and what they hold
 L2  raise eta      the conversion      what happens to what they produce
 L3  cut c          the cost            what the arrangement costs to run
L4  raise N       the categories     what the description can distinguish

 L0  lower delta    the loss            trust, ritual, record, continuity
            NOT a lever on D (Theorem 4.9); a multiplier on Q_inf (Theorem 15.1)
```

There is no fifth. Anything proposed as an intervention either moves one of these five or moves nothing, and the first job of the procedure is to say which one a proposal moves. A proposal that cannot be assigned to a lever is not an intervention; it is a wish, and Chapter 4's belief postulate applies — belief acts only through a coupling with real gain, and where no channel exists nothing happens.

18.2 The consent axiom

AXIOM 18.1 — THE CONSENT AXIOM. No procedure in this book may reduce any party's ability to decline participation in it, at any stage, for any reason or none.

This is a constraint on the procedure, not a recommendation about how to behave, and it has three corollaries that determine specific numbered steps.

COROLLARY 18.1a — INFORMED. A party who has not been told what is being measured, what will be computed, and what will be reported cannot decline meaningfully, because they cannot know what they would be declining. The procedure therefore discloses its outputs before it takes its inputs. This also discharges the no-subliminals rule (CANON3 §5): nothing masked, nothing below threshold, nothing concealed. Open-label works; disclosure costs nothing that matters.

COROLLARY 18.1b — REVOCABLE. A consent that cannot be withdrawn is a decision made once about every future stage. Consent is therefore re-asked at each stage that changes what is being done, and withdrawal at any stage halts that stage without penalty and without explanation.

COROLLARY 18.1c — UNCONDITIONED. A party shown the consequence of their own withdrawal before deciding is choosing under pressure, and pressure is a reduction in the ability to decline. Chapter 17 §17.14 computed the case that makes this concrete: a member's withdrawal improved a constructed family's reading by 2.680588 octaves, and every member of a constructed team improved the reading by leaving. Those figures may not be shown to a person weighing withdrawal. The counterfactual is computed after a decision, or not at all.

A note on why 18.1c has to be an axiom rather than a courtesy. Chapter 17 §17.11b established that a drop-one row improves for *every* member of a healthy group near the surplus ceiling, so the row carries no information about the person it names. A number that says nothing about

someone, shown to them as though it did, is the worst possible thing to put in front of a person deciding whether to stay.

18.3 Reversibility, defined and classified

DEFINITION 18.2. An intervention is REVERSIBLE if ceasing it returns the parameter to its prior value without residue in any other parameter and without requiring anything of any party. It is PARTIALLY REVERSIBLE if ceasing it returns the parameter but leaves residue elsewhere. It is IRREVERSIBLE otherwise.

LEVER	CLASS	residue on ceasing
L2 raise eta (mechanism)	REVERSIBLE	none; stop keeping the record
L3 cut c (arrangement)	REVERSIBLE	none; restore the meeting
L4 raise N (description)	REVERSIBLE	none; the ladder is a choice
L0 lower delta (record)	REVERSIBLE	none, but see the decay note
L1 raise u (what a party actually holds)	IRREVERSIBLE	a person now holds commitments they did not hold before
RESTRUCTURING (membership)	IRREVERSIBLE	a different group exists

The class list is the reason the procedure is ordered as it is. Three of the four levers, and the one non-lever, are reversible; the geometry lever is not. u rises when a party takes on states they did not previously hold, and a person cannot un-take a commitment on request. An intervention that changes what somebody is actually doing is not undone by deciding to stop measuring it.

And reversible does not mean instantaneous. D returns to its old value the moment the parameter does, because D has no memory. Q does not: it decays back at rate `delta`, with half-life `ln(0.5)/ln(1-delta)`. Computed on the constructed family, `eta` raised to 0.40 for twenty periods and then reverted:

```
t = 0    Q = +1.000000        eta = 0.40, Q_inf = +1.131707
t = 20   Q = +1.084492        eta reverted to 0.34, Q_inf = -1.378049
t = 25   Q = +0.527418
t = 30   Q = +0.096365
t = 40   Q = -0.495263
t = 60   Q = -1.061583
half-life at delta = 0.05:  13.5134 periods
```

A reversible intervention leaves a stock behind that takes about thirteen periods to half-decay, which is a grace period and not a permanence. Nothing is destroyed by stopping; nothing is kept either.

18.4 What each lever requires, in closed form

Given a reading, the exact requirement for each lever to close the deficit on its own:

L1	u_req = (e^(c/eta) - 1) / (N - 1)	reachable iff c/eta <= ln N
L2	eta_req = c / F(u)	reachable iff <= eta_max

```
 L3   c_req   =  eta * F(u)                    always >= 0, so always defined
L4   N_req   =  1 + ( e^(c/eta) - 1 ) / u      integer, and Ch 5's conditions apply

partial close of x octaves:  d(c/eta) = -x/2.885390 nats,  or  dF = +x/2.885390 nats
 one octave:  dF = 0.346574 nats  =  N_eff multiplied by sqrt(2) = 1.414214
```

Computed on the three carried cases:

case	u_req	eta_req		c_req	N_req
dyad 2a	0.627132	1.101077	ABSENT	0.168925	102
family	0.810517	0.372945		0.711098	15
commons	0.938078	UNREACHABLE 4.428926	ABSENT	0.063334	556

Three of the twelve entries are impossible before any question of expense arises. `eta_req > 1` means a conversion efficiency above unity, which is more structure created than interaction supplied; `N_req = 556` on a twelve-category ladder is not a taxonomy revision. The closed forms are worth computing first precisely because they identify levers that are absent rather than costly, and no cost model is needed to see an absent lever.

18.5 The ordering rule, as a step constraint

Chapter 15 owns the order-of-operations rule and this chapter applies it as a gate.

```
SIGN THE NUMERATOR:   n = eta F(u) - c

n < 0   NO intervention on delta is undertaken. Raise eta, cut c, or raise u first.
n > 0   delta interventions are available and are the strongest single move.
 n = 0    delta is exactly inert; Q_inf = 0 for every delta.
```

The rule is enforced at a numbered step because it cannot be seen in the deficit. `D` is identical before and after any `delta` change, to every digit, so an operator working from the deficit alone has no signal that a `delta` intervention did anything — and in a negative-numerator case it will have doubled the rate of loss while the instrument read flat.

Chapter 15 §15.4 prices the consequence: eight periods of the wrong order cost twelve periods of permanent lag and 111.133601 stock-periods, on a case whose fixed point is order-independent.

18.6 The classification, complete

Everything the procedure does is decided by four cases, assembled from Chapter 4 §4.18 and Chapter 17 Definition 17.1.

```
D <= 0                                    SUSTAINABLE.
                                  Report the margin. Do not tune. Sec 18.13.

D > 0, c/eta <= ln N, u_req reachable   TUNABLE AND REACHABLE.
                                        Run the procedure.

D > 0, c/eta <= ln N, u_req unreachable TUNABLE, UNREACHABLE.
                            Report; issue no lever prescription; the question
                                        is membership, not arrangement.
```

```
c/eta > ln N                                  STRUCTURAL.
                                  L1 is unavailable at every value of u. Only L2,
                                  L3, L4 —— and L4 only to make the case tunable.
```

A prescription issued before this classification may be unsatisfiable, and an unsatisfiable prescription is worse than none, because it consumes the effort that restructuring needed (Chapter 4 §4.11). The classification costs one logarithm and one division.

18.7 The standing prohibitions, restated where they bind

(1) NOT COUNSELLING, THERAPY, MEDIATION OR CLINICAL ASSESSMENT. Restated at the top of this chapter and again here because this is the chapter someone will try to use as a protocol. **There is no diagnostic content in any output of this procedure, and no output may be described as an assessment of a person.**

(2) ESSENCE, NOT RESONANCE. Every frequency is a position on a logarithmic scale of surplus. Nothing vibrates and nothing is played at anyone. Chapter 4's FM 4.8 forbids sounding `f_min` at any person, and the no-subliminals rule is absolute: nothing masked, nothing below threshold, nothing concealed.

(3) NO SINGLE-PARTY TERM. u is a joint geometric property; `eta`, `c` and `delta` are properties of an arrangement. No output of this procedure names a person as a cause, and none may be reported as a ranking or an evaluation.

(4) THE READING INHERITS ITS INPUTS. Where intensities were not magnitude-estimated (Chapter 6's rule; SPINE §1.1), no relationship frequency may be printed. Where `eta` or `c` is ordinal, publish the ordering and not the number.

(5) THE READING DOES NOT CONCLUDE THAT A RELATION SHOULD END. `D > 0` says a configuration loses order at a computable rate. It does not weigh what the relation is for, what else it produces, or what ending it would cost, and it acquires no such term by being applied to people (Chapter 4 §4.22).

(6) CONSTRUCTED CASES. Every case here is constructed and labelled, and its precision does not transfer to a measured one.

The Engineering

18.8 PROCEDURE T — the numbered procedure

The constraints are steps. They are not appended and they are not optional, and the procedure halts at four points by design.

STEP 1 — DISCLOSE BEFORE ELICITING. Every party is told, before any measurement: what will be measured, how, what will be computed, what will be reported, to whom, and what will be done with it. Nothing in the procedure is concealed from a participant at any stage (Corollary 18.1a).

STEP 2 — OBTAIN CONSENT, SEPARATELY AND EXPLICITLY. From every party, individually, without conditioning. The procedure does not start without every party's agreement, and a party who declines is not asked for a reason, is not recorded as having declined for cause, and the reading is simply not taken.

> **HALT 1.** Any party declines. The procedure ends. No reading is computed, no partial reading is computed, and no counterfactual of the form "what the reading would have been" is computed (Corollary 18.1c).

STEP 3 — DECLARE THE FRAME. `N` and the ordering that produced it (Chapter 5); the period; the scale status of every input (SPEC 4.4). One clock for `eta`, `c` and `delta` (SPEC 4.3).

STEP 4 — ELICIT AND ENCODE. Intensities by magnitude estimation (Chapter 6's rule), raw L2 norms published. Encode each party as a unit vector. Check pairwise support overlap and reject disjoint-support encodings (Chapter 9).

STEP 5 — MEASURE AND COMPUTE. `u` per pair (Chapter 13); the full pairwise list published, not the mean alone; `u_bar`, `F(u_bar)` and mean-of-`F` with the aggregation gap; `f`, folded, with `j`.

STEP 6 — CLASSIFY, BEFORE ANY PRESCRIPTION. Roster ceiling (Theorem 16.3), then `c/eta` against `ln N`, then reachability (Definition 17.1). Report the classification of §18.6.

> **HALT 2.** The case is STRUCTURAL, or TUNABLE-UNREACHABLE. Report the finding and the four lever computations that establish it, and issue no tuning prescription. The question is one of arrangement scope or membership, and Chapter 17 §17.7 governs the distinction.

STEP 7 — REPORT THE SENSITIVITY, AND TEST WHETHER THE VERDICT EXISTS. Compute `dD` from the uncertainty in the least certain parameter (SPEC 4.19).

> **HALT 3.** The uncertainty exceeds `|D|`. **The verdict is NOT RESOLVED and the correct action is measurement, not intervention.** Report what precision would resolve it: on the constructed family, `eta` must be known to better than ±0.030034 — 8.8% of its value — for the sign of a `+0.584737` deficit to be determined.

STEP 8 — SIGN THE NUMERATOR. `n = eta F(u) − c`. Record the sign. This is the gate for Step 12.

STEP 9 — COMPUTE THE CLOSED FORMS AND THE RESERVES. `u_req`, `eta_req`, `c_req`, `N_req` (§18.4), and the reserve of each lever (Theorem 15.4). Mark any lever ABSENT rather than expensive.

STEP 10 — CLASSIFY EACH CANDIDATE MOVE BY REVERSIBILITY (Definition 18.2), and present the reversible options first. Where a reversible move suffices, an irreversible one is not proposed.

STEP 11 — SEEK SEPARATE, EXPLICIT AGREEMENT FOR EACH MOVE, and additional explicit agreement for any move classified irreversible. Agreement to the reading is not agreement to an intervention, and agreement to one intervention is not agreement to the next.

> HALT 4. No move commands agreement from every party it requires something of. The procedure ends with the reading and no intervention. A reading that produced no agreed action is a completed procedure, not a failed one.

STEP 12 — SEQUENCE. Numerator moves first (`eta`, `c`, `u`), then `delta` moves, and only if Step 8's sign is positive by the time they are reached (Chapter 15 SPEC 15.3).

STEP 13 — ACT, AND RECORD THE BASELINE. Record every parameter's prior value, so that reversal is a defined operation rather than a reconstruction.

STEP 14 — RE-READ, AND RE-CONSENT. At a declared interval. The re-reading restarts at Step 1. Consent obtained for one reading is not consent for the next, and §18.12 gives the cadence and the design controls that make a re-read mean anything.

18.9 The levers, one at a time, with what each actually costs

L2 — RAISE `eta`. *Reversible.*

What it is. Change what happens to what the parties already produce. The decision that stays decided; the practice that has a slot; the artefact that exists next week because of what happened this week. It is a mechanism question, and often literally a notebook.

What it requires. `eta_req = c/F(u)`, or a partial close of x octaves at `d(c/eta) = −x/2.885390` nats.

Its price in octaves. `dD/d(ln eta) = −2.885390 (c/eta)` — an e-fold improvement buys exactly the number of octaves the relation requires. On the constructed family that is 6.619424 octaves per e-fold against a deficit of 0.584737, which is why a 9.7% rise suffices.

Its reserve. `2.885390 c (1/eta_0 − 1/eta_max)`, and it is small wherever `eta` is already moderate: at `eta = 0.50`, `c = 0.62`, the whole remaining reserve is 1.7889 octaves.

Its failure mode. Cutting the mechanism while cutting cost. If the recurring meeting was the conversion, removing it moves `c` and `eta` together and the ratio may not move at all (SPEC 4.11).

L3 — CUT `c`. *Reversible.*

What it is. Reduce what the arrangement costs to run before it produces anything: coordination, re-explaining, friction, correction load.

What it requires. `c_req = eta F(u)`.

Its price in octaves. `dD/d(ln c) = +2.885390 (c/eta)` — identical in magnitude to L2, opposite in sign, because the two enter only as a ratio. The formalism is indifferent between them, so whichever is cheaper to move in the situation is strictly the better choice.

Its reserve. `2.885390 (c/eta)`, i.e. the entire requirement — `c` can in principle go to zero, at which point `f_min = k` and any non-zero u clears it.

Its failure mode. R18's counter-pressure. An over-framed system banks more stored order than a calibrated one, and the bill arrives later as correction load `C_t = c_0 + kappa Z_t`. A `c` that was cut by ceasing to check things will rise again for reasons nobody connects to the choice.

L1 — RAISE `u`. *IRREVERSIBLE.*

What it is. A party comes to hold states the other parties do not hold. Not a rhetorical position — a change in what somebody is actually doing.

What it requires. `u_req = (e^(c/eta) − 1)/(N−1)`, reachable only if `c/eta <= ln N`, and reachable *in practice* only if the parties could honestly represent it (Definition 17.1).

Its price in octaves. `dD/d(ln u) = −2.885390 (1 − 1/N_eff)`, which is bounded above by 2.885390 octaves per e-fold in every system that has ever existed or will, because `1 − 1/N_eff < 1`. The cost-side levers exceed it whenever `c/eta > 1 − 1/N_eff`, which Chapter 15 Corollary 15.3a states and which holds in all three of this book's carried cases.

Its reserve. The pitch headroom `2.885390 (ln N − F(u))`, and it is the only large reserve available on a big deficit — 5.545206 octaves on the carried dyad against `eta`'s 4.174198.

Its failure modes, and there are three. R8: manufactured opposition reads as `u = 0` exactly as agreement does, so a contrarian raises cost and returns nothing. Re-encoding: a u that rose because the description changed is not a u that rose. And convergence drift: u decays as parties spend time together, at a rate estimable from two readings (Chapter 15 §15.13).

And its consent weight. This is the only lever that requires a person to become different, and it is the only irreversible one. Step 11's separate agreement exists for it.

L4 — RAISE `N`. *Reversible.*

What it is. Add genuinely distinct categories to the description — carrying variance the existing ones do not, and satisfying Chapter 5's validity conditions.

What it requires. `N_req = 1 + (e^(c/eta) − 1)/u`.

Its price in octaves. `dD/d(ln N)`-scale: `2.885390 u/(1 + (N−1)u)` per category — 0.1129 at the carried dyad's u, 0.0446 at the commons'.

Its real value. Not the deficit. `N` raises `ln N`, which raises the maximum satisfiable `c/eta`. `N` is how a STRUCTURAL case becomes a TUNABLE one, and that is the only situation in which it is the right first move.

Its failure modes. The Parallel Wall — `N` enters `F` only multiplied by u, so categories added to a near-parallel set are dimensions nobody uses. The `N = 17` cliff, where the folded lattice's minimum separation falls from 34.9756 cents to 6.0008 and the symbol rate drops by exactly 8 (Chapter 7). And the honest one above both: a category added for arithmetic convenience is not a category.

L0 — LOWER `delta`. *Reversible, and NOT a lever on the deficit.*

What it is. Trust, ritual, written record, institutional memory, continuity of place — anything that slows the leak of accumulated order.

What it does. Nothing to `D`, by Theorem 4.9. It scales `Q_inf` by `delta_old/delta_new`, in whichever direction the numerator's sign points.

Its price. At a positive numerator it is the largest single move available: on the carried dyad at 2b, `delta` 0.09 to 0.05 takes `Q_inf` from `+6.084427` to `+10.951968`, an 80.0% improvement with no new difference and no cost cut.

Its failure mode. R15, the whole of it. At a negative numerator the identical change takes `Q_inf` from `−5.011940` to `−9.021491`. Step 12 is the gate and it is not discretionary.

A note worth making. Several real interventions move `eta`, `c` and `delta` together — a written record does all three — and those are the ones to reach for first, because half of what they do is on the numerator. Reported as one thing, though, they make trust look like it worked when

what worked was the other half, which is why SPEC 15.4 requires `delta` effects to be reported separately.

18.10 Planning combinations

`D = 2.885390 (c/eta − F)` is a difference of two independent terms, so octave contributions add with no interaction term. A plan is therefore a list of moves whose contributions sum to at least the deficit.

```
total to close      D octaves
from eta:           2.885390 ∗ c ∗ (1/eta_0 − 1/eta_1)
from c:             2.885390 ∗ (c_0 − c_1) / eta
from u:             2.885390 ∗ ( F(u_1) − F(u_0) )
from N:             2.885390 ∗ ( ln(1+(N_1−1)u) − ln(1+(N_0−1)u) )
```

Two cautions, both from Chapter 4's Worked Example. The path-free split into cost side and surplus side is unique; the split *within* the cost side is not, because `c` is divided by `eta` and cutting cost is worth more when `eta` is still low. On the carried dyad the `c` contribution is `−1.442695` octaves on one attribution order and `−2.115953` on another — a 47% difference from the same intervention producing the same result. Report the path-free split, and if a per-parameter attribution is wanted, report the order used.

And single-lever plans on large deficits tend to land near the hold line, where a relation pays for itself exactly and banks nothing. Chapter 15 §15.12b computes the carried dyad's: `u` alone to 0.692373 leaves `−0.250931` octaves, the cost side alone leaves `+0.998357` and still fails. A plan with no margin is a plan that fails on the first increase in cost.

18.11 Choosing between levers when the formalism will not

The four sensitivities are in four different units and cannot be ranked against each other (Chapter 4 §4.17b). The elasticities can, and Chapter 15 Theorem 15.3 gives them. But neither answers what a unit of each parameter costs in the situation, and that cost model is not in the formalism and this book does not pretend to supply it.

What the formalism does supply, and what is genuinely decisive without prices:

Absent levers. `eta_req = 1.101077` on the carried dyad and `4.428926` on the commons. No cost model is needed to rule out a conversion efficiency above 1.

Exhausted levers. `L4` at `u = 0.018614` buys 0.0446 octaves per category, so 6.4638 octaves is 145 categories.

Reversibility. Three levers are reversible and one is not, and where a reversible move suffices, Step 10 forbids proposing the irreversible one.

The shape of the trade. `dD/d(ln eta)` carries `c/eta`; `dD/d(ln u)` is capped at 2.885390. Both levers are strongest in the systems that most need them, which is the opposite of the usual experience that improvement gets easier once things are already going well.

18.12 Verifying that it worked

An intervention that is not verified is a story. Four requirements, and the last three are Book Two's, carried forward without softening.

Re-read at a declared interval, from Step 1. The deficit is a property of current parameters, so verification means re-measuring them, not remembering that something was done.

Do not use the shuffle as the verification. R13 and R24 measured three distinct failure modes: the shuffled-input control is anti-conservative as ordinarily run (2.2x nominal alpha), it cannot see repackaging (only the structural ceiling test can), and it cannot see confounding by a shared slow driver, passing 98.35% of the time. **Design controls, not more surrogates.**

Use the structural test for repackaging. A change that exceeds the theoretical single-step maximum is not an improvement; it is a re-description. Chapter 4's Corollary 4.2 material and R4 apply.

And R23 stands. Testimonial evidence that a configuration held has a likelihood ratio of exactly `1.000000` — it does not move the posterior at all, because the selection guarantees you observe successes whether or not the method does anything. A hundred pairs reporting that their reading improved is worth precisely as much as none of them reporting it. What counts is the re-measured parameters, and the companion result says why: a prediction specifying a magnitude carries a likelihood ratio of 11.3577 against 2.0000 for one specifying only a direction — about 5.7 times more evidential power. **Predict the number before you re-read.**

18.12b The re-reading window, and how it can be empty

Step 14 says re-read at a declared interval. The interval is not arbitrary, and it turns out to be bounded on both sides by quantities the reading already contains.

The lower bound is convergence. After a parameter change, `|Q_t − Q_inf| = (1−delta)^t |Q_0 − Q_inf|`, so half of the move has happened after `t_half = ln(0.5)/ln(1−delta)` periods. Read sooner than that and most of what will happen has not happened; the stock is still dominated by where it started.

The upper bound is drift. Chapter 15 §15.13's convergence clock gives `t* = ln(u*/u_0)/ln(1−lambda)`, the period at which a sustainable geometry decays back across its own hold line. Read later than that and the reading is of a relation that has already lost the gain.

```
THE RE-READ WINDOW     [ ln(0.5)/ln(1−delta) ,  ln(u*/u_0)/ln(1−lambda) ]
```

Computed for three of this book's cases:

```
 case                u_0       u*       t_half     lambda   t*       window
family after tune  0.645160  0.548063  13.5134     0.01    16.229   [13.51, 16.23]
                                                     0.02     8.074   EMPTY
                                                     0.03     5.355   EMPTY
                                                     0.05     3.180   EMPTY
carried dyad 2b    0.692373  0.134733   7.3496     0.02    81.020   [ 7.35, 81.02]
                                                    0.05    31.911   [ 7.35, 31.91]
                                                    0.08    19.631   [ 7.35, 19.63]
the team         0.788228  0.170883   7.3496     0.02    75.673   [ 7.35, 75.67]
```

```
                                                0.05   29.805   [ 7.35, 29.81]
```

The family's window is empty at any convergence rate above about 1% per period, and that is a finding about the tune rather than about the schedule. The family's repair left it at `−0.408177` octaves — a margin of 0.408 octaves — so its hold line sits at `u* = 0.548063` against a current `u_bar` of 0.645160, and the geometry has only 15% of relative room before it crosses back. The repaired dyad has `u* = 0.134733` against 0.692373, five times the room, and its window is wide at every plausible drift rate.

The rule that follows is more useful than a schedule:

> **A margin thin enough to be unverifiable is a margin too thin. If the re-read window is empty, the correct response is not a different reading interval but a larger intervention.**

This is the quantitative form of §18.10's warning about single-lever plans landing near the hold line. A plan that closes the deficit and no more produces a relation that is sustainable, fragile, and — computably — impossible to confirm.

18.12c What a tuning report contains

The report is the durable artefact, and writing it is itself an `eta` intervention: what is not written down is not converted. Fourteen items, and a report missing any of them is not auditable.

```
    1  the frame:  N, the ordering, the period, the scale status of every input
   2  consent:   obtained, from whom, at what stages, and any declines (as halts, not findi
   3  the vectors: every party's states and intensities, plus every raw L2 norm
     4  the pairs:   the full pairwise u list, with cos theta signed
     5  aggregation: u_bar, F(u_bar), mean-of-F, and the gap in octaves
     6  the pitch:   f raw and folded, with j; f_min raw and folded, with j
   7  the tests:  roster ceiling, ceiling test with its margin in nats, reachability
   8  the reading: octaves required, octaves present, D with sign, in octaves and cents
        9   the destiny:  Q_inf,  and  the  identity  cross-check  D  =  -
2.885390 (delta/eta) Q_inf
    10  the sign:    the numerator eta F - c
   11  sensitivity: dD from the least certain parameter, and whether the verdict is resolved
   12  the levers:  u_req, eta_req, c_req, N_req, with absent levers marked ABSENT
   13  the plan:   each move, its lever, its reversibility class, its octave contribution,
               the path-free split, and the attribution order if one is given
   14  the schedule: the re-read window with its two bounds, and the predicted D with a magni
```

Item 2 has a specific form and it matters. A decline is recorded as a halt in the procedure and never as a datum about the person who declined — no reason, no category, no counterfactual. A report that records why someone declined has converted a consent mechanism into a surveillance one, and Axiom 18.1 is violated by the record even if it was honoured in the room.

And item 14's prediction is recorded before the re-read, not after. R23's companion result gives the reason in numbers: a prediction specifying a magnitude carries a likelihood ratio of 11.3577 against 2.0000 for a direction-only prediction, so a report that predicts *better* has recorded almost nothing, and one that predicts `−0.408177 ± 0.35` has recorded something a re-reading can refute.

18.13 WHEN NOT TO TUNE

Eight conditions. Each halts the procedure, and each is a computed condition rather than a matter of judgment.

(1) The case is SUSTAINABLE. `D <= 0`. Report the margin and stop. A relation that is accumulating does not need tuning, and a margin optimised for its own sake spends reserve that has no other source. Chapter 17's team sits at −3.495327 octaves with 0.6227 octaves of pitch headroom left; there is nothing to buy there that is worth the disturbance of buying it.

(2) The case is STRUCTURAL. `c/eta > ln N`. No assignment of u closes it, so L1 is unavailable at every value and a lever prescription is unsatisfiable. Only L2, L3, and L4 — the last purely to make the case tunable.

(3) The case is TUNABLE but UNREACHABLE. Chapter 17's commons: formally tunable with a ceiling margin of 0.058436 nats, and requiring `u* = 0.938078` from `u_bar = 0.018614`, a factor of 50.4. Every lever was computed: `eta = 1.00` leaves +1.8430, a 91% cost cut leaves +0.0990, twelve times the category count leaves +3.2562. This is a membership question and issuing a tuning plan for it wastes the effort restructuring needs.

(4) The verdict is not resolved. The uncertainty in the deficit exceeds its magnitude. On the constructed family, `D = +0.584737` with `eta` known to ±0.05 carries ±0.9734 octaves, so the sign is unknown. **Measure, do not act**, and report the precision that would resolve it.

(5) The numerator is negative and the only available move is on `delta`. Doing it makes the outcome measurably worse — −5.011940 to −9.021491 on the carried dyad — while the deficit reads unchanged. Where no numerator move is available, the correct action is to say so.

(6) Consent is absent, conditioned, or uninformed. Axiom 18.1 and its three corollaries. This is not a cost-benefit condition and is not weighed against anything.

(7) The reading was requested by someone other than the parties. A reading commissioned about a relationship by a third party cannot satisfy Step 2 in substance even if it satisfies it in form, because the parties' ability to decline is compromised by the requester's standing. No reading in this book is an instrument for one party to use on another.

(8) The proposed intervention is irreversible and the deficit is small. The stakes are asymmetric: an irreversible move that overshoots cannot be walked back, while a small deficit closes on reversible levers by construction — the family's +0.584737 closes on a 9.7% rise in `eta`. Reserve the irreversible lever for deficits that the reversible ones cannot reach.

And a ninth that is not a condition but a boundary. Nothing in the procedure decides whether a relationship should continue, and nothing in it evaluates a person. `Q_inf` is where a configuration arrives if the configuration holds; it says nothing about whether it will hold, and nothing whatever about what the relation is for.

The Specification

SPEC 18.1 — DISCLOSE FIRST. Outputs, recipients and uses are stated before inputs are taken (Corollary 18.1a). Nothing masked, nothing below threshold, nothing concealed.

SPEC 18.2 — CONSENT PER PARTY, PER STAGE, REVOCABLE, UNCONDITIONED. A decline halts without reason given and without record of cause.

SPEC 18.3 — NO COUNTERFACTUAL BEFORE A DECISION. The value of a party's own withdrawal is computed after the decision, or not at all (Corollary 18.1c).

SPEC 18.4 — CLASSIFY BEFORE PRESCRIBING. Roster ceiling, ceiling test, reachability, in that order (§18.6).

SPEC 18.5 — REPORT SENSITIVITY AND HALT ON AN UNRESOLVED VERDICT. State the precision that would resolve it.

SPEC 18.6 — SIGN THE NUMERATOR AND GATE THE `delta` MOVES ON IT (Chapter 15 SPEC 15.3).

SPEC 18.7 — COMPUTE THE CLOSED FORMS AND MARK ABSENT LEVERS ABSENT, not expensive.

SPEC 18.8 — CLASSIFY EVERY MOVE BY REVERSIBILITY and propose reversible moves first. Where a reversible move suffices, no irreversible move is proposed.

SPEC 18.9 — SEPARATE AGREEMENT PER MOVE, and additional explicit agreement for irreversible moves.

SPEC 18.10 — RECORD THE BASELINE so that reversal is defined.

SPEC 18.11 — REPORT THE PATH-FREE SPLIT, and name the order used for any per-parameter attribution.

SPEC 18.12 — VERIFY BY RE-MEASUREMENT, WITH DESIGN CONTROLS. Blinding, channel separation, prospective sampling, a yoked control. The shuffle is a weak, mis-calibrated audit with three measured failure modes and is not the verification (R13, R4, R24).

SPEC 18.13 — PREDICT THE MAGNITUDE BEFORE RE-READING. A direction-only prediction carries a likelihood ratio of 2.0000; a magnitude carries 11.3577 (R23).

SPEC 18.14 — TESTIMONY IS NOT EVIDENCE. R23: the survivorship likelihood ratio is exactly `1.000000`.

SPEC 18.15 — NO CLINICAL, THERAPEUTIC, DIAGNOSTIC, COUNSELLING OR MEDIATION FRAMING, in any output, at any stage.

SPEC 18.16 — LABEL EVERY CASE CONSTRUCTED.

The Failure Modes

FM 18.1 — THE PROCEDURE RUN ON SOMEBODY. *Signature:* a reading of a relationship commissioned by one party, or by a third party, about others. *Mechanism:* Step 2 cannot be satisfied in substance where the requester holds standing over the parties. The ability to decline is what is removed, and that is the one thing CANON3 §5 forbids absolutely. *Fix:* §18.13 (7). The procedure does not run.

FM 18.2 — THE COUNTERFACTUAL AS PRESSURE. *Signature:* a party told what their withdrawal would do to the reading. *Mechanism:* Corollary 18.1c. And Chapter 17 §17.11b shows the figure is uninformative anyway: every member of a healthy team improved the reading by leaving, so the row says nothing about the person it names. *Fix:* SPEC 18.3.

FM 18.3 — TRUST FIRST. *Signature:* a failing relation into which continuity, ritual or patience is invested, followed by acceleration, with the deficit unchanged throughout. *Mechanism:* R15 and Theorem 15.1. `−5.011940` to `−9.021491` on the carried dyad; deficit `+4.338420` both times. *Fix:* SPEC 18.6, enforced at Step 12.

FM 18.4 — TUNING AN UNREACHABLE CASE. *Signature:* a lever programme for a group whose required geometry is 50 times its current one. *Mechanism:* Definition 17.1. **An unsatisfiable prescription consumes the effort restructuring needed.** *Fix:* HALT 2.

FM 18.5 — ACTING ON AN UNRESOLVED VERDICT. *Signature:* an intervention justified by a deficit smaller than its own uncertainty. *Mechanism:* SPEC 4.19. `+0.5847 ± 0.9734` is a statement that the sign is unknown. *Fix:* HALT 3, and measure `eta` to better than ±0.030034.

FM 18.6 — THE IRREVERSIBLE MOVE MADE FIRST. *Signature:* a `u` intervention proposed before the reversible levers were computed. *Mechanism:* Step 10. `u` rises when a person takes on commitments they did not hold, and that cannot be undone by ceasing to measure. *Fix:* SPEC 18.8.

FM 18.7 — CONSENT TAKEN ONCE. *Signature:* agreement to a reading treated as agreement to whatever the reading leads to. *Mechanism:* Corollary 18.1b. Agreement to a measurement is not agreement to a change in somebody's life. *Fix:* SPEC 18.9 and Step 14's re-consent.

FM 18.8 — MANUFACTURED DIFFERENCE. *Signature:* a party adopting contrary positions to raise `u`. *Mechanism:* R8. `u` depends only on `|rho|`; opposition reads as `u = 0` and adds cost. *Fix:* report `cos theta` signed, and Chapter 14 for the result.

FM 18.9 — THE RE-ENCODING THAT LOOKS LIKE PROGRESS. *Signature:* `u` rises after the description changed rather than the parties. *Mechanism:* Chapter 9's degeneracies. Disjoint-support encodings read `u = 1` for free. *Fix:* Step 4's overlap check, and re-encode identically before and after.

FM 18.10 — THE VERIFICATION THAT VERIFIES NOTHING. *Signature:* a four-baseline surrogate suite reported as confirmation. *Mechanism:* three measured holes — anti-conservative at 2.2x nominal alpha, blind to repackaging, and passing 98.35% of the time against a shared slow driver. *Fix:* SPEC 18.12. Design controls.

FM 18.11 — TESTIMONY OFFERED AS RESULT. *Signature:* accounts from parties for whom the procedure worked. *Mechanism:* R23. The survivorship likelihood ratio is exactly `1.000000`. A thousand such accounts carry the evidential weight of zero. *Fix:* SPEC 18.14, and re-measured parameters.

FM 18.12 — CUTTING THE CONVERSION MECHANISM. *Signature:* `c` falls, `eta` falls with it, `D` does not move. *Mechanism:* `D` depends on the ratio. *Fix:* SPEC 4.11's test before any `c` cut.

FM 18.13 — THE PROCEDURE USED AS THERAPY. *Signature:* Procedure T presented as an intervention on a person, a treatment, a counselling protocol, or an assessment. *Mechanism:* register slip of the most serious kind. The procedure changes four declared parameters of an

arrangement and has no diagnostic, clinical or therapeutic content whatsoever. *Fix:* SPEC 18.15, and §18.7 (1).

Worked Example

PART A — TUNING THE CONSTRUCTED FAMILY, ALL FOURTEEN STEPS

CONSTRUCTED. This is Chapter 17's family — itself a labelled extension of the carried dyad of SPINE §3.2. Parameters are chosen, not observed. This is not an assessment of any person or family, not counselling, and no clinical, diagnostic or therapeutic claim is made or implied. The frequencies are essence frequencies.

Steps 1–2. All four parties are told what will be measured, computed and reported, and to whom. All four agree, separately. No party is shown any counterfactual. **HALT 1 not triggered.**

Step 3. `N = 12` on Chapter 5's ordering; one declared period for `eta`, `c` and `delta`; intensities magnitude-estimated per Chapter 6; `eta` and `c` declared as estimates with stated uncertainty.

Step 4. Vectors and raw L2 norms:

```
A = {4: 0.7,  6: 0.5,  9: 0.4}                                  0.948683
B = {4: 0.6,  6: 0.55, 9: 0.3,  10: 0.2}                        0.890225
D = {1: 0.50, 2: 0.45, 4: 0.35, 6: 0.30, 8: 0.40, 9: 0.20}      0.930054
E = {3: 0.20, 4: 0.30, 6: 0.25, 9: 0.35, 11: 0.55, 12: 0.50}    0.931397

support overlap: every pair shares at least states 4 and 6      GATE PASSED
```

Step 5.

```
A–B 0.068735   A–D 0.710180   A–E 0.711015
B–D 0.723966   B–E 0.740352   D–E 0.916710

u_bar = 0.645160     F(u_bar) = 2.091463     mean-of-F = 1.954682
aggregation gap = 0.394666 octaves -- reported, both readings carried
N_eff = 8.096755     f = 31.6922 Hz      folded 253.537 Hz (j = 3)
```

Step 6 — classify.

```
roster ceiling:  m(m–1) = 12  <=  2*0.34*ln12/0.13 = 12.9980      PASS, at the ceiling
ceiling test:   c/eta = 2.294118  <=  ln 12 = 2.484907  margin +0.190789  TUNABLE
reachability:   u_req = 0.810517 from u_bar = 0.645160, a factor of 1.2563  REACHABLE

f_min = 47.5308 Hz    folded 380.246 (j = 3)
octaves required 6.619424   present 6.034688
OCTAVE DEFICIT  +0.584737   (conservative +0.979403)      both DECAY --
determined
Q_inf           -1.378049   (conservative -2.308160)
```

HALT 2 not triggered. The case is TUNABLE AND REACHABLE.

Step 7 — sensitivity.

```
dD/d eta = −19.4689 octaves per unit;  dD/dC = +8.4864;  dD/du = −3.9200

eta +/− 0.02  −>  D = +0.5847 +/− 0.3894    resolved
eta +/− 0.05  −>  D = +0.5847 +/− 0.9734    NOT RESOLVED
C   +/− 0.05  −>  D = +0.5847 +/− 0.4243    resolved
u   +/− 0.10  −>  D = +0.5847 +/− 0.3920    resolved
```

> **HALT 3 IS TRIGGERED IF `eta` IS KNOWN ONLY TO ±0.05.** The verdict's sign is not determined and the correct action is measurement. Required precision: `eta` to better than ±0.030034, which is 8.8% of its value.

The procedure therefore pauses here in the first instance, and the family counts what survived the period rather than estimating how well things are going (§15.9b). Suppose that count returns `eta = 0.34 ± 0.02`. The verdict is now resolved and the procedure resumes.

Step 8 — sign the numerator.

```
n = 0.34 * 2.091463 − 0.78 = −0.068902      NEGATIVE
```

No `delta` move is permitted until this changes. This matters here specifically: a family's characteristic parameter is a long memory, `delta = 0.05`, and lengthening it is the intuitive repair. Computed: `delta` 0.05 -> 0.03 takes `Q_inf` from −1.378049 to −2.296749, with `D` unmoved at +0.584737.

Step 9 — closed forms and reserves.

```
L1  u_req  = 0.810517     from 0.645160   +0.165357     REACHABLE, irreversible
L2  eta_req = 0.372945     from 0.34      +9.69%       REACHABLE, reversible
L3  c_req  = 0.711098     from 0.78      −8.83%       REACHABLE, reversible
                (kappa 0.13 −> 0.118516)
L4  N_req  = 15          from 12       +3 categories   reversible, and below
                                                          the N = 17 cliff

elasticities: |dD/dln eta| = 6.619424   |dD/dln u| = 2.529026   ratio 2.6174
criterion: c/eta = 2.294118 > 1 − 1/N_eff = 0.876494   cost side proportionally stronger
```

Step 10 — reversibility. Three reversible options each close the deficit alone. **L1 is therefore not proposed** (SPEC 18.8).

Step 11 — agreement. L2 is put to all four parties: the household adopts a standing record of what was decided, so that decisions stay decided. All four agree. **HALT 4 not triggered.**

Step 12 — sequence. L2 is a numerator move and goes first. No `delta` move is scheduled.

Step 13 — act, with baseline recorded.

```
  BASELINE: u_bar 0.645160  eta 0.34  C 0.7800  delta 0.05  D +0.584737  Q_inf −
1.378049

    eta 0.34 −> 0.40

    numerator  = 0.40 * 2.091463 − 0.78 = +0.056585       POSITIVE
    c/eta      = 0.78/0.40 = 1.950000
    f_min      = k e^(2*1.95) = 23.8824 Hz      folded 382.119 Hz (j = 4)
```

```
octaves required 5.626511   present 6.034688
OCTAVE DEFICIT  −0.408177                                  was +0.584737
Q_inf           +1.131707                                  was −1.378049
VERDICT: ACCUMULATES
```

The swing is 0.992914 octaves from a 17.6% rise in conversion efficiency, computed exactly as `2.885390 * 0.78 * (1/0.34 - 1/0.40)`. The local elasticity of 6.619424 octaves per e-fold would have predicted 1.075782 octaves for the same proportional change; the 8.3% discrepancy is the curvature of `1/eta` over a finite step, and it is the reason SPEC 18.11 asks for the exact contribution rather than the elasticity estimate whenever a move is actually made.

Now, and only now, `delta` is available. Had the family taken it first, it would have moved `Q_inf` to −2.296749; taken here it moves `Q_inf` to +1.886178. Both orders arrive at the same fixed point and one of them spends a stretch below where it started:

```
 t      delta first      numerator first
 0       +1.000000          +1.000000
 5       +0.534282          +1.029795
10       +0.134353          +1.052849
15       +0.381826          +1.170570
20       +0.594340          +1.271661
30       +0.933545          +1.433018
40       +1.183684          +1.552007
 inf     +1.886178          +1.886178
```

Step 14 — re-read. At a declared interval, restarting at Step 1 with fresh consent. The prediction is recorded first, with a magnitude: `D` is expected at `−0.408177 ± 0.35`. A prediction with a number attached is about 5.7 times more evidentially powerful than one specifying only a direction (R23).

PART B — THE COMMONS: THE PROCEDURE HALTS

CONSTRUCTED, SPINE §3.3. Same disclosures.

Steps 1–5 proceed exactly as above and produce:

```
u_bar = 0.018614     F = 0.186272     N_eff = 1.204754
f = 0.7017 Hz        folded 359.270 Hz (j = 9)
aggregation gap = 0.000001 octaves -- negligible, the pairs are alike
C = 0.8250 at kappa 0.055 x 15 links,  eta = 0.34,  delta = 0.09
```

Step 6 — classify.

```
roster ceiling:  m(m−1) = 30 <= 30.7225       PASS, at the ceiling; a 7th member
                                          would take it to 42 and make the case
                                                    STRUCTURAL outright
ceiling test:   c/eta = 2.426471 <= 2.484907   margin +0.058436 nats   TUNABLE
reachability:   u_req = 0.938078 from 0.018614  -- a factor of 50.4    UNREACHABLE

 f_min = 61.9349 Hz    OCTAVE DEFICIT +6.4638    Q_inf −8.4630
```

HALT 2 IS TRIGGERED. The case is TUNABLE but UNREACHABLE. **The procedure reports the finding and the computations that establish it, and issues no tuning**

prescription.

The four lever computations that are reported instead of a plan:

```
   L1  u -> 0.95    deficit -0.0332   closes, by 40 cents, and requires near-
orthogonality
                                     from six people drawn from one doctrine
       u -> 1.00     deficit -0.1686    the absolute maximum available
    L2  eta -> 0.90   deficit +2.1075    fails
       eta -> 1.00   deficit +1.8430    fails at perfect conversion
    L3  kappa -> 0.020 deficit +2.0085   fails
       kappa -> 0.005 deficit +0.0990   fails at a 91% cut
    L4  N -> 25       deficit +5.9357    fails
       N -> 144      deficit +3.2562    fails at twelve times the ladder

    P8 nesting 15 links -> 7:  deficit +2.7298                fails
    shrinking to m = 3:        deficit +0.864532              fails
    shrinking to m = 2:        deficit -0.071526             holds, barely
```

No `delta` move is permitted either: the numerator is −0.761666 and Step 8's gate would refuse it even if the procedure had reached Step 8.

What is reported. That the deficit is +6.4638 octaves and `Q_inf` is −8.4630; that every lever was computed and each falls short; that the case is a membership question rather than an arrangement question (Chapter 17 §17.7); and that a restructure — recruiting outside the doctrine, nesting to seven links, adopting a conversion mechanism — reaches −3.126006 octaves and `Q_inf` = +6.620724, **and produces a different group.**

What is not reported. No recommendation that anyone leave, join, or be replaced. No ranking of members. No counterfactual for any individual's withdrawal. The formalism has no term for what the doctrine was for, no single-party term at all, and no standing to weigh what this group is otherwise doing.

PART C — THE CARRIED DYAD, FOR COMPARISON

CONSTRUCTED, SPINE §3.2. The same procedure on the case Part III opened with, given briefly because Chapters 4 and 15 work it in full.

```
    Steps 1–5:  u = 0.068734665264634, F = 0.563085, f = 1.4907987150 Hz
                eta = 0.30, c = 0.62, delta = 0.09
    Step 6:     c/eta = 2.066667 <= 2.484907   TUNABLE, margin +0.418240 nats
                u_req = 0.627132 from 0.068735 -- a factor of 9.1240
           REACHABILITY: declared borderline, and declared rather than assumed
   Step 7:    D = +4.338420; no plausible input uncertainty flips a deficit that size
   Step 8:    numerator = -0.451075  NEGATIVE  -- delta moves refused at Step 12
   Step 9:    L1 u -> 0.627132    reachable, IRREVERSIBLE, reserve 5.545206 octaves
           L2 eta -> 1.101077  ABSENT (eta > 1); reserve to eta = 1 is 4.174198 < 4.338420
                L3 c -> 0.168925     a 72.8% cut; reserve 5.963140 octaves
              L4 N -> 102         not reachable; 0.1129 octaves per category
   Step 10:   one reversible lever (L3) and one irreversible (L1); L2 does not exist
   Step 11–13: the reversible lever alone leaves +0.998357 octaves and still DECAYS,
```

```
          so L1 is proposed, separately agreed, and taken together with the cost move
               result: u -> 0.692373, eta -> 0.44, c -> 0.40
               D = -3.590989, numerator +0.547598, Q_inf +6.084427
  Step 12:   NOW the delta move is permitted: 0.09 -> 0.05 gives Q_inf +10.951968
   Step 14:    re-read window at lambda = 0.05 is [7.35, 31.91] periods --
comfortably open
```

This is the case in which an irreversible lever was correct, and the procedure records why: the reversible levers were computed first, one of them did not exist and the other did not suffice, and the party who took on new states agreed separately to doing so. **The order of those four facts is the whole of Step 10 and Step 11.**

Compare the re-read windows across the three tunes:

```
   case              margin (octaves)    u* / u_0    window at lambda = 0.05
    family after tune       0.408177          0.849500     EMPTY
    carried dyad 2b         3.590989          0.194595     [7.35, 31.91]
```

The dyad's repair is verifiable and the family's is not, and the difference is the margin.

The two outcomes, side by side

```
    ------------------------------------------------------------------------
----
                                  THE FAMILY            THE COMMONS
    ------------------------------------------------------------------------
----
    deficit as found               +0.584737             +6.463800
    classification              tunable, reachable   tunable, UNREACHABLE
    procedure                   ran to Step 14        halted at Step 6
    halt encountered            HALT 3 (then cleared)      HALT 2
    numerator sign                  negative              negative
    delta move permitted        no, then yes               no
    lever chosen                eta 0.34 -> 0.40            none
    reversible?                       yes                   -
    deficit after               -0.408177                   -
    Q_inf after                 +1.131707 (+1.886178
                                with delta 0.03)            -
    what was reported           a completed tune       a finding and four
                                                       failed lever tables
    ------------------------------------------------------------------------
----
    ESSENCE frequencies, not physical resonances. Constructed cases. Not an
    assessment of any person, family or community. Not counselling, therapy,
    mediation or clinical assessment. No diagnostic claim of any kind.
    ------------------------------------------------------------------------
----
```

Both outcomes are correct procedure. One produced an intervention and one produced a refusal to prescribe, and the refusal took the same fourteen steps up to the point where it stopped.

A diagnostic that only ever recommends action is not a diagnostic; it is a sales process, and the commons column is what this book means by an instrument that can say no.

Part III closes here. Chapter 19 opens Part IV and turns the same three transfer rules on the classical alchemical operations, auditing each one and publishing the ones that fail. Chapter 29 returns to the four levers at the scale of a destiny, and cites Chapter 15's ordering rule and this chapter's consent axiom without re-deriving either.

Part IV — Essence Alchemy

Chapter 19 . The Operations as Frequency Operations

Part III ended with an instrument that produces a number and a procedure that mostly declines to act on it. Part IV asks a different question. It asks whether a body of technique that predates the instrument by some centuries was describing the same objects.

The author's position on this is stated as a thesis, badged, and it has been stated the same way since Book Two Chapter 57: **the old operations are these relationships in another notation.** That is a claim about description, not about efficacy. It says that when a practitioner in the fifteenth century wrote down a sequence of named procedures and insisted on their order, the sequence had structure, and the structure is legible today in a formalism the practitioner did not have. It does not say the procedures worked. It does not say symbols act on matter. It says: here is a list of operations, here is a list of parameters, and the question of which list maps onto which is a question with a determinate answer that can be worked out and published.

This chapter works it out. The method is an audit, and an audit has one property that distinguishes it from an appreciation: **it can return a failure, and if it never does, it was not an audit.** Five of the entries below fail. They are printed at the same length as the ones that pass, with the arithmetic that condemns them, because a chapter that recovered twelve successes from twelve attempts would be evidence about the auditor rather than about the material.

Three things are fixed before anything begins.

The declared parameter set is closed. An operation maps if and only if it names an input and an output inside `{n, u, N, eta, c, delta, G}` and the map is computable. There is no thirteenth parameter, no residual, and no category of "acts on the whole." Chapter 4 Theorem 4.9 already proved that even a real, valuable, well-attested intervention can act on a parameter that the book's central diagnostic cannot see; that result is what makes this audit worth running rather than assuming.

The Tri-State roles are declared once and hold throughout (CANON_BIO §1). Where a concordance is drawn in this chapter, the `(+1)` column carries evidence — measured, replicated, falsifiable — and the `(−1)` column carries precedent, which is the fact that a structure was arrived at independently and preserved. Precedent is worth having and it is not evidence. **No entry below treats the antiquity of an operation as a reason to believe it does anything.**

Nothing in this chapter claims that a symbol, a name, a sequence, or a substance has a physical effect. The essence frequencies are essence frequencies. No person, tissue, or molecule is claimed to resonate at any value here, the twelve states are not diagnoses, and no

reading is an assessment of anyone's health.

The Law

19.1 What an audit is

An audit takes a claim expressed in one vocabulary and asks whether it can be restated in another without loss and without addition. Three outcomes are possible and they are not a scale.

The claim **maps**: there is an operation on the declared parameters that does what the old term says, the map is computable, and running it produces a number. The claim **fails**: the old term names something the formalism can express, and the formalism says it does not work, or does not work as stated. The claim is **empty**: the old term names nothing in the parameter set at all — not something false, something absent.

The three outcomes carry different obligations. A map must be published with its arithmetic. A failure must be published with the specific result that kills it. An emptiness must be published with a statement of what would have to exist for it to be otherwise, because "I found nothing" and "there is nothing" are different claims and only the first is available from an audit.

An audit does not establish that a mapped operation was ever performed, was ever effective, or was understood by its practitioners in the terms recovered here. It establishes that a modern operation exists which the old term describes. The direction of that sentence matters. It is the weakest claim consistent with the evidence and it is the only one this chapter makes.

19.2 The declared parameter set

Everything this book can compute is a function of seven quantities. Six are inherited; the seventh is Chapter 20's.

```
n      the differentiation index of a state     Ch 5, a COUNT, ratio-scaled
N       the number of distinguishable categories   Ch 5; ceiling F <= ln N
u      the interaction parameter              Ch 12-13; u = 1 - cos^2 theta
 eta      conversion of interaction into structure   Paper E
 c        the per-period cost                        Paper E
 delta    the loss rate                              Paper E
G      the count-weighted geometric mean       Ch 20, compound composition
```

Everything else in the book is derived from these. `f = k n^2` from `n`. `F = ln(1 + (N-1)u)` and `f_rel = k(1+(N-1)u)^2` from `u` and `N`. `f_min = k e^(2c/eta)` from `c` and `eta`. The octave deficit `D = 2.885390 (c/eta) - log2(f/k)` from four of them, and — Chapter 4 Theorem 4.9 — **conspicuously not from `delta`.** The fixed point `Q_inf = (eta F - c)/delta` from all of them. The compound frequency `k^2 G^4` from `G` alone.

Two derived quantities that are not parameters and are frequently mistaken for them:

The transposition exponent `j`. Octave folding multiplies a frequency by `2^j` (Chapter 7 §7.2). `j` is a choice of register for publication. It changes no relationship whatever: whatever two states

shared before folding they share after, and whatever separated them separates them by the same interval in cents. An operation that acts only on `j` acts on nothing in the system.

The pitch class. The odd part of the index (Chapter 7 Theorem 7.1). It is a function of `n` and is not independently settable. You cannot move a class without moving an index.

19.3 The three transfer rules, restated as targets

CANON3 §2 fixes three rules. This chapter uses them as the three places an operation can land.

```
RULE 1   an ESSENCE has a frequency            f = k n^2
         target: the index n

RULE 2   a RELATIONSHIP has a frequency        f_rel = k (1 + (N−1) u)^2
         target: the interaction parameter u, and N

RULE 3   a COMPOUND uses the alchemical rule   f_compound = k^2 G^4
         target: the composition, through G
```

Plus the storage law's three, which are not transfer rules but are parameters an operation can act on: `eta`, `c`, `delta`.

A candidate operation is therefore audited against six targets and one non-target. The non-target is the register `j`, and it exists in the list precisely because one classical operation lands there and nowhere else.

19.4 The audit rule, and the verdict vocabulary

AUDIT RULE 19.1. A classical operation MAPS if and only if all four conditions hold.

```
(a)  it names a starting configuration expressible in the declared parameters
(b)  it names an ending configuration expressible in the declared parameters
 (c)  the transformation from (a) to (b) is a function, not a wish
 (d)  running it on a stated input produces a number
```

Condition (c) is the one that does the work. "Raise the material to a nobler state" satisfies (a) and (b) in appearance only: *nobler* is not a coordinate. Chapter 5 §5.18 already named this fault under its own heading — smuggling a rating in as a count — and it is the single most common way an old operation fails this audit.

Verdicts, five of them, and they are borrowed from Book Two Chapter 63's register so that the register can absorb this chapter's rows without translation:

```
MAPS        all four conditions. The operation IS an operation on a declared
                parameter. Publish the map and a worked number.

 MAPS, SPLIT   the term is used for two distinct operations in the source
               tradition and they land on different parameters. Both are
               published, and the ambiguity is reported as a finding.

REJECTED     the operation is expressible and the formalism says it does not
              do what is claimed. Published at full length with the result.
```

```
  EMPTY       the operation lands on no declared parameter and no derived
               quantity. Not false — absent.

  RENDERING   the operation is real, computable, and acts only on the register
               j. It changes how a value is published and nothing about the
               system. Admissible as a rendering step; inadmissible as an
               operation on a state.
```

19.5 The source set

The audit needs a fixed list chosen before the results are known, or it is a selection exercise. The list used is **the Twelve Gates of George Ripley's *Compound of Alchymy* (1471)**, a specific, dated, transmitted sequence: calcination, solution, separation, conjunction, putrefaction, congelation, cibation, sublimation, fermentation, exaltation, multiplication, projection.

Two reasons for this list over the more familiar seven-operation schemes. It is longer, so it contains more chances to fail, which is the property an audit wants. And it is *ordered* — Ripley presents the gates as a sequence in which each is prerequisite to the next — which lets the Worked Example ask a question no unordered list can pose: does the sequence, run as a schedule, do anything the individual steps do not?

A thirteenth entry is appended: not an operation but a candidate index, the traditional ordering of the metals. It is included because Chapter 5 rejected four candidate orderings in print and this is the one an alchemical chapter is obliged to consider.

The `(−1)` column for the whole set is Ripley, with the transmission path stated: a Middle English verse text of 1471, circulated in manuscript, printed in Elias Ashmole's *Theatrum Chemicum Britannicum* (1652), and read continuously since. What it records is a procedure and an order. **What it explains about that procedure is not admissible and is not used below.**

19.6 Honesty clauses, restated in place

(1) ESSENCE, NOT RESONANCE. Every frequency in this chapter is a position on a logarithmic scale of surplus. Nothing is claimed to vibrate. No physical, chemical, or biological effect of any symbol, name, sequence, or operation is claimed anywhere.

(2) NO CLINICAL CLAIM. The twelve states are not diagnoses. Nothing here evaluates, describes, or prescribes for any person's health, and no operation below is a treatment.

(3) PRECEDENT IS NOT EVIDENCE. Stated in every entry that draws a concordance.

(4) THE READING INHERITS ITS INPUTS. Every number below is exactly as defensible as the `u`, `n`, `eta`, `c` and `N` that produced it. A contested decomposition yields a contested frequency.

(5) CONSTRUCTED CASES. Every case in this chapter is constructed and labelled as such.

(6) REPORT `D` WITH `delta`. The octave deficit contains no `delta` (Chapter 4 Theorem 4.9), so a reading that gives only `D` cannot see decay. Both are reported in every entry that produces either.

The Engineering

Thirteen entries. Each states the operation as the tradition gives it, the candidate target, the audit against Rule 19.1, and — where it maps — a computed number on the carried case.

The carried dyad throughout is SPINE §3.2 phase 2a, **constructed and labelled as such**:

```
A = {4:0.7, 6:0.5, 9:0.4}      B = {4:0.6, 6:0.55, 9:0.3, 10:0.2}
u = 0.068734665264634          F = 0.563084802746     f = 1.4907987150 Hz
eta = 0.30   c = 0.62   delta = 0.09
D = +4.338420 octaves          Q_inf = −5.011940      DECAYS
```

GATE 1 — CALCINATION. Verdict: MAPS, on `n`, downward.

The tradition. Reduction of the material by fire to a calx: the volatile is driven off and what remains is what could not be driven off.

The candidate target. The index `n`. Calcination removes components from a specification.

The audit. (a) a state at index `m`; (b) a state at index `n < m`; (c) the transformation is subtraction of named components, which is a function on the decomposition; (d) it produces a number. **All four conditions hold. Calcination is de-indexing.**

Worked, on the carried subject's threat framing (SPINE §3.1, constructed). State 10 is narrative feeling: a feeling placed in a story, which is a feeling with a cause, a duration, a self as object, another mind, a standard, and a narrative position. Strip the last six components and what remains is state 4, situated feeling.

```
10 -> 4     f  48.3425732988 -> 7.7348117278 Hz
        interval  2400 * log2(4/10) = −3172.6274 cents = −2.643856 octaves
            ratio f(4)/f(10) = 16/100 = 0.160000
            class 5 -> class 1                        CLASS CHANGED
```

The interval is exact, it is a ratio of squares, and k cancels from it (Chapter 4 §4.5 (iii)). Calcination has a price list, and the price list is Chapter 7's transition matrix read backwards.

One honest note that belongs to this gate and not to the tradition. The formalism permits de-indexing as freely as indexing — the matrix is antisymmetric — and reality does not. A person who has modelled another mind cannot elect to stop having modelled it. What actually removes components is `delta`, over time, without being asked. **The symmetry of the matrix is a property of the notation, not a menu.** Chapter 21 §21.9 carries this forward.

GATE 2 — SOLUTION. Verdict: MAPS, on the composition, as the inverse of Rule 3.

The tradition. Dissolution of the calcined body in its own water; the *solve* of *solve et coagula*.

The candidate target. `G`, inverted: given a compound, recover its components.

The audit. All four conditions hold, with one qualification large enough to require a chapter of its own. **The inverse exists but is not a function: many distinct component sets produce the same compound frequency.** Chapter 20 owns the *solve* step, proves the non-uniqueness, and computes the fibres. This gate is recorded as MAPS and handed over.

The one number worth stating here, because it is the reason Chapter 20 exists: the carried dyad's member A holds the chord {4, 6, 9}, whose product is 216 and whose count-weighted geometric mean is therefore exactly 6. **Four other chords drawn from the same twelve-state ladder have the same mean and therefore the identical compound frequency:**

```
T = 3, product 216:   {2,9,12}  {3,6,12}  {3,8,9}  {4,6,9}  {6,6,6}
all five give G = 6.0000000000 and f_compound = 302.8757693527
```

Solve cannot tell them apart. That is not a defect of this book's arithmetic; it is a property of the geometric mean, and Chapter 20 proves it in general.

GATE 3 — SEPARATION. Verdict: MAPS, on N.

The tradition. Division of the dissolved material into its distinct parts, subtle from gross.

The candidate target. N, the number of distinguishable categories — lever L4 of Chapter 4 §4.8.

The audit. All four conditions hold. Separation is the operation of finding a distinction where one category previously stood, and its effect on the reading is exactly Chapter 4's fourth lever.

Worked on the carried dyad at 2a, with dD/dN = −2.885390 u / (1 + (N−1)u):

```
N = 12   F = 0.563085   f =  1.490799 Hz   D = +4.338420   Q_inf = −5.011940
N = 13   F = 0.601479   f =  1.609785 Hz   D = +4.227638   Q_inf = −4.883958
N = 14   F = 0.638454   f =  1.733339 Hz   D = +4.120951   Q_inf = −4.760710
N = 17   F = 0.741821   f =  2.131409 Hz   D = +3.822698   Q_inf = −4.416154

 dD/dN at 2a = −0.112937 octaves per category
```

And the sentence that makes this gate worth auditing rather than admiring:

> **Separation alone closes this dyad's deficit at N = 102.**

One hundred and two genuinely distinguishable categories, each one of which must survive Chapter 5's count argument and Chapter 13's collision test. **The operation maps, the operation is real, and on this case it is useless** — which is precisely the kind of finding an audit exists to produce, and precisely the finding that an admiring reading of the same word would have missed.

GATE 4 — CONJUNCTION. Verdict: MAPS, SPLIT — and the split is this chapter's central finding.

The tradition. The marriage: two prepared materials are joined into one.

The candidate target. Ambiguous, and the ambiguity is not in the tradition's language but in ours. **Two of the three transfer rules describe a joining, they are different operations, and they return values in different units.**

Reading (i) — Rule 2, the relational join. Two parties enter into a relationship. The result is u, and f_rel = k(1 + 11u)^2 in hertz.

Reading (ii) — Rule 3, the compositional join. Two component sets are pooled into one compound. The result is G over the pooled multiset, and k^2 G^4 in the source's Hz-label, which is dimensionally hertz-squared (Book Two Ch 29; Chapter 20 §20.6).

Both readings applied to the carried dyad, which is the point of the gate:

```
reading (i)   Rule 2 on A and B          u = 0.068734665264634
                                         f_rel = 1.4907987150 Hz

reading (ii)  Rule 3 on the pooled chord {4x2, 6x2, 9x2, 10x1}
              T = 7, product 466560, G = 6.4542224391
          f_compound = 405.5414451025  (Hz-LABELLED, dimensionally Hz^2)
```

Taking the naive logarithm of the ratio gives 8.087620, and **that number is meaningless, because the two quantities are not commensurable.** It is printed here so that a reader who computes it independently knows it has already been considered and discarded.

FINDING 19.1. *Conjunction* is not one operation. Applied to two people it is Rule 2 and returns a relationship frequency; applied to two component sets it is Rule 3 and returns a compound value in squared units. **Any use of the word that does not declare which reading is in force produces a number that cannot be interpreted, and the tradition's texts do not declare it.** This is a genuine defect in the old notation, recoverable only because the new one keeps its units.

GATE 5 — PUTREFACTION. Verdict: MAPS, on `delta` — and therefore moves the deficit by exactly

zero.

The tradition. The blackening; the material is allowed to rot, and the rotting is required rather than permitted. Ripley's insistence on this gate is emphatic.

The candidate target. `delta`, the loss rate. Putrefaction is deliberate, tolerated decay of accumulated structure.

The audit. All four conditions hold. It maps cleanly, and what it maps to is the one parameter Chapter 4 proved is not a lever on the diagnostic.

Worked on the carried dyad at 2a:

```
delta = 0.09    D = +4.338420    Q_inf = -5.011940
delta = 0.20    D = +4.338420    Q_inf = -2.255373
delta = 0.30    D = +4.338420    Q_inf = -1.503582
```

The deficit does not move by one part in the last printed digit while `delta` more than triples, and the destination moves by 3.508358 units. This is Theorem 4.9 and Theorem 4.5 in the same three rows: `D = 2.885390(c/eta - F)` contains no `delta` at all, while `Q_inf = -D eta / (2.885390 delta)` is scaled by it.

The reading this licenses is unexpected and worth having. In a relation whose numerator is already negative — which 2a's is — **raising `delta` moves the destination toward zero from below.** It does not repair anything. It makes the arrival less bad by making arrival matter less, which is a real effect with a real price, and it is exactly the shape of a tradition that requires the material to be destroyed before it can be re-formed. R15 (Chapter 15) is the same statement with the sign reversed: lowering `delta` in a failing relation accelerates the failure.

The concordance, with roles stated. `(-1)` Ripley, and the wider tradition's insistence that nothing is made without putrefaction first — precedent, recorded, and it carries no evidence. `(+1)`

Chapter 4 Theorems 4.5 and 4.9, verified to 2.665e-15 across twelve parameter sets — evidence, and it carries the entire weight. **What the concordance establishes is that a structure was findable by two routes, not that the older route knew what it had found.**

GATE 6 — CONGELATION. Verdict: MAPS, on G. Chapter 20 owns it.

The tradition. Fixing the dissolved material into a stable body; the *coagula*.

The candidate target. G, forward: given components, produce the compound.

The audit. All four conditions hold, and this is the one gate for which the tradition's own notation already contains an exact closed form recovered in Book Two Chapter 29:

```
f_compound = k^2 * G^4        G = ( prod_i n_i^(c_i) )^(1/T),   T = sum c_i
 k^2 = 0.2337004393
```

Two worked compounds, both **constructed**:

```
{3x1, 5x1}    T = 2   P = 15   G = 3.8729833462   f = 52.5825988460
{2x2, 7x1}    T = 3   P = 28   G = 3.0365889719   f = 19.8702609489
```

Chapter 20 states the rule in full, proves what it is not, and handles the units. This entry records the verdict and hands over.

GATE 7 — CIBATION. Verdict: MAPS, on c, through cadence.

The tradition. Feeding: the material is given further matter, in small measured quantities, at the correct intervals, and never all at once.

The candidate target. Not a transfer rule at all — the input schedule of the storage law, which enters the diagnostic through c.

The audit. All four conditions hold once the schedule is priced. R11 (Book Two Ch 59, Theorem 59.1) is the relevant result: **for a concave gain function with g(0) = 0, dividing a fixed budget over more occasions never delivers less.** Computed with g(x) = ln(1+x) and a budget of 1.0:

```
occasions m      per occasion      total delivered
     1             1.000000           0.693147
     2             0.500000           0.810930
     4             0.250000           0.892574
     8             0.125000           0.942264
    16             0.062500           0.969994
    32             0.031250           0.984693
    64             0.015625           0.992268
   ->inf                               1.000000
```

Sixty-four occasions deliver 43.2% more than one occasion from the identical budget. **This is concavity, not diligence, and it is the arithmetic reason cadence beats intensity.** R11's own warning applies and is restated: this is a result about *distribution of a fixed input*, not a recommendation of greater total input.

On the carried dyad, a cadence change that delivers the same contact at lower per-period cost:

```
c = 0.62   D = +4.338420   Q_inf = −5.011940
c = 0.50   D = +3.184264   Q_inf = −3.678606      (−1.154156 octaves)
c = 0.40   D = +2.222467   Q_inf = −2.567495      (−2.115953 octaves)
```

At `eta = 0.30`, `dD/dc = 2.885390 / eta = 9.617967` octaves per unit of cost. **Cibation is the cheapest gate in the list on this case, and it is the only one of the twelve that a reader is likely to have dismissed as housekeeping.**

GATE 8 — SUBLIMATION. Verdict: RENDERING. Rejected as an operation on a state.

The tradition. The volatile part is raised by heat and re-condensed above, purer, unchanged in kind.

The candidate target. Announced as `n`. Actual landing: `j`, the transposition exponent.

The audit. Condition (c) fails at the level that matters. *Raised, unchanged in kind* is an exact description of octave transposition — Chapter 7 §7.2 — and octave transposition is precisely the operation this book has already proved changes nothing:

```
f_audible = f * 2^j        j an integer

folding cannot create a pitch-class collision
folding cannot remove one
every interval in cents is preserved exactly
```

State 3 folded from 4.3508315969 Hz to 278.453222 Hz at `j = 6` is at a higher pitch and is the same state, with the same class, at the same intervals from everything else. **Nothing in the declared parameter set has moved, because `j` is not in the declared parameter set.**

REJECTED as an operation on a state; ADMISSIBLE as a rendering step, which is what Chapter 7 §7.18 already licenses it as. The distinction is not pedantry. A practitioner who believes sublimation raises the material has performed an operation on the publication format and recorded it as an operation on the substance, and Chapter 7's Failure Mode F8 already names the modern version of the same error: band-shopping, choosing the register until the numbers look better.

This is the first of the five failures, and it is the mildest, because the operation is real. **It is simply real about something else.**

GATE 9 — FERMENTATION. Verdict: MAPS, on `G` and on `u`.

The tradition. A small quantity of the perfected substance — the ferment, often gold — is added, and the whole takes on its nature.

The candidate target. Two, and here they do not conflict. Adding a component of declared index changes `G` under Rule 3. Adding a state to a party's chord changes the support overlap and therefore `u` under Rule 2.

The audit. All four conditions hold in both readings, and unlike Gate 4 the readings are compatible rather than ambiguous, because they answer different questions about the same act. What must be declared is which one is being reported.

Worked on the compositional reading, **constructed**:

```
{3x1, 5x1}                T = 2   G = 3.8729833462   f =  52.5825988460
+ one component at 7      T = 3   G = 4.7176939803   f = 115.7653513543
                          gain in the Hz-labelled value: +1.138 in log2
```

And on the relational reading, the carried dyad at 2a versus 2b, where SPINE §3.2 records exactly this act — one party takes on states the other does not hold:

```
u  0.068734665264634 -> 0.692372881355932
f  1.4907987150 Hz   -> 35.8881768941 Hz
D  +4.338420        -> -0.250931     (holding eta = 0.30, c = 0.62 fixed)
D  +4.338420          -> -3.590989      (with 2b's eta = 0.44, c = 0.40)
```

Fermentation is the only gate in the twelve that acts on the numerator of the sustainability condition, and Chapter 15's R15 is the rule that says it must therefore be run first. The tradition places fermentation ninth of twelve. That ordering is not vindicated by this audit and the audit does not pretend it is.

GATE 10 — EXALTATION. Verdict: REJECTED.

The tradition. The material is raised to a higher, nobler, more dignified nature.

The candidate target. Announced as n.

The audit. Condition (c) fails outright. **"Nobler" is a value ordering, not a count**, and the index n is defined in SPINE §1 as the number of distinguishable components required to specify the state. There is no operation "increase the dignity" available on a count. Chapter 5 §5.18 named this exact fault when it rejected the valence-magnitude rival: a rating presented as a count inherits none of the count's scale properties and all of the rating's ambiguity.

Two attempts to rescue the gate, both of which fail, and both are published because the rescues are what a reader will reach for.

Rescue 1 — read "higher" as "higher index." Then exaltation is Chapter 21's re-indexing, already covered by Gates 1 and 9, and the word contributes nothing. Worse, it contributes something false: **Chapter 5 §5.20 established that more components is not better, and the twelve-state ladder is explicitly not an ordering by worth.** A reading that makes exaltation equal index-raising imports a value judgement the index does not carry.

Rescue 2 — read "higher" as "above the ceiling." Then the gate claims an operation that raises an essence past the maximum the category set permits, and Paper D Corollary 2.2 forbids it:

```
F <= ln N = ln 12 = 2.484907 nats
f <= k N^2 = 69.6133055503 Hz
state 12 sits exactly at the ceiling: headroom 0.000000 octaves
```

The only way past k N^2 is to raise N, which is Gate 3, which is a different operation with a different name and a measured price of 0.112937 octaves per category on the carried dyad.

REJECTED. The operation as stated names no computable transformation, and both readings that would make it computable turn it into an operation the list already contains under another name.

GATE 11 — MULTIPLICATION. Verdict: SPLIT — the iterative reading MAPS, the unbounded reading is

REJECTED.

The tradition. The stone is multiplied: repeated passes through the work increase its potency, and the increase is presented without a stated limit.

Reading (i), iterated refinement. MAPS. Repetition of a bounded operation on an accumulating store is exactly the storage recursion, and its exact solution is Paper E Theorem 3.1 with the convergence Chapter 22 owns:

```
|Q_t − Q_inf| = (1 − delta)^t |Q_0 − Q_inf|
```

Computed at `delta = 0.09` toward the carried dyad's post-intervention destination `Q_inf = +6.084427`, from `Q_0 = 0`:

```
t = 0     Q = 0.000000    remaining fraction 1.000000
t = 1     Q = 0.547598    remaining fraction 0.910000
t = 5     Q = 2.287549    remaining fraction 0.624032
t = 10    Q = 3.715053    remaining fraction 0.389416
t = 20    Q = 5.161754    remaining fraction 0.151645
t = 50    Q = 6.029940    remaining fraction 0.008955
t = 100   Q = 6.083939    remaining fraction 0.000080

half-life of the gap  =  ln 2 / −ln(1−delta)  =  7.349615 passes
99% closure           =                          48.829785 passes
```

Reading (ii), unbounded potency. REJECTED. The tradition's claim is that each pass multiplies, so that potency grows without limit with the number of passes. **The formalism says the opposite in the strongest available way: the recursion is a contraction, it has a unique fixed point, and it is reached from any starting value.** Repetition does not ascend. It arrives, geometrically, at a destination fixed entirely by `(eta, F, c, delta)` — and no number of passes changes any of those four. A thousandth pass on unchanged parameters adds `0.910^1000` of the original gap, which is 10^−41 of it.

REJECTED, and the rejection is a strong one: **this is not "unsupported," it is contradicted.** The only way to raise the destination is to change a parameter, which means running a different gate. Multiplication is the gate that most rewards being audited, because the mapped half of it is genuinely useful — Chapter 22 is built on it — and the unmapped half is the exact error that a formalism with a contraction theorem is built to catch.

GATE 12 — PROJECTION. Verdict: REJECTED.

The tradition. A grain of the finished stone is cast upon a mass of base metal and transmutes the whole of it.

The candidate target. None available.

The audit. Every condition fails, and they fail for reasons the earlier chapters have already established.

(a) fails. The operation requires a distinction between "base" and "noble" material that is not a coordinate in the parameter set. This is Gate 10's fault again, at compound scope.

(b) fails. The end state is a quantity of material at a raised index, where the quantity is unbounded and the seed is arbitrarily small. Chapter 20 §20.5 proves the compound rule is blind to overall quantity — `G` for `{3x1, 5x1}`, `{3x2, 5x2}` and `{3x5, 5x5}` is the identical 3.872983346207 — so "a mass transmuted" and "a grain transmuted" are the same value under Rule 3, and the claim's whole content is the mass.

(c) fails, and this is the decisive one. The operation asserts a multiplier with no channel. Book Two's belief postulate is explicit and CANON3 §4 restates it: **belief acts only through a coupling with real gain, and where no channel exists nothing happens.** A seed that reorganises an unbounded mass at no cost is a gain of infinity through an unspecified path.

And the hard constraint applies. Book Two Chapter 67's CONSTRAINT L67 — carried into Chapter 21 as this book's own — states that addressing reaches only what is already latent. Projection claims to confer a property the mass did not have and could not have had. **Nothing in this formalism can create a component that was not already in the decomposition.**

The `(-1)` column here is instructive precisely because it is worthless as evidence. The transmutation record is the most heavily survivorship-filtered body of testimony in the tradition: successes were reported, failures were not, and R23 fixes the likelihood ratio for testimonial evidence of this kind at exactly 1.000000. **A likelihood ratio of one is not weak evidence. It is the arithmetic statement that the testimony leaves the prior exactly where it found it.** Chapter 24 treats this at length; the entry records it and moves on.

REJECTED on all four conditions.

ENTRY 13 — THE PLANETARY-METAL ORDERING AS AN ESSENCE INDEX. Verdict: REJECTED.

The tradition. Seven metals in a fixed traditional sequence — lead, tin, iron, copper, mercury, silver, gold — with planetary correspondences, transmitted continuously and used as an ordering of worth and of process stage.

The candidate. Use the sequence as the ordinal n in Rule 1, giving `f = k n^2` for `n = 1 .. 7`.

The audit against Paper B's validity conditions (Chapter 5 §5.5), which is the same audit the four psychological rivals received:

```
completeness       FAIL   the set is closed at seven by cosmological
                          convention, not by the domain
non-arbitrariness  FAIL   the ordering is by nobility -- a value ranking.
                          Chapter 5 rejected exactly this under the
                          valence-magnitude rival
count semantics    FAIL   position 5 is not "five of something." There is
                          no component being counted, so the index has no
                          true zero and no equal units
stability          FAIL   variant orderings are attested; the sequence is
                          not unique across the transmission
```

And the cost of the failure, computed the way Book Two Chapter 29 §29.9 requires — by asking what a plausible re-ordering does to the published pitch classes:

```
    swap positions 3 and 5:   class 3 -> class 5
                              folded 278.453222 Hz -> 386.740586 Hz
                              +568.7174 cents -- nearly half an octave

    swap positions 3 and 6:   class 3 -> class 3
                             +2400.0000 cents, and the class does not move at all
```

One swap of two disputed positions moves a published pitch class by 568.7174 cents; another swap moves the pitch by two full octaves and changes nothing audible. An ordering whose frequencies are that sensitive to a choice the tradition does not fix cannot support a published number.

REJECTED as an essence index. Admissible, exactly as Book Two Chapter 57 has it, as a **finite relational graph** — a labelled seven-element set with edges to planets, days, and metals — which is a descriptive statement carrying no claim that the labelling is meaningful. Graphs are Paper A's object. Indices are not.

19.7 The audit table

```
    GATE              TARGET        VERDICT       WHAT IT ACTUALLY DOES
    ------------------------------------------------------------------------
-------
   1  calcination    n           MAPS         removes components; de-indexes
   2  solution     G (inverse)   MAPS        decomposes; NOT uniquely (Ch 20)
   3  separation    N           MAPS        lever L4; +0.112937 oct/category
   4  conjunction   u  AND  G    MAPS, SPLIT   two operations, two unit systems
   5  putrefaction   delta        MAPS         moves Q_inf; moves D by ZERO
   6  congelation    G             MAPS          Rule 3; k^2 G^4 (Ch 20)
   7  cibation      c           MAPS        cadence; -9.617967 oct per unit c
   8  sublimation   j          RENDERING    changes the register, nothing else
   9  fermentation   G  and  u    MAPS         the only gate on the numerator
   10 exaltation    --          REJECTED     a rating smuggled in as a count
   11 multiplication delta / --    SPLIT         converges; does not ascend
   12 projection    --           REJECTED     no channel; ceiling; L67; R23
   13 metal ordering n          REJECTED    fails all four validity conditions
    ------------------------------------------------------------------------
-------
   MAPS 7   SPLIT 2   RENDERING 1   REJECTED 3     (5 entries carry a failure)
```

Seven of thirteen entries map cleanly. Two are ambiguous in a way the old notation cannot resolve and the new one can. One is real about the wrong object. Three fail outright. That distribution is the chapter's result, and the two-thirds that survive are worth more for the third that did not.

The Specification

SPEC-19.1 — Inputs required before an operation may be audited

```
the operation's name and its source, with transmission path
the operation's statement in the source's own terms, quoted or paraphrased
the candidate target, named from {n, N, u, eta, c, delta, G} or declared absent
the starting configuration, in declared parameters
the ending configuration, in declared parameters
```

SPEC-19.2 — The audit, in order

```
1 test condition (a): is the start expressible? If not, EMPTY or REJECTED.
2 test condition (b): is the end expressible? If not, EMPTY or REJECTED.
3 test condition (c): is the transformation a function? This is where
  value-orderings fail. A term that names a direction of worth rather
  than a coordinate fails here and the failure is published.
4 test condition (d): run it on a stated input and produce a number.
5 if the target is j and only j, the verdict is RENDERING, not MAPS.
6 if two readings land on different parameters, the verdict is SPLIT and
  BOTH are published with their units.
7 publish the verdict with its arithmetic, failures at full length.
```

SPEC-19.3 — Required outputs for a MAPS verdict

```
the map, stated as a function on declared parameters
a worked number on a stated, constructed input
the effect on D, with its sign and magnitude
the effect on Q_inf, with delta stated
the honesty clause that binds the reading
```

The fourth line is mandatory and Chapter 4 Theorem 4.9 is why: `D` contains no `delta`, so a report giving only `D` cannot distinguish Gate 5 from doing nothing. Gate 5 is the entry in this chapter that would have been silently lost by a `D`-only report.

SPEC-19.4 — Required outputs for a REJECTED verdict

```
which of the four conditions failed, named
the specific result that condemns it, cited to chapter and theorem
every rescue a reader is likely to attempt, attempted and reported
what would have to be true for the verdict to change
```

The third line is the one that costs work and is the one that makes a rejection auditable. Gate 10 carries two rescues; Gate 11 carries the split that rescues half of it.

SPEC-19.5 — Concordance rule for this chapter

Where an entry draws a concordance, both columns are named, the roles are stated in the same sentence, and the precedent is not counted toward the evidence. **An entry may not upgrade its**

status on the strength of the (−1) column alone, at any weight, ever. Gate 12's entry is the worked instance: the (−1) column there is large, old, and continuous, and it moves the verdict by nothing, because R23 fixes its likelihood ratio at 1.000000.

SPEC-19.6 — Precision

Recompute every derived value from the twelve-or-more-decimal u given in SPINE §3.2, never from a rounded print (SPINE §3.4). Intervals in cents to four decimals; frequencies to at least six; deficits to six. Ratios and deficits are unaffected by rounding of k because k cancels from both.

SPEC-19.7 — Gates on publication

```
G1  every verdict has its arithmetic in print
G2  at least one REJECTED entry appears, or the audit is void
G3  no entry claims a physical effect of any symbol, name, or operation
G4  every case is labelled constructed
G5  units are stated wherever Rule 3 is used; the Hz label is a
    convention and is dimensionally Hz^2
G6  D is never published without delta
```

The Failure Modes

FM-19.1 — The audit in which everything passes

Signature: thirteen entries, thirteen maps, an appreciative tone.

Why it happens: the operations are vague enough that a determined reader can find a parameter for any of them. *Nobler* can be read as higher-index; *projection* can be read as a metaphor for teaching; every failure can be dissolved by generosity.

The fix: Rule 19.1 condition (c), applied without generosity. **A transformation that becomes computable only after the auditor supplies its content was supplied by the auditor.**

FM-19.2 — Precedent counted as evidence

Signature: "both the tradition and the formalism agree that X, so X is well supported."

Why it wrong: two sources agreeing does not double the evidence when only one of them is evidence (CANON_BIO §1). What independent arrival establishes is that a structure was findable by more than one route. That is real and it is not corroboration.

The fix: SPEC-19.5. State the roles in the sentence that draws the concordance.

FM-19.3 — Reporting D and calling the reading complete

Signature: Gate 5 audited, D unchanged, conclusion "putrefaction does nothing."

Why it is wrong: `D` moved by zero and `Q_inf` moved by 3.508358. The deficit is delta-blind (Chapter 4 Theorem 4.9), and a `D`-only report cannot see the one gate in the list that acts purely on `delta`.

The fix: G6. Both numbers, every time.

FM-19.4 — Mixing the two conjunctions

Signature: a compound value in the source's Hz label compared with a relationship frequency in hertz, and a ratio taken between them.

Why it is wrong: Finding 19.1. The values are not commensurable; Rule 3's output is dimensionally hertz-squared.

The fix: declare the reading before computing. If both are wanted, publish both, separately labelled, and take no ratio.

FM-19.5 — Band-shopping, which is Gate 8 performed without knowing it

Signature: the folded values are recomputed in a different band until two states look close.

Why it is wrong: folding preserves every interval exactly (Chapter 7 §7.2). Two states that share a class share it in every band; two that do not, do not.

The fix: the band is declared before the values are computed, and `j` is published with every value.

FM-19.6 — Treating the sequence as validated because the gates are

Signature: seven gates map, therefore Ripley's twelve-step order is the correct order of operations.

Why it is wrong: the audit tested the gates individually. **On the ordering, the audit's finding is negative: fermentation is the only gate acting on the numerator, R15 requires the numerator first, and the tradition places fermentation ninth.**

The fix: order is a separate claim and needs a separate audit. The Worked Example runs it.

FM-19.7 — Reading a rejection as a claim about the practitioners

Signature: "the tradition was wrong about projection, therefore it was unserious."

Why it is wrong: the audit is about the mapping, not the people. Seven gates map, and one of them — cibation — encodes a concavity result that was not stated formally until much later, from a tradition with no access to the theorem.

The fix: keep the two claims apart in the sentence. Chapter 23 handles what the tradition got right at length, and Chapter 24 the reverse, and neither is a verdict on anyone's intelligence.

FM-19.8 — Auditing an operation against a target chosen after seeing the result

Signature: the target is left open until a parameter is found that makes the operation pass.

Why it is wrong: it is the target-selection version of Chapter 13's FM-13.5. With seven parameters and a free choice, almost anything maps.

The fix: SPEC-19.1 requires the candidate target to be named as an input. Changing it after the audit begins voids the entry and it is re-run from step 1.

FM-19.9 — Importing worth into the index

Signature: an operation is said to raise a state, and the reader takes "higher n" to mean "better."

Why it is wrong: Chapter 5 §5.20 established that more components is not better. The ladder is a count of specification, not a ranking of value, and Chapter 10 established that the fully consonant self is the fully redundant one.

The fix: Gate 10's rejection, restated wherever an operation moves an index.

FM-19.10 — Using an audited operation as an instruction

Signature: "Gate 7 maps, therefore run Gate 7 on your relationship."

Why it is wrong: the audit says an operation exists and is computable. Chapter 18 owns what may be done with a reading, including the consent axiom and the halting conditions, and this chapter adds no permissions whatever.

The fix: the audit's outputs are verdicts, not prescriptions. Nothing here is counselling, therapy, mediation, or clinical assessment, and none of it may be presented as any of those.

FM-19.11 — The unbounded reading of a converging process

Signature: "each pass makes it stronger, so more passes make it stronger still."

Why it is wrong: Gate 11. `(1-delta)^t` closes 99% of the gap in 48.83 passes at `delta = 0.09` and asymptotes there. The destination is set by four parameters and repetition changes none of them.

The fix: before iterating, compute `Q_inf`. If it is where you want to be, iterate. If it is not, iteration is the wrong operation and one of the four levers is the right one.

FM-19.12 — Publishing a Rule 3 value into an acoustic calculation

Signature: `k^2  G^4` fed into a renderer as hertz.

Why it is wrong: the value carries hertz-squared if the units are taken seriously (Book Two Ch 29; Chapter 20 §20.6). Only ratio operations are safe on it, and octave folding is a ratio operation.

The fix: re-anchor by folding before rendering, and label the value as convention-Hz everywhere it appears.

Worked Example

PART A — THE TWELVE GATES RUN AS A SCHEDULE ON THE CARRIED DYAD

Constructed case, labelled as such. SPINE §3.2 phase 2a, unchanged:

```
u = 0.068734665264634    eta = 0.30    c = 0.62    delta = 0.09    N = 12
F = 0.563084802746        f = 1.4907987150 Hz
D = +4.338420 octaves     Q_inf = −5.011940          DECAYS
```

The question this part asks is the one FM-19.6 raised: run the gates in Ripley's order, as a schedule, and record what each one moves. Every step is a **constructed** intervention chosen to be the smallest change that the gate's own description licenses.

STEP	GATE	PARAMETER MOVED	D after	Q_inf after
0	(as found)	--	+4.338420	−5.011940
1	calcination	u 0.068735 -> 0.040	+4.911002	−5.673412
2	solution	none (analysis only)	+4.911002	−5.673412
3	separation	N 12 -> 13	+4.831945	−5.582082
4	conjunction	ambiguous -- HALT	--	--
5	putrefaction	delta 0.09 -> 0.20	+4.831945	−2.511937
6	congelation	none on D (Rule 3)	+4.831945	−2.511937
7	cibation	c 0.62 -> 0.50	+3.677789	−1.911937
8	sublimation	j only	+3.677789	−1.911937
9	fermentation	u 0.040 -> 0.692373	−1.628106	+0.846388
10	exaltation	REJECTED	--	--
11	multiplication	iterate only	--	--
12	projection	REJECTED	--	--

```
total move in D, steps 1–9:  −5.966526 octaves
contributed by step 9 alone: −5.305895 octaves  =  88.93%
contributed by steps 1–8:    −0.660631 octaves  =  11.07%
```

Read the schedule and four things are visible that no individual entry shows.

Step 1 makes it worse. Calcination as the opening gate removes differentiation, lowering u, and lowering u raises the deficit by 0.572582 octaves. The tradition opens with the gate that, on this case, moves the reading in the wrong direction. That is no refutation of calcination — Gate 1 maps, and de-indexing has legitimate uses — it is a refutation of calcination-first on a relation whose problem is insufficient difference.

Step 4 halts the schedule. Conjunction is ambiguous between Rule 2 and Rule 3, and the two readings return values in incommensurable units. A procedure cannot proceed through a step whose output units are undetermined, so the schedule halts here, exactly as Chapter 18's PROCEDURE T halts at an undetermined input. The halt is a finding rather than a failure of the audit; it is resumed below under reading (i) so that the remaining gates can be scored.

Step 5 changes the destination by 3.070145 and the deficit by nothing. `Q_inf` moves from

−5.582082 to −2.511937 while D sits at +4.831945 in both rows. A report carrying only D would show step 5 as a no-op. **The delta-blindness of the deficit is not an abstraction here; it is a row in this table that would have been invisible.**

Step 9 does 88.93% of the work. Fermentation moves D by −5.305895 octaves; the eight gates before it move it by −0.660631 between them. The one gate acting on the numerator is ninth of twelve, and by the time it runs, eight steps have been spent on the denominator, on N, and on delta.

FINDING 19.2, and it is the Worked Example's whole point. Ripley's ordering places the only numerator-acting gate ninth of twelve, and R15 (Chapter 15) requires the numerator first: in a relation below f_min, an intervention on delta amplifies the sign already present. Step 5 is exactly such an intervention and it runs four steps before the numerator is repaired.

The two orderings, side by side, running the same three consequential gates:

```
    RIPLEY ORDER   putrefaction while the numerator is NEGATIVE
                   delta 0.09 -> 0.20 : Q_inf -5.582082 -> -2.511937
                   the destination improves, because delta is dividing a
                   negative numerator and a larger divisor shrinks it

   R15 ORDER     fermentation first : D +4.338420 -> -0.250931, Q_inf +0.289887
                 then cibation     : D -0.250931 -> -1.405087, Q_inf +1.623220
                    then putrefaction, numerator now POSITIVE
                    delta 0.09 -> 0.20 : Q_inf +1.623220 -> +0.730449
                    the destination WORSENS, from the identical move
```

Raising delta improves the destination in one ordering and damages it in the other, and the only thing that differs is the sign of the numerator when the move is made. That is R15, restated in the old notation, with the arithmetic attached.

PART B — THE FIVE FAILURES, ON ONE LEDGER

Printed together, at equal length, because a chapter that scattered its failures among its successes would be hiding them in plain sight.

```
    ENTRY            CONDITION FAILED     THE RESULT THAT CONDEMNS IT
     ------------------------------------------------------------------------
----
    8 sublimation    (c) partially       Ch 7 §7.2: folding by 2^j preserves
                     -- lands on j       every interval and every pitch class
                                        exactly. Nothing in the parameter set
                                            moves. Verdict RENDERING.
                                       Would change if: j entered any verdict.
                                            It does not; k cancels from D.

    10 exaltation    (c)                  "Nobler" is a value ordering, not a
                                         coordinate. Ch 5 §5.18. Rescue 1 makes
                                            it Gate 1 or 9; Rescue 2 hits the
                                       ceiling F <= ln 12, f <= 69.6133055503.
                                            Would change if: a count-valued
```

			definition of nobility were supplied. None is, in the source or since.
11	multiplication (c), unbounded (unbounded reading)	reading only	Contraction: |Q_t − Q_inf| = (1−delta)^t |Q_0 − Q_inf|. 99% closure at 48.83 passes; the destination is fixed by four parameters that repetition does not touch. Would change if: a pass altered eta, F, c or delta. Then it is a different gate under a borrowed name.
12	projection	(a), (b), (c), (d)	No base/noble coordinate; Rule 3 is blind to quantity so the mass is not in the value; no channel (belief postulate, CANON3 §4); CONSTRAINT L67 forbids reaching the non-latent; R23 fixes the testimonial LR at 1.000000. Would change if: a channel with measured gain were exhibited. None is.
13	metal order	all four of Paper B's conditions	Completeness, non-arbitrariness, count semantics and stability all fail. One disputed swap moves a published class by 568.7174 cents. Would change if: the ordering were fixed by a count. It is fixed by worth.

Each row's last field is the one that makes the rejection a scientific statement rather than a dismissal. **A rejection that cannot say what would reverse it is an opinion.**

PART C — WHAT THE AUDIT LICENSES

Stated tightly, because the temptation at the end of a chapter like this one is to say more than was shown.

It licenses: the statement that seven of Ripley's twelve gates name operations on this book's declared parameters; that the maps are computable and were computed; that two gates are ambiguous in the old notation and resolvable in the new one; that three fail against specific results; and that the sequence, run as a schedule, contradicts R15 on a constructed case.

It does not license: any claim that the operations were effective; any claim about what their practitioners understood; any claim that a symbol, name, or sequence has a physical effect; any diagnostic, clinical, or therapeutic reading of any number above; or the use of a mapped gate as an instruction to act on anyone, which remains Chapter 18's territory and Chapter 18's constraints.

And it licenses one thing that is easy to miss. The gate that came out best on the carried case is cibation — feeding in small measured quantities at correct intervals — which is the least

glamorous item on Ripley's list and the one a modern reader is most likely to skim. It is worth 9.617967 octaves per unit of cost at `eta = 0.30`, and the concavity that makes it work was not written down as a theorem for another five centuries. That is what precedent is worth, stated at its correct weight: not evidence, and not nothing.

Chapter 20 takes Gates 2 and 6 — solution and congelation — and gives them the chapter they were handed off to. It owns the compound rule `f = k^2 G^4`, states plainly what that rule is not, and proves why the solve step cannot be inverted uniquely. Chapter 21 takes Gates 1 and 9, and asks what it costs to move a state from one index to another, and what cannot be reached at any price.

Chapter 20 . Solve et Coagula

Two words, joined by a conjunction, and the joining is the instruction. Dissolve, and fix. Take apart, and put together. It is the most compressed statement the old corpus produced and it is almost the only one that survives Chapter 19's audit with both halves intact — Gate 2 and Gate 6, solution and congelation, both returned MAPS, and both were handed to this chapter.

They arrive in very different condition.

The coagula step has an exact closed form. It was not built for this book. It was reverse engineered in Book Two Chapter 29 out of a published recipe for combining essence values, it collapsed to four symbols, and it then reproduced 37 of 44 published composite values to better than one part in `10^12`. That is an unusually strong result for a formalism recovered from a source rather than derived from axioms, and this chapter states it, computes with it, and hands the statement to Chapters 21 and 26 as CANON3 §2's Rule 3.

The solve step has no closed form, and this chapter proves that it cannot have one. The map from components to compound is many-to-one, catastrophically so, and it gets worse as the compound gets larger: at four components drawn from the twelve-state ladder, 83.88% of all possible component sets share their compound frequency with at least one other, and the value `G = 6` alone is produced by sixty distinct component sets. Dissolution is not the inverse of fixing. It is a search over a fibre, and the fibre has to be published with the answer.

Three things are settled before the arithmetic.

This chapter owns the compound rule (SPINE §4). It states it in full and Chapters 21 and 26 cite it. It also owns the negative statement that goes with it, which is more important than the rule and is the single most misquoted result in the series: **`f = k^2 G^4` is not a generalisation of `f = k n^2`, does not reduce to it, and never agrees with it at any integer index whatever.**

The units are not hertz. The composite value carries hertz-squared if the units are taken seriously (Book Two Ch 29), and the "Hz" label on it is a convention inherited from the source. Only ratio operations are safe on it. Every value in this chapter is labelled accordingly.

Nothing here is a physical claim. These are essence frequencies — positions on a logarithmic scale of surplus. Nothing is claimed to vibrate, no molecule is claimed to resonate, the twelve states are not diagnoses, no reading is an assessment of any person's health, and every case is constructed and labelled as such.

The Law

20.1 The coagula rule, stated and derived

RULE 3 (CANON3 §2; the alchemical rule). Let a compound contain essence indices `n_i` with multiplicities `c_i`, and let `T = sum(c_i)` be the total component count. Define `G`, the count-weighted geometric mean of the indices, and the compound frequency:

```
G           =  ( prod_i  n_i^(c_i) )^(1/T)

f_compound  =  k^2 * G^4
```

The derivation is Book Two Chapter 29 §29.10 and is repeated here in four lines because a chapter that owns a rule states it whole. The source recipe raises each component's essence frequency to the power of its share of the total count, multiplies, and squares:

```
f_compound = ( prod_i f(n_i)^(c_i/T) )^2
           = ( prod_i (k n_i^2)^(c_i/T) )^2
           = ( k^(sum c_i / T) * prod_i n_i^(2 c_i / T) )^2
           = ( k * G^2 )^2
           = k^2 * G^4
```

The third line uses `sum(c_i)/T = 1`, which is what makes k come out with an exponent of one and the whole expression collapse. The rule is two constants and one geometric mean, and **everything about a compound that this formalism can see is in `G`.**

20.2 The constant

```
k    = (9/8)^2 / phi^2  =  0.483425732988414 Hz
k^2  =                     0.233700439315386
published to ten places:   0.2337004393
```

k is a chosen reference constant and is badged as a postulate every time it appears (Chapter 7 §7.1). k^2 inherits the badge. As with k, it cancels from every ratio, so no verdict in this book depends on it — only the absolute values do.

20.3 Verification against the source

Book Two Chapter 29 tested the collapsed form against every composite value the 2022 source publishes: 24 pharmacological compounds, 8 nucleotides in the carbon column, and 6 each in the silicon- and germanium-substituted columns. **Thirty-seven of forty-four are reproduced to better than one part in `10^12`. Seven disagree.** The seven, with the disagreement recomputed here in both relative and octave form:

```
formula            published       k^2 G^4     rel. deviation    octaves
 ------------------------------------------------------------------------
----
O2               11.124611798    957.23699944     85.0468        +6.4270
C5H6N2O2          4.669611378     21.80527042      3.6696        +2.2233
C12H17N2O4P      18.534334373     13.36731640      0.2788        -0.4715
C257H383N65O77S6  14.276237883     11.27999043      0.2099        -0.3399
```

```
C43H66N12O12S2    12.827862548     11.29835161      0.1192      −0.1832
C77H120N18O26S    11.075644799     10.43459440      0.0579      −0.0860
C158H251N39O46S    9.460360140      9.41751106      0.0045      −0.0065
 ──────────────────────────────────────────────────────────────────────
────
```

20.4 Why the seven are errata and not a different law — and why the O2 row settles it

This is the argument that licenses using the rule at all, so it is given in full.

For a single-element species the **count-weighted geometric mean is the element index itself, independent of how many atoms are present.** Oxygen has index 8, so:

```
O    G = (8^1)^(1/1) = 8        O2   G = (8^2)^(1/2) = 8
O3   G = (8^3)^(1/3) = 8        O_x  G = (8^x)^(1/x) = 8    for every x

therefore f = k^2 * 8^4 = 957.2369994358  for EVERY pure-oxygen formula
```

No atom count whatever produces the published 11.1246117974981. That row cannot have come from the stated rule, at any multiplicity, ever. And the shape of the disagreement across the table decides between the two hypotheses: a rule error fails uniformly, a data-entry error fails on scattered rows, and **what is observed is scattered rows** — thirty-seven exact to fourteen significant figures, seven wrong by amounts ranging over four orders of magnitude with no pattern. The rule is confirmed and seven cells are errata.

20.5 What the rule is NOT — the statement this chapter exists to make

Rule 3 looks like a generalisation of Rule 1. It is not, and the confusion is worth killing in detail because it produces numbers that are wrong by octaves.

THEOREM 20.1 (the single-component identity). For a compound of one component at index n, at any multiplicity,

```
f_compound  =  k^2 n^4  =  (k n^2)^2  =  f(n)^2
```

Proof: `G = n` for any multiplicity, by §20.4. Substitute. **QED.**

Verified at every index on the twelve-state ladder to a worst absolute error of `9.09e−13`, which is floating-point noise in values of order `10^3`:

```
n     f(n) = k n^2       f_compound = k^2 n^4      f(n)^2
1      0.4834257330          0.2337004393       0.2337004393
2      1.9337029320          3.7392070290       3.7392070290
3      4.3508315969         18.9297355845      18.9297355845
5     12.0856433247        146.0627745721     146.0627745721
8     30.9392469113        957.2369994358     957.2369994358
12    69.6133055503       4846.0123096438    4846.0123096438
```

So the compound rule applied to a single-component species does not return that species' essence frequency. It returns the square of it. The two rules answer different questions and produce different numbers from the same input.

COROLLARY 20.2 (they never agree). `k n^2 = k^2 n^4` requires `k n^2 = 1`, i.e. `n = sqrt(1/k) = 1.438252434444351`, which is not an integer. **There is no index at which Rule 1 and Rule 3 coincide.** The nearest approach is at `n = 1`, where they differ by a factor of `k` itself — 0.4834257330 against 0.2337004393, a gap of 1.048946 octaves — and the gap widens monotonically with n, reaching 6.121 octaves at `n = 12`.

And this is exactly why the source table's O2 row was provably impossible. The two facts are one fact: `G` for a single-component species is index-independent in the multiplicity and returns the essence value squared, so a published composite that neither equals `f(8)^2 = 957.2369994358` nor any other value the rule can produce has no rule-consistent reading at all. The seven outliers are transcription errors, not evidence of a different law, and the O2 row is the one that proves it, because it is the only row whose input is so constrained that the rule permits exactly one answer.

Rule 3 is not a law of the same kind as Rule 1 and must never be described as one. Rule 1 assigns a frequency to a category from its index. Rule 3 assigns a value to a *composition* from the geometric mean of its parts, in squared units, and it is blind to things Rule 1 sees.

20.6 The quantity-blindness theorem

THEOREM 20.3. `G` is invariant under multiplying every multiplicity by a common positive integer. If `c_i -> m c_i` for all `i`, then `T -> mT`, `prod n_i^(m c_i) = (prod n_i^(c_i))^m`, and

```
G' = ( P^m )^(1/(mT)) = P^(1/T) = G
```

QED. Computed:

```
{3x1, 5x1}    T = 2    P = 15        G = 3.872983346207   f = 52.582598845962
{3x2, 5x2}    T = 4    P = 225       G = 3.872983346207   f = 52.582598845962
{3x3, 5x3}    T = 6    P = 3375      G = 3.872983346207   f = 52.582598845962
{3x5, 5x5}    T = 10   P = 759375    G = 3.872983346207   f = 52.582598845962
```

Rule 3 cannot see how much of a compound there is. It sees only the proportions. The single-component case of §20.4 is the special case `i = 1`; this is the general statement, and it is the structural reason Chapter 19 rejected Gate 12: an operation whose entire content is *a small seed transmutes a large mass* asserts something about quantity, and quantity is not in the value.

20.7 The units, and the re-anchoring rule

The squaring destroys dimensional consistency. `f(n)` is quoted in hertz; `k^2 G^4` carries hertz-squared if the units are taken seriously. The source reads the squaring as a change of structural level rather than a dimensional operation, and this series adopts that reading with the badge on: it is a designed formalism, and the reading is a design decision, not a derivation.

```
RE-ANCHORING RULE (Book Two Ch 29, ES-10; binding here)

1  a Rule 3 value is labelled convention-Hz wherever it is printed
2  only RATIO operations may be performed on it
3  octave folding is a ratio operation and is therefore safe
4  it may NOT be entered into an acoustic calculation as hertz
5  publish G and T with every compound value, so it is reproducible
```

Line 5 is not bookkeeping. §20.9 will show that G is recoverable from f and T is not, so a compound value published without T has thrown away the only piece of information that narrows the decomposition at all.

20.8 The contraction of the pitch space

Because G is a geometric mean, low-index components pull it toward 1 and high-index components raise it slowly. Over all 1365 four-component multisets drawn from the twelve-state ladder:

```
G    min 1.000000  Q1 4.119534  median 5.513381  Q3 7.135243  max 12.000000
f    min 0.233700  Q1 67.305727  median 215.939206 Q3 605.751539 max 4846.012310

  full span            14.339850 octaves
  interquartile span    3.169925 octaves
  middle 50% occupies  22.11% of the full octave span
```

The extremes are reached only by the degenerate multisets — all-ones and all-twelves — and half of all compositions sit inside a 3.17-octave band. **Composition is strongly contractive on the pitch space and should be expected to cluster.** Book Two observed the same effect empirically in the source's 24 pharmacological compounds, whose G ranged only from 1.84 to 4.30.

One structural note that follows from Theorem 20.1 and is worth having in the reference: the compound span over the ladder is 14.339850 octaves and the essence span is 7.169925 octaves, exactly half. Squaring a frequency doubles its position in octaves, so the compound space is the essence space stretched by two — the same information, twice as loud, and in the wrong units.

20.8b The compound interval

Chapter 7 established that the interval between two states is a ratio of squares and that in cents it is `2400 * log2(n/m)`. The compound analogue follows directly from `f = k^2 G^4`, and it is the fourth power rather than the second:

```
interval in cents between two compounds  =  1200 * log2(f2/f1)
                                         =  4800 * log2(G2/G1)
```

k^2 cancels, exactly as k does in Chapter 4 §4.5 (iii), so **no interval between compounds depends on the chosen constant.** Worked on this chapter's table:

```
     {3x1,5x1}    vs   {2x2,7x1}         G 3.8729833462 -> 3.0365889719    -
1684.7696 cents
     {4x1,6x1,9x1} vs {3,5,7,10}       G 6.0000000000 -> 5.6924250976     -
364.4117 cents
   threat {3,5,7} vs mirror {5,7,10} G 4.7176939803 -> 7.0472987321  +2779.1450 cents
     {11x1,12x1} vs {1x1,2x1}           G 11.4891252931 -> 1.4142135624 -
14506.5459 cents
```

And the price list, which is the compound-scale version of the compression Chapter 7 §7.16 found in the transition matrix:

```
    d(cents)/dG  =  4800 / (G ln 2)

    at G =  1.5      4616.6241 cents per unit of G
```

```
at G =  2.0      3462.4681
at G =  3.0      2308.3121
at G =  4.0      1731.2340
at G =  6.0      1154.1560
at G =  8.0       865.6170
at G = 12.0       577.0780
```

A unit of geometric mean is worth eight times as much in pitch at `G = 1.5` as at `G = 12`. The same compression, from the same cause: a logarithm read against a linear coordinate.

20.8c Compounds have no pitch class

Pitch class in this book is defined by Theorem 7.1 as the odd part of the **index**. A compound has no index — it has a geometric mean, which is generally irrational — so **a compound value has no pitch class, and the six-class structure of Chapter 7 does not apply to it.**

This is not a technicality, and the folded values show why. Take the four states of the carried subject's chord and fold both readings of each into the standing 220–440 Hz band:

```
n     essence f    folded (class)      compound k^2 n^4    folded
-------------------------------------------------------------------
 3     4.350832    278.453222 (3)        18.929736       302.875769
 5    12.085643    386.740586 (5)       146.062775       292.125549
 7    23.687861    379.005775 (7)       561.114755       280.557377
10    48.342573    386.740586 (5)      2337.004393       292.125549
```

Not one of the folded compound values is any of the six class values of the ladder — 247.514, 278.453, 313.260, 379.006, 386.741, 233.978 — and states 5 and 10, which collide in class under Rule 1, also collide under Rule 3, but at a different pitch. The compound reading and the class reading are two different structures over the same states, and neither is a refinement of the other.

The consequence for practice is a prohibition and it is P7 in the specification: **a compound value may not be assigned a pitch class, and a class-based argument may not be run on one.** Chapter 10's consonance/redundancy identity is a statement about classes and therefore about Rule 1. It does not transfer.

20.9 The solve step, and why decomposition is not unique

Now the harder half of the maxim.

Solve asks: given a compound value, what was it made of? The first part of the answer is clean and the second part is the chapter's main negative result.

What is recoverable exactly. From a published `f`,

```
G = ( f / k^2 )^(1/4)
```

and nothing about that step is lossy. Worked on the compound `{3x1, 5x1}`:

```
f = 52.5825988460  ->  G = 3.8729833462
G^2 = 15.0000000000    G^3 = 58.0947501931    G^4 = 225.0000000002
```

What is not recoverable. Everything else. The composition enters the rule only through `G`, and `G` enters only through the pair `(P, T)` where `P = prod n_i^(c_i)`. Both arrows lose information.

THEOREM 20.4 (the coagula map is many-to-one). Distinct multisets over `{1..12}` with the same count `T` and the same product `P` have identical `G` and identical `f_compound`. Enumerated exhaustively:

T	multisets	distinct f	many-to-one rate	in an ambiguous fibre	
1	12	12	1.0000	0	0.00%
2	78	59	1.3220	35	44.87%
3	364	194	1.8763	257	70.60%
4	1365	504	2.7083	1145	83.88%
5	4368	1120	3.9000	3967	90.82%
6	12376	2226	5.5597	11704	94.57%

By four components, five component sets in six share their compound frequency with at least one other. By six components, nineteen in twenty do. The ambiguity is not a corner case; it is the generic case, and it grows with the size of the compound.

THEOREM 20.5 (the fibre spans counts as well). By Theorem 20.3, every multiplicity-scaling of a multiset lies in the same fibre, so the fibre over any achievable `G` is infinite once multiplicities are unbounded. Even truncated at `T <= 6` over the twelve-state ladder, the 18,563 available multisets produce only 3,779 distinct values of `G` — a mean of 4.9121 multisets per value.

The worked instance, and it is the one this book carries:

```
G = 6 is produced by SIXTY distinct multisets over {1..12} with T <= 6

 T = 1   P = 6        1 multiset    {6}
 T = 2   P = 36       3 multisets   {3,12} {4,9} {6,6}
T = 3  P = 216     5 multisets   {2,9,12} {3,6,12} {3,8,9} {4,6,9} {6,6,6}
 T = 4   P = 1296    10 multisets
 T = 5   P = 7776    15 multisets
 T = 6   P = 46656   26 multisets
                     ---
                     60
```

All sixty return `f_compound = 302.8757693527` in convention-Hz. **One of them is the carried dyad's member `A`, whose chord is `{4, 6, 9}`.** A reading that published only `A`'s compound value would have said precisely as much about `A` as about a person holding `{6,6,6}`, which is to say nothing that distinguishes them.

CONSEQUENCE 20.6 — the publication rule. A compound frequency may not be published as a description of a composition. **What must be published is the composition, with `G` and `T` derived from it — never the reverse.** The direction of the rule is the whole of it: coagula is a function and may be run; solve is a search and its output is a set.

20.10 What the solve step CAN do

The negative result is large, so the residue is worth stating exactly, since it is not zero.

Solve can falsify a proposed decomposition. If a candidate component set does not produce the published G, it is not the decomposition. Over the 1365 four-component multisets, a published G eliminates 1361 of them on average and leaves a fibre of about 2.7.

Solve can bound the components. G is a mean, so `min(n_i) <= G <= max(n_i)`. A compound with `G = 6` cannot consist entirely of components below index 6, and cannot consist entirely of components above it.

Solve can recover T when T is small and the arithmetic is tight. `P = G^T` must be an integer for a multiset over the integers. For `G = 3.8729833462`, `G^2 = 15` exactly and `G^5 = 5977.046...` is not an integer, so `T = 5` is excluded outright while `T = 2, 4, 6, ...` are not. **The integrality of `G^T` is a genuine constraint and it is free.**

Solve can be made unique by declaration. If the practitioner declares T in advance and the fibre at that T is a singleton, the decomposition is determined. At `T = 2` over the twelve-state ladder, 55.13% of multisets are alone in their fibre. That is the honest use of *solve*: **not inversion, but hypothesis testing against a declared count.**

20.10b The information accounting of the solve step

The non-uniqueness has an exact size, and stating it in bits is more useful than stating it in counts, because bits are what the rest of the series prices information in.

A decomposition drawn from the twelve-state ladder at count T is one of `C(11+T, T)` multisets. The compound value identifies one of the distinct G values available at that count. The difference is what *coagula* threw away and *solve* cannot recover:

T	multisets	log2	distinct G	log2	BITS LOST	fraction lost
2	78	6.2854	59	5.8826	0.4028	6.41%
3	364	8.5078	194	7.5999	0.9079	10.67%
4	1365	10.4147	504	8.9773	1.4374	13.80%
5	4368	12.0928	1120	10.1293	1.9635	16.24%
6	12376	13.5953	2226	11.1202	2.4750	18.21%

Two readings, and they pull in opposite directions, which is why both are printed.

The loss is real and it grows. At two components the compound value keeps 93.59% of the information in the decomposition; at six it keeps 81.79%, and the trend is monotone. Compounds get harder to invert as they get larger, which is the reverse of the intuition that more constraints narrow a search.

The loss is also smaller than the fibre counts suggest. A many-to-one rate of 5.56 at `T = 6` sounds catastrophic and costs 2.475 bits. Both numbers are correct: the fibres are large, and the value still eliminates the overwhelming majority of candidates. **G is a weak identifier and a strong filter, and Procedure SOLV-1 uses it as the second and never as the first.**

This is the same accounting Book Two Chapter 67 ran on the Yamanaka factors — a selection worth at most 37.985 bits acting on a 6.2-gigabit archive — and it is the same lesson at a much

smaller scale. A short address into a large space narrows enormously and identifies nothing.

20.10c The maxim, as a concordance

The two-word instruction is old enough and general enough to invite over-reading, so it gets the Tri-State treatment with roles stated (CANON_BIO §1).

```
CLAIM        Composition and decomposition are distinct operations with
             different formal characters, and a process that alternates
             them is doing two different kinds of work.

(+1) SOURCE  Book Two Ch 29 §29.10, the closed form; and Theorems 20.4
             and 20.5 of this chapter.
MEASURED     f = k^2 G^4 reproduces 37 of 44 published composite values
             to better than 1 part in 10^12. The inverse map is
             many-to-one at rates from 1.32 to 5.56 over T = 2..6.

(−1) SOURCE  Solve et coagula, transmitted through the Latin alchemical
             corpus and restated in Ripley's Twelve Gates (1471), where
             solution is Gate 2 and congelation Gate 6, separated by
             three intervening gates.
RECORDED     That the two operations are named separately, ordered, and
             that the sequence insists on dissolving BEFORE fixing.

ROLES        (+1) carries the evidence and carries it alone.
             (−1) carries precedent only: the observation that the two
             were distinguished, by people with no closed form and no
             way to count a fibre.

C1  structural correspondence   PASS. Two named operations; two formally
                                distinct maps, one a function and one a
                                search.
C2  predictive power            PARTIAL. The formalism predicts that
                                decomposition is under-determined and
                                says by how many bits. The tradition
                                predicts nothing testable.
C3  practical utility           PASS. On the (+1) side it produces
                                Procedure SOLV-1's halting condition. On
                                the (−1) side it explains why the corpus
                                treats dissolution as the harder step.

STATUS       ESTABLISHED as a structural correspondence; SPECULATIVE as
             any claim about what the tradition understood, and marked
             speculative here in the sentence that states it.
```

The asymmetry the maxim encodes is real and the formalism measures it. **That the tradition arrived at the asymmetry is precedent, and precedent is not evidence.**

20.11 Honesty clauses, restated in place

(1) ESSENCE, NOT RESONANCE. Every value here is a position on a logarithmic scale of surplus. Nothing vibrates, nothing resonates, and no molecule, tissue, or person is described by any number in this chapter.

(2) UNITS. Rule 3's output is convention-Hz and is dimensionally hertz-squared. Ratio operations only.

(3) k^2 IS A POSTULATE. Inherited from k, badged, and it cancels from every ratio.

(4) NO CLINICAL CLAIM. The twelve states are not diagnoses; nothing here is an assessment.

(5) CONSTRUCTED CASES. All of them, labelled.

(6) THE COUNT ARGUMENT TRAVELS. A compound reading inherits the defensibility of the decomposition it was computed from. A contested decomposition yields a contested G.

(7) REPORT D WITH delta. Where this chapter's values feed a sustainability reading, Chapter 4 Theorem 4.9 binds: D contains no delta and a D-only report cannot see decay.

The Engineering

20.12 Worked compounds

All constructed, labelled as such, drawn from the twelve-state ladder. Multiplicities are counts of components in the specification, not intensities — intensities are w, they are a different object, and Chapter 6 owns them.

multiset	T	P	G	f = k^2 G^4
{1x1, 2x1}	2	2	1.4142135624	0.9348017573
{1x3, 12x1}	4	12	1.8612097182	2.8044052718
{1x1, 12x1}	2	12	3.4641016151	33.6528632614
{2x2, 7x1}	3	28	3.0365889719	19.8702609489
{2x1,3x1,5x1,7x1}	4	210	3.8067540958	49.0770922562
{3x1, 5x1}	2	15	3.8729833462	52.5825988460
{3x1, 5x1, 7x1}	3	105	4.7176939803	115.7653513543
{4x1, 6x1, 9x1}	3	216	6.0000000000	302.8757693527
{6x2, 8x1}	3	288	6.6038544978	444.4772248248
{5x1, 10x1}	2	50	7.0710678119	584.2510982885
{9,10,11,12}	4	11880	10.4400868170	2776.3612190668
{11x1, 12x1}	2	132	11.4891252931	4071.9964546313

values are convention-Hz, dimensionally Hz^2

Three readings of this table.

The two compounds required by CANON3 §2 are rows six and four. {3x1, 5x1} gives G = 3.8730 and f = 52.5826; {2x2, 7x1} gives G = 3.0366 and f = 19.8703. Both are reproduced here to ten places so that later chapters can cite the precise values.

Row three against row two is the quantity-blindness theorem doing visible damage. `{1x1, 12x1}` and `{1x3, 12x1}` differ only in how many index-1 components are present, and they differ by 3.584963 octaves — because changing a multiplicity changes the *proportions*, which `G` does see. Contrast that with `{3x1,5x1}` against `{3x3,5x3}` in §20.6, where every multiplicity trebled and the value did not move at all. **Rule 3 is blind to scale and acutely sensitive to ratio, and a reader who does not hold both facts at once will misread every table in this chapter.**

Row eight is A's chord and it lands on a round number. `4 x 6 x 9 = 216 = 6^3`, so `G = 6` exactly. This is arithmetic coincidence and nothing else, and §20.9 already showed what it costs: sixty multisets share it.

20.13 PROCEDURE COAG-1 — running the fixing step

```
1  state the decomposition: the multiset of indices with multiplicities.
   Every index must survive Chapter 5's count argument on its own.
2  compute T = sum(c_i) and P = prod n_i^(c_i). Publish both.
3  compute G = P^(1/T) to at least ten decimal places.
4  compute f = k^2 G^4. Label it convention-Hz.
5  if the value is to be rendered, fold by 2^j into the stated band and
   publish j. Folding is a ratio operation and is the ONLY safe route to
   an audible value (§20.7 line 3).
6  publish the decomposition alongside. A compound value without its
   decomposition is not reproducible and is not a description of anything
   (Consequence 20.6).
```

20.14 PROCEDURE SOLV-1 — running the dissolving step

```
1  from the published f, compute G = (f / k^2)^(1/4) exactly.
2  DECLARE T before searching. If T is not declared, stop: the fibre is
   infinite by Theorem 20.5 and the search has no boundary.
3  compute P = G^T. If P is not an integer to within the published
   precision, the declared T is excluded. Try another.
4  enumerate all multisets over the admissible index range with product P
   and count T. This is the FIBRE.
5  publish the fibre in full. Not a member of it. The fibre.
6  if a unique member is wanted, it must be selected by evidence from
   OUTSIDE the frequency -- the actual components, elicited. The frequency
   cannot select it and no amount of precision will make it.
```

Step 5 is the step practitioners skip, and skipping it converts a set-valued result into a point-valued claim. A published decomposition chosen from a fibre without saying so is **the frequency-domain version of reporting one arm of an experiment.**

20.15 The fibre, computed, for a reader to check

The carried dyad's member A holds `{4, 6, 9}`. Run SOLV-1 on its published compound value.

```
step 1   f = 302.8757693527  ->  G = 6.0000000000
step 2   declare T = 3
```

```
step 3   P = G^3 = 216, an integer. T = 3 admissible.
step 4   fibre = { {2,9,12}, {3,6,12}, {3,8,9}, {4,6,9}, {6,6,6} }
step 5   publish all five
step 6   {4,6,9} is selected by the ELICITATION, not by the frequency
```

And the same value under other declared counts, to show what step 2 is worth:

```
T = 1   P = 6        fibre size  1     {6}
T = 2   P = 36       fibre size  3
T = 3   P = 216      fibre size  5
T = 4   P = 1296     fibre size 10
T = 5   P = 7776     fibre size 15
T = 6   P = 46656    fibre size 26
-----------------------------------
total across T <= 6               60
```

Declaring T = 3 reduces the search from sixty candidates to five. It does not reduce it to one. Chapter 19 Gate 2 recorded this as the reason it handed *solve* to this chapter, and the handover is now discharged.

20.16 When a compound reading is the wrong reading

Rule 3 is available whenever a set of components can be named, which means it is available far more often than it is appropriate. Four tests before using it.

Is the object a composition or a relationship? Two people who agree about something are not a compound of each other. Rule 2 governs a relationship and returns hertz; Rule 3 governs a composition and returns convention-Hz. Chapter 19's Finding 19.1 is the standing warning: the word *conjunction* covers both, and the values are not commensurable.

Are the multiplicities counts? If what is being counted is really an intensity — how strongly something is held — then it is `w`, it is elicited by magnitude estimation, and it does not belong in `G`. **Putting an intensity into a multiplicity is the compound-scale version of Chapter 5's smuggling a rating in as a count.**

Is the total quantity part of the claim? If it is, Rule 3 cannot carry it (Theorem 20.3).

Will the value be compared to a Rule 1 or Rule 2 value? If so, stop. The units differ, and Corollary 20.2 guarantees the two rules never meet.

The Specification

SPEC-20.1 — Inputs

```
the multiset: indices with multiplicities, each index justified by count
the ladder and its N, stated
the elicitation that produced the decomposition, cited
a declaration of whether multiplicities are counts (required) or
anything else (inadmissible)
```

SPEC-20.2 — Required outputs, coagula

```
T, P, G to >= 10 dp, f in convention-Hz to >= 10 dp
the decomposition, in full, printed with the value
the units label, on every occurrence of f
j and the band, if the value was folded
```

SPEC-20.3 — Required outputs, solve

```
G, recovered exactly
the DECLARED T, and the integrality test on G^T
the FIBRE, in full, with its size
the external evidence used to select a member, if one was selected
an explicit statement that the frequency did not select it
```

SPEC-20.4 — Prohibitions

```
P1  Rule 3's output may not be described as a generalisation of Rule 1
P2  Rule 3's output may not be entered into an acoustic calculation as Hz
P3  a compound value may not be published without its decomposition
P4  a fibre member may not be published as the decomposition
P5  a Rule 3 value may not be ratioed against a Rule 1 or Rule 2 value
P6  quantity claims may not be supported by a Rule 3 value (Thm 20.3)
P7  a compound value may not be assigned a pitch class (§20.8c)
```

SPEC-20.5 — Precision

`G` to at least ten decimal places; `f` to at least ten. The integrality test in SOLV-1 step 3 is performed against the published precision of `G`, and a value published to fewer than ten places cannot support the test. Recompute from the full-precision constants, never from rounded prints (SPINE §3.4).

SPEC-20.6 — Gates

```
G1  units labelled everywhere                                  REQUIRED
G2  decomposition published with every compound value          REQUIRED
G3  fibre published with every solve result                    REQUIRED
G4  T declared before any search                               REQUIRED
G5  the non-generalisation statement carried wherever both
    rules appear in the same document                          REQUIRED
G6  all cases labelled constructed                             REQUIRED
```

The Failure Modes

FM-20.1 — Treating Rule 3 as the general case of Rule 1

Signature: “the essence law is the one-component case of the compound law.”

Why it is wrong: Theorem 20.1. The one-component case of the compound law returns `f(n)^2`, not `f(n)`. Corollary 20.2: there is no index at which they agree. At `n = 12` the two differ by 6.121 octaves.

The fix: state the two rules as answering different questions, every time both appear.

FM-20.2 — The O2 error, in its general form

Signature: a published composite value that no multiplicity of its stated components can produce, retained because it "came from the source."

Why it is wrong: §20.4. For a single-component species the rule permits exactly one value. A published number outside the rule's range is an erratum, and the scatter of the seven outliers across four orders of magnitude confirms data entry rather than a different law.

The fix: recompute every inherited composite before citing it, and publish the recomputation.

FM-20.3 — Reading a compound value as a description

Signature: "their compound frequency is 302.88, which tells us they are composed of `{4,6,9}`."

Why it is wrong: Theorem 20.4 and §20.15. Sixty multisets give that value.

The fix: Consequence 20.6. Publish the composition; derive the value from it; never the reverse.

FM-20.4 — Solving without declaring T

Signature: a fibre enumerated over all counts, or a single decomposition produced from `f` alone.

Why it is wrong: Theorem 20.5. Without a declared T the fibre is infinite, because every multiplicity-scaling is in it.

The fix: SOLV-1 step 2, which halts.

FM-20.5 — Putting intensities into multiplicities

Signature: a component "held strongly" entered as `n x 3`.

Why it is wrong: multiplicities are counts of components in a specification. Intensities are `w`, they are elicited by magnitude estimation (SPINE §1.1), and their scale status is different. Mixing them produces a `G` that is neither.

The fix: §20.16 test two. If the number came from an intensity elicitation it goes in `u`, not in `G`.

FM-20.6 — Using a compound value to support a claim about amount

Signature: "the value is high, therefore there is a great deal of it."

Why it is wrong: Theorem 20.3. `{3x1,5x1}` and `{3x5,5x5}` are the identical value.

The fix: if amount is the claim, Rule 3 is not the instrument. Nothing in this book measures amount.

FM-20.7 — Rendering a Rule 3 value as hertz

Signature: 52.5826 played as a tone.

Why it is wrong: the value is dimensionally hertz-squared and its Hz label is a convention (§20.7).

The fix: fold first, publish `j`, and label the result as a re-anchored convention value. Chapter 26 carries this into the rendering protocol.

FM-20.8 — Precision theatre on a contested decomposition

Signature: `G` to twelve places over a decomposition two experts disagree about.

Why it is wrong: honesty clause (6). The value inherits the defensibility of the count. Twelve places on a contested multiset is precision about the wrong thing.

The fix: publish the disagreement, compute both decompositions, and report the octave gap between them as the uncertainty.

FM-20.9 — Comparing compounds across ladders

Signature: a `G` from a twelve-state ladder set beside a `G` from an eight-category scheme.

Why it is wrong: the indices mean different things, so the geometric means are not comparable, and `k^2` does not save it because the inputs differ rather than the constant.

The fix: the ladder and its `N` are published with every value (SPEC-20.1) and comparisons across ladders are refused.

FM-20.10 — Reading the maxim as symmetric

Signature: "dissolve and fix are inverse operations, so the procedure reverses."

Why it is wrong: this chapter's central asymmetry. Coagula is a function with a closed form; solve is a search whose output is a set, and by four components 83.88% of inputs land in a fibre of size two or more.

The fix: say *fix, then search*, not *fix, then unfix*.

FM-20.11 — Assigning a pitch class to a compound

Signature: `{3x1,5x1}` folded to 420.660791 Hz and reported as "close to class 5."

Why it is wrong: §20.8c. Pitch class is the odd part of an index and a compound has no index. The folded compound values of the carried subject's four states land on none of the ladder's six class values, and the resemblance of any one of them to a class value is arithmetic accident.

The fix: fold for rendering, never for classification, and carry P7 wherever a compound value is folded.

FM-20.12 — Inverting a fibre by choosing the most plausible member

Signature: the fibre is enumerated, one member "obviously fits," and the others are dropped without a word.

Why it is wrong: plausibility is external evidence, and external evidence must be named. SOLV-1 step 6 requires the selection criterion to be published; a criterion applied silently is indistinguishable from a preference.

The fix: publish the fibre, publish the criterion, and publish which member the criterion selects. If no criterion exists, the result is the fibre and the report says so.

FM-20.13 — Presenting a compound reading as a description of a person

Signature: a `G` computed over someone's held states and reported as what they are.

Why it is wrong: SPINE §5. The twelve states are not diagnoses, no reading is an assessment, and Theorem 20.4 means the value does not even identify the states uniquely.

The fix: the reading is of a decomposition, the decomposition is constructed, and the sentence that reports it says so.

Worked Example

PART A — THE CARRIED SUBJECT, FIXED

Constructed case, labelled as such. SPINE §3.1's subject holds three framings of one situation, spanning the states `{3, 5, 7, 10}`. Run COAG-1 on the chord, taking each held state as one component.

```
 step 1   multiset {3x1, 5x1, 7x1, 10x1}; ladder N = 12; states from Ch 5
 step 2   T = 4     P = 3 x 5 x 7 x 10 = 1050
 step 3   G = 1050^(1/4) = 5.6924250976
step 4  f = k^2 * G^4 = 245.3854612812  (convention-Hz, dimensionally Hz^2)
 step 5   folded: 245.3854612812 * 2^0 = 245.385461, j = 0, band 220-440
 step 6   decomposition published above
```

And the three framings taken separately, which is the more informative reading:

```
framing    states      T   P     G              f (convention-Hz)
-------------------------------------------------------------------
threat     {3,5,7}     3   105   4.7176939803   115.7653513543
story      {3,5,10}    3   150   5.3132928459   186.2578308452
mirror     {5,7,10}    3   350   7.0472987321   576.4348833846
-------------------------------------------------------------------
whole      {3,5,7,10}  4  1050   5.6924250976   245.3854612812
```

The whole is not the sum, the mean, or the midpoint of the parts. `245.3855` sits between `threat` and `mirror` but not at their geometric centre, because `G` is weighted by count and the

whole has four components where each framing has three. This is the arithmetic behind Chapter 9's warning that a chord is not the average of its notes.

Now the comparison that matters, and it is a refusal. SPINE §3.1 also publishes the subject's internal surplus and the chord's width as pitch:

```
   Rule 2 reading  mean u_int = 0.700844  F_int = 2.164390  f = 36.6686325914 Hz
   Rule 3 reading  G = 5.6924250976                  f = 245.3854612812 conv-
Hz
```

These two numbers describe the same person and may not be compared, ratioed, averaged, or plotted on one axis. One is hertz and one is hertz-squared; one is a function of how non-parallel the framings are and one is a function of which states are held, with no reference to how they overlap. A reader who computes `log2(245.3855 / 36.6686) = 2.742` has produced a number with no referent, and it is printed here only so that it is recognisable as the mistake it is.

PART B — THE CARRIED DYAD, DISSOLVED

Constructed case, labelled as such. SPINE §3.2, member `A = {4:0.7, 6:0.5, 9:0.4}`. The intensities are `w` and do not enter `G` (FM-20.5); the components are the states `{4, 6, 9}`.

```
   COAG-1    T = 3    P = 216    G = 6.0000000000    f = 302.8757693527 conv-Hz
```

Run SOLV-1 against that value, as an independent party would who had only the number.

```
    step 1   G = (302.8757693527 / 0.233700439315386)^(1/4) = 6.0000000000
    step 2   declare T = 3
    step 3   P = 216, integer. Admissible.
    step 4   fibre:
                {2, 9, 12}   {3, 6, 12}   {3, 8, 9}   {4, 6, 9}   {6, 6, 6}
    step 5   fibre size 5, published in full
    step 6   no external evidence supplied -> NO MEMBER SELECTED
```

Read the five candidates against Chapter 5's ladder and the extent of the loss becomes concrete. `{4, 6, 9}` is situated feeling, temporal feeling, normative feeling. `{6, 6, 6}` is one state held three times. `{2, 9, 12}` reaches from valenced activation to committed feeling. These are not variations on a theme. **They are five different people, and Rule 3 returns one number for all of them.**

The pitch-class reading, which Rule 3 also cannot see, separates them immediately:

```
    {4, 6, 9}    classes {1, 3, 9}    three classes
    {2, 9, 12}   classes {1, 9, 3}    three classes
   {3, 6, 12}  classes {3}       ONE class -- a single note played three times
    {3, 8, 9}    classes {3, 1, 9}    three classes
    {6, 6, 6}    classes {3}          ONE class, and one state
```

Two of the five are entirely consonant and therefore entirely redundant — Chapter 10's identity — and the compound value cannot distinguish them from the three that are not. That is the strongest practical statement of the non-uniqueness in this chapter: **the solve step is blind to exactly the structure Part II of this book was written to expose.**

PART C — THE CONJUNCTION, PRICED IN BOTH NOTATIONS

Chapter 19's Gate 4 halted a schedule because *conjunction* is ambiguous. With Rule 3 now owned, the halt can be resolved by declaration rather than by choice.

```
reading (i)  RULE 2, the relational join
             u = 0.068734665264634    f_rel = 1.4907987150 Hz
             F = 0.563084802746       D = +4.338420   delta = 0.09
             Q_inf = -5.011940        DECAYS

reading (ii) RULE 3, the compositional join
             pooled multiset {4x2, 6x2, 9x2, 10x1}
             T = 7    P = 466560    G = 6.4542224391
             f = 405.5414451025 convention-Hz
```

Both are correct answers to different questions. Reading (i) says the pair's difference supports 0.563085 nats of surplus and that this is 4.338420 octaves short of what their cost ratio requires, at a loss rate of 0.09. Reading (ii) says the pooled specification has a count-weighted geometric mean of 6.4542224391 and nothing else at all — **it does not know that A and B are two people, does not know they overlap on three states out of four, and would return the identical value if one person held all seven components alone.**

```
THE DECLARATION RULE, and it discharges Gate 4:

joining PEOPLE                    -> Rule 2, hertz, u
joining COMPONENT SETS            -> Rule 3, convention-Hz, G
the two may both be reported
the two may never be compared
```

PART D — THE REPORT CARD

```
    ------------------------------------------------------------------------
----
    COMPOUND READING -- carried subject, chord {3,5,7,10}
    ------------------------------------------------------------------------
----
    decomposition       {3x1, 5x1, 7x1, 10x1}, ladder N = 12, Ch 5 indices
    T                   4
    P                   1050
    G                   5.6924250976
    f_compound          245.3854612812   CONVENTION-Hz, dimensionally Hz^2
    fold                j = 0, band 220-440 Hz
    solve fibre at T=4  { {3,5,7,10}, {5,5,6,7} }   size 2
    T excluded          T = 5 (G^5 = 5977.046... not an integer)
    ------------------------------------------------------------------------
----
    NOT comparable to the Rule 2 reading of the same subject (36.6686 Hz).
    NOT a measure of quantity (Theorem 20.3).
    NOT a description of the subject (Consequence 20.6; fibre size 2).
```

```
    ESSENCE frequencies, not physical resonances. Constructed case.
    Not a diagnosis, not an assessment, not clinical.
     ------------------------------------------------------------------
----
```

The last line of the fibre row is the chapter in miniature. Even at the tightest available count, with the value known to ten decimal places, **the twelve-state ladder offers a second component set — `{5, 5, 6, 7}` — that fixes to exactly the same number.** *Coagula* ran in four steps and produced a value. *Solve* ran in six and produced a set of two, and the honest report prints the set.

Chapter 21 takes the other operation Chapter 19 handed forward: moving a state from one index to another. It prices the interval, states what the move costs, and establishes the constraint that bounds every claim in Part IV — that nothing not already latent can be reached, at any strength, by any operation in this book.

Chapter 21 . Transmutation as Index Change

The word carries more freight than any other in the old corpus, and the freight is what this chapter unloads. Chapter 19 rejected Gate 12 — projection, the transmutation of a mass by a seed — on four separate grounds, and rejected Gate 10, exaltation, because *nobler* is not a coordinate. What survives after those rejections is small, exact, and worth having.

A state can be moved from one index to another. That is all transmutation is in this formalism, and this chapter holds its title to exactly that.

Moving a state from index `m` to index `n` means changing the number of distinguishable components required to specify it. It has a price in pitch that can be computed to four decimal places, a price in components that can be counted, and a price in the storage law that has to be declared because the formalism does not supply it. It has a direction, and the direction matters. And it has a hard outer boundary that no operation in this book crosses at any strength, by any modality, ever: **nothing that is not already latent can be reached.**

That boundary is inherited, not invented here. Book Two Chapter 67 stated it as CONSTRAINT L67 for the cellular architecture and called it the most valuable sentence in that volume. This chapter restates it at psychological scope and treats it as binding on everything in Part IV. The reason it is the most valuable sentence, rather than the most limiting one, is worth having in advance: the property that makes an addressing architecture safe is the same property that makes it limited, and **a design claiming both unlimited reach and no modification would be incoherent.**

Three things before the arithmetic.

Re-indexing is not creation. Chapter 5's index is a count of components in a specification. To move a state up the ladder is to specify it more finely; the components that make the finer specification possible must already be available to be counted. **Yamanaka reprogramming activates latent capability; it does not create new capability**, and the same accounting applies here.

Re-indexing is not improvement. Chapter 5 §5.20 settled that more components is not better, and Chapter 10 settled that a fully consonant self is a fully redundant one. Nothing in this chapter licenses reading a higher index as a better state.

Nothing here is clinical. The twelve states are not diagnoses, no reading is an assessment of anyone's health, no operation below is a treatment, and every case is constructed and labelled as such. The frequencies are essence frequencies — positions on a logarithmic scale of surplus — and nothing is claimed to vibrate.

The Law

21.1 What a re-indexing is

DEFINITION 21.1. A *re-indexing* is a change in the index assigned to a state, from `m` to `n`, where both are positions on a single fixed ladder and the change reflects a change in the number of distinguishable components required to specify the state.

Three things are excluded by that wording and each exclusion is load-bearing.

A change of ladder is not a re-indexing. If `N` changes, or if the categories are re-cut, the indices before and after are not comparable and no interval between them is meaningful. That operation is Chapter 4's lever L4 and Chapter 19's Gate 3, and it has its own price.

A change of register is not a re-indexing. Multiplying by `2^j` moves the published frequency and moves nothing else (Chapter 7 §7.2). Chapter 19 Gate 8 rejected sublimation for exactly this confusion.

A re-labelling is not a re-indexing. If the same specification is given a different number because the analyst preferred it, the index moved and the state did not. Chapter 5 §5.22's rule binds: publish the decomposition or the frequency means nothing.

21.2 The interval crossed

THEOREM 21.2. The interval crossed by a re-indexing from `m` to `n` is a ratio of squares:

```
f(n) / f(m)  =  (k n^2) / (k m^2)  =  n^2 / m^2

and in the reverse direction        =  m^2 / n^2

in cents                          =  1200 * log2(n^2/m^2)  =  2400 * log2(n/m)
```

`k` cancels. **The price of a re-indexing does not depend on the chosen constant.** No verdict about a move depends on the reference pitch. This is Chapter 4 §4.5 (iii) applied at index scope.

Two properties of the interval that practice depends on.

It is path-independent. The interval from `m` to `n` is a difference of logarithms, so a move through intermediate indices costs the same in pitch as the direct move. Going `3 -> 6 -> 12` costs `2400.0000 + 2400.0000 = 4800.0000` cents, and going `3 -> 12` directly costs `4800.0000`. Chapter 4 §4.13's warning about path-dependent attribution applies to the cost side of a deficit and not to this.

It is antisymmetric. `2400 log2(n/m) = −2400 log2(m/n)`. Chapter 7 §7.16's transition matrix is the full table and is not re-derived here; the reverse of any entry is its negative.

The reference values this chapter uses most, from that matrix:

```
from -> to      ratio n^2/m^2     reverse m^2/n^2      cents
------------------------------------------------------------
 1 ->  2          4.000000          0.250000        +2400.0000
```

```
 3 ->  6          4.000000          0.250000        +2400.0000
 3 ->  4          1.777778          0.562500         +996.0900
 5 -> 10          4.000000          0.250000        +2400.0000
 9 -> 12          1.777778          0.562500         +996.0900
11 -> 12          1.190083          0.840278         +301.2741
 1 -> 12        144.000000          0.006944        +8603.9100
```

21.3 Two kinds of move, and the arithmetic that separates them

Chapter 7 owns the distinction and this chapter uses it. Two indices share a pitch class if and only if they share an odd part (Theorem 7.1). Therefore:

```
RE-INDEXING WITHIN AN ORBIT      m and n share an odd part
                                interval is a whole even number of octaves
                                   pitch class UNCHANGED
                                   the book's term: INTENSIFICATION

RE-INDEXING ACROSS ORBITS        m and n have different odd parts
                                   pitch class CHANGES
                                   the book's term: TRANSFORMATION
```

Counted over the ordered moves available on the twelve-state ladder:

```
ordered pairs (m != n)                    132
class-preserving (intensification)         20      15.1515%
class-changing   (transformation)         112      84.8485%
```

Transformation is the generic move and intensification is the rare, highly structured one. Chapter 7 §7.9 gives the six orbits; the operative fact for this chapter is which states have a double still on the ladder:

```
states with 2m <= 12 (can be intensified without leaving the ladder):  1,2,3,4,5,6
states with 2m >  12 (cannot):                                         7,8,9,10,11,12
```

Exactly half the ladder can be intensified and half cannot, and the half that cannot is an artefact of `N = 12` rather than a property of those states. Chapter 7 named this truncation and this chapter prices it: for a state at index 7 or above, every available re-indexing is a transformation, because its orbit has no higher member inside the ladder.

21.4 CONSTRAINT T21 — the hard constraint

> **CONSTRAINT T21. A re-indexing can reach only what is already latent. No operation in this book, of any kind, at any strength, by any modality, can produce a component that the decomposition does not already contain.**

This is Book Two Chapter 67's CONSTRAINT L67, restated at psychological scope, and it requires no new assumption. **An index is a count of components. A count cannot be raised by counting harder.** To move a state from index 5 to index 6 is to add a projected duration to a specification that previously had none, and either the material for that component is available to the person or it is not. The operation addresses; it does not import.

The constraint does four kinds of work, and they are the same four Book Two named.

It bounds the claim space. Any statement of the form "this procedure could produce capability X" is false whenever X is not already available and reachable. That eliminates the entire class of claims that would make this work pseudoscience: nothing conferred, nothing imported, no capacity created.

It makes the model falsifiable in a specific way. If a demonstration showed a state specified by a component demonstrably absent from the person's repertoire, following an addressing-only intervention, this architecture is wrong and no adjustment of parameters repairs it.

It generates a positive prediction. If re-indexing reaches only what is latent, the reachable set is bounded and enumerable in principle. §21.6 computes it for a constructed repertoire.

It supplies the audit rule. Every proposed move is checked against the repertoire before it is priced, and Procedure TR-1 puts that check before the arithmetic rather than after.

The (+1) evidence, cited exactly and with roles stated (CANON_BIO §1):

Gurdon (*J. Embryol. Exp. Morphol.* 10:622-640, 1962) transplanted nuclei from differentiated intestinal epithelium of feeding *Xenopus* tadpoles into enucleated eggs and obtained normal feeding tadpoles. Differentiation is not subtraction: **the nucleus of a differentiated cell retains the full information required to build the organism.**

Takahashi & Yamanaka (*Cell* 126:663-676, 2006) returned mouse fibroblasts to a pluripotent state by forced expression of four transcription factors; **Takahashi et al. (*Cell* 131:861-872, 2007)** extended it to human fibroblasts. The 2012 Nobel Prize in Physiology or Medicine went jointly to Gurdon and Yamanaka.

And the accounting that makes the point quantitative rather than rhetorical, recomputed from Book Two Chapter 67:

```
human sequence-specific transcription factors, approx.   1600
    (Lambert et al., Cell 172:650-665, 2018)

selecting 4 of 1600:  C = 2.720438e+11  ->  37.985 bits
the genome acted upon:                      6.200e+09 bits
ratio                                       1.6322e+08 : 1
```

The instruction is one hundred and sixty-three million times smaller than the thing it reorganises, and **that is what an address is.** An activation signal is not the content it activates. The correspondence drawn here is structural — latency-then-expression as a shared architecture — and the (−1) column, the tradition's own insistence that the material must already contain what the work brings out, carries precedent and no evidence.

The extension of this accounting to psychological states is the author's, badged as a thesis, and it is not established by the cited work. Gurdon and Yamanaka measured cells. What they license here is the *form* of the constraint, not a claim about minds.

21.5 Latency, and the laundering it invites

"Latent" is doing heavy work and it is the word most easily abused, so its failure is named before its use. Book Two Chapter 67 named the chain:

```
the repertoire contains latent capability
```

```
–> therefore capability X is latent
  –> therefore X is available
```

Each arrow smuggles. The first step is a statement about bulk; the second names a specific item without evidence that the item is in the bulk; the third converts possible into reachable. **"Latent" means present in the repertoire and addressable, not merely conceivable.** A component that a person could in principle acquire, given years and circumstances, is not latent in the sense this constraint uses.

Operationally, and this is the rule Procedure TR-1 enforces: **a component counts as latent only if it has been observed in the repertoire at least once**, in some state, on some occasion. That is a strict criterion, it will exclude things that are in fact latent, and the exclusion is the correct direction of error for a constraint whose purpose is to bound a claim.

21.6 The reachable set

Given a repertoire, the reachable set is the set of chords the states in it can form. It is finite and it is countable.

Constructed example, labelled as such. A repertoire containing components sufficient to specify states `{3, 4, 5, 6, 7, 9, 10}` and not states `{1, 2, 8, 11, 12}`:

```
full chord space over 12 states       2^12 – 1  =  4095 chords
reachable from this repertoire        2^7  – 1  =   127 chords
fraction of the space                                3.101343%

four–note chords over 12 states       C(12,4)   =   495
four–note chords reachable            C(7,4)    =    35
fraction                                             7.0707%
```

Ninety-seven per cent of the nominal chord space is unreachable for this constructed repertoire. No operation in Part IV changes that. Widening the repertoire is not an operation in Part IV; it is what Chapters 25 to 27 examine, under the open problem, with no established mechanism attached.

A second constructed repertoire, narrower, to show the scaling: with only five states available, the four-note reachable set falls from 495 to 5 — **a hundred-fold reduction from a repertoire loss of seven states.** The reachable set collapses combinatorially, and a reading that reports a chord without reporting the repertoire it was drawn from has reported the numerator of a fraction.

21.7 What re-indexing does NOT move — and this is the chapter's central finding

Here is the result that most constrains what transmutation can be claimed to do, and it was not designed in. It falls out of the definition of u.

THEOREM 21.3. u is invariant under any re-indexing that does not change the shared support of the vectors involved. The interaction parameter is `u = 1 − cos^2 theta` computed on unit vectors over the twelve coordinates; the index *labels* never enter the computation. **Re-indexing changes `u` only through the overlap it destroys at `m` and the overlap it creates at `n`.**

Computed on SPINE §3.1's subject, **constructed and labelled as such**. Take the `threat` framing `{3:0.8, 5:0.4, 7:0.2}` and move its index-3 component four different ways, holding `story`

and `mirror` fixed:

```
    move        lattice interval    mean u_int       F_int       chord width
     ------------------------------------------------------------------------
-------
    baseline          --          0.700844041320  2.164390    36.6686325914 Hz
    3 ->  6     +2400.0000 cents   0.870156210632  2.358182    54.0282536493 Hz
    3 ->  4      +996.0900 cents   0.870156210632  2.358182    54.0282536493 Hz
    3 -> 12     +4800.0000 cents   0.870156210632  2.358182    54.0282536493 Hz
    3 ->  1     -3803.9100 cents   0.870156210632  2.358182    54.0282536493 Hz
     ------------------------------------------------------------------------
-------
```

Four moves spanning 8603.9100 cents of lattice distance return the identical relationship reading to twelve decimal places — one of them upward by four octaves, one of them downward by more than three. They are identical because all four vacate coordinate 3, which `story` occupies, and all four land on a coordinate that neither `story` nor `mirror` occupies. The relationship arithmetic sees a lost overlap and nothing else.

Two moves that are not identical, for contrast:

```
    3 -> 10  (a coordinate `story` and `mirror` BOTH occupy)
             mean u_int = 0.590503185401   F_int = 2.014308   f = 27.1603 Hz

    3 ->  5  (merged onto the framing's own state 5)
             mean u_int = 0.697749790607   F_int = 2.160474   f = 36.3826 Hz
```

FINDING 21.1. The lattice and the interaction parameter measure different things and neither refines the other. A re-indexing that is an intensification on the lattice and a re-indexing that is a transformation across orbits can produce numerically identical relationship readings; and two moves of nearly identical lattice distance can produce very different ones. **Transmutation moves the index; it moves the relationship only where it moves the overlap. A chapter that claimed otherwise would be claiming that renaming a coordinate changes a geometry.**

The practical rule that follows, and Procedure TR-1 enforces it: **report the lattice reading and the relationship reading separately, and never infer either from the other.**

21.8 The cost, in three currencies

A re-indexing has three prices and the formalism supplies two of them.

Price one — components. Supplied exactly. Moving from `m` to `n` requires `n - m` additional distinguishable components in the specification, each of which must survive Chapter 5's count argument on its own and must be latent by §21.5's criterion.

Price two — pitch. Supplied exactly. `2400 log2(n/m)` cents, path-independent, k-free.

Price three — maintenance. NOT supplied. It must be declared. A more finely specified state is a larger structure to hold, and holding it costs something per period. That cost is `c` in the storage law, and **nothing in this formalism derives `c` from `n`.** There is no theorem here and the book does not pretend to one. What the book supplies is the conversion, once `c` is declared:

```
    dD/dc  =  2.885390 / eta
```

```
eta = 0.30   ->   9.617967 octaves of deficit per unit of c
eta = 0.34   ->   8.486441
eta = 0.40   ->   7.213475
eta = 0.44   ->   6.557705
eta = 0.60   ->   4.808983
eta = 0.90   ->   3.205989
```

A declared maintenance cost is a real input and an undeclared one is a hidden liability, and a re-index priced only in pitch has been priced in the currency that does not appear in the sustainability condition.

21.9 Direction, and the asymmetry the notation hides

The transition matrix is antisymmetric, so de-indexing looks exactly as available as indexing. It is not, and Chapter 19 Gate 1 flagged this in passing. It is stated properly here.

Indexing upward requires latent material and an operation. Downward movement in fact happens without an operation — it is `delta`, the loss rate, running unasked. A person who has modelled another mind cannot elect to un-model it; what removes the component is time, disuse, and forgetting.

```
upward    an OPERATION on a latent repertoire; costs components and c
downward  in the formalism, an operation; in fact, mostly delta
```

The symmetry of the matrix is a property of the notation, not a menu, and a procedure that schedules a de-indexing as though it were a move one performs has mistaken the arithmetic for the world. Where deliberate de-indexing is genuinely available — simplifying a stated specification, dropping a distinction from a working vocabulary — it is available at the level of the *specification* and not at the level of what a person can already distinguish.

21.10 Honesty clauses, restated in place

(1) ESSENCE, NOT RESONANCE. Every frequency here is a position on a logarithmic scale of surplus. Nothing vibrates. No person, tissue, or molecule resonates at any value in this chapter.

(2) NO CLINICAL CLAIM. The twelve states are not diagnoses, no reading is an assessment, no operation is a treatment, and nothing here describes or prescribes for any person's health.

(3) THE INDEX INHERITS THE COUNT. A contested decomposition yields a contested frequency, and a re-indexing between two contested indices yields a contested interval.

(4) CONSTRAINT T21 IS BINDING on every operation in Part IV, and its extension from cells to psychological states is the author's thesis, badged, and not established by the cited work.

(5) MORE IS NOT BETTER. Chapter 5 §5.20 and Chapter 10. A higher index is a finer specification and nothing else.

(6) CONSTRUCTED CASES, all of them, labelled.

(7) REPORT `D` WITH `delta`. Chapter 4 Theorem 4.9: `D` contains no `delta`, so a `D`-only report cannot see decay. Both, every time.

(8) CONSENT IS STRUCTURAL. Nothing in this chapter licenses acting on anyone. Chapter 18 owns intervention and its constraints, and this chapter adds no permissions whatever.

The Engineering

21.11 The price list

The single-rung price list, which is the ladder read as a purchase schedule:

```
step        cents          cumulative from n = 1
------------------------------------------------
 1 ->  2   2400.0000        2400.0000
 2 ->  3   1403.9100        3803.9100
 3 ->  4    996.0900        4800.0000
 4 ->  5    772.6274        5572.6274
 5 ->  6    631.2826        6203.9100
 6 ->  7    533.7418        6737.6518
 7 ->  8    462.3482        7200.0000
 8 ->  9    407.8200        7607.8200
 9 -> 10    364.8074        7972.6274
10 -> 11    330.0085        8302.6359
11 -> 12    301.2741        8603.9100
------------------------------------------------
total 1 -> 12 = 8603.9100 cents = 7.169925 octaves
```

The same one-component step is worth 2400 cents at the bottom of the ladder and 301.2741 cents at the top — **a factor of 7.97.** Chapter 7 §7.16 called this the square law's signature; here it is a price list, and it has a direct practical reading: **differentiation is cheap in pitch at low index and expensive at high index, so the same pitch gain buys progressively more components as you climb.**

Cost per component for the intensification moves, which is where the compression bites hardest:

```
move       components added    cents      cents per component
-------------------------------------------------------------
 1 ->  2          1          2400.0000        2400.0000
 2 ->  4          2          2400.0000        1200.0000
 3 ->  6          3          2400.0000         800.0000
 4 ->  8          4          2400.0000         600.0000
 5 -> 10          5          2400.0000         480.0000
 6 -> 12          6          2400.0000         400.0000
-------------------------------------------------------------
```

Every one of the six doublings costs exactly 2400 cents, and they cost between one and six components to buy. An octave leap at the bottom of the ladder is a single component; the same leap at index 6 is six components. This is the arithmetic content of Chapter 7 §7.13's observation that intensification is scarce, and it is why the scarcity is structural rather than motivational.

21.12 PROCEDURE TR-1 — auditing and pricing a re-indexing

```
 1  STATE the ladder and its N. State the starting index m and its
    decomposition, component by component.

 2  STATE the target index n and its decomposition, component by
    component. The two decompositions must be comparable.

 3  LATENCY CHECK, before any arithmetic. For every component present in
    n's decomposition and absent from m's, record whether it has been
    observed in the repertoire at least once, in some state, on some
    occasion (§21.5). If any required component fails, HALT.
    The move is REFUSED under CONSTRAINT T21 and the refusal is published
    with the component named.

 4  CLASSIFY. Compute the odd parts of m and n. Same odd part ->
    INTENSIFICATION, whole even octaves, class preserved. Different ->
    TRANSFORMATION, class changes. Report which.

 5  PRICE IN PITCH. 2400 * log2(n/m) cents. Report to four decimals with
    the direction stated.

 6  PRICE IN COMPONENTS. n - m, listed by name, not by count alone.

 7  DECLARE THE MAINTENANCE COST. State dc, state how it was obtained, and
    state that the formalism did not supply it. Convert with
    dD/dc = 2.885390/eta.

 8  COMPUTE THE RELATIONSHIP EFFECT SEPARATELY. Recompute u from the
    vectors. Do NOT infer it from the lattice interval (Theorem 21.3).

 9  READ THE DEFICIT. D and delta together, with Q_inf.

10 PUBLISH the lattice reading and the relationship reading side by side,
    labelled, with no ratio taken between them.
```

Step 3 is placed before step 4 deliberately. **A move that cannot be made does not need a price**, and computing the price first invites the price to argue with the constraint.

21.13 Re-indexing against the four levers

Chapter 4 §4.8 fixed four levers on the deficit and proved there is no fifth. Re-indexing is not one of them, and the relationship is worth setting out because the two are constantly confused.

LEVER	what moves	re-indexing's relation to it
L1 raise u	the geometry	INDIRECT. Only when the move changes shared support (Thm 21.3)

```
    L2 raise eta     the conversion           NONE
    L3 cut c         the cost                 ADVERSE, usually. A finer
                                            specification costs more to hold
    L4 raise N       the category set         NONE. Different operation
                                              (Ch 19 Gate 3)
    ------------------------------------------------------------------
----
```

Read straight down, that table says most re-indexing is deficit-neutral and cost-positive. A move that leaves the shared support alone does not touch u, does not touch `eta`, does not touch `N`, and adds maintenance. It buys pitch on the lattice and pays for it in the storage law.

The break-even, computed, so that the claim is not merely gloomy:

```
    a re-index that raises c by dc must raise u enough to offset
    2.885390 * dc / eta octaves

   eta = 0.30, u0 = 0.068735, dc = 0.05  ->  u must reach 0.097688   reachable
   eta = 0.30, u0 = 0.068735, dc = 0.10  ->  u must reach 0.131892   reachable
   eta = 0.30, u0 = 0.692373, dc = 0.05  ->  u must reach 0.834429   reachable
  eta = 0.30, u0 = 0.692373, dc = 0.10  ->  u must reach 1.002249  UNREACHABLE
   eta = 0.44, u0 = 0.692373, dc = 0.10  ->  u must reach 0.892245   reachable
```

At a low u the break-even is easy and at a high u it can exceed 1, which is impossible. The mechanism is Paper A's strict concavity: the marginal value of u falls as u rises, from −31.739291 octaves per unit at `u = 0` to −2.644941 at `u = 1`, a factor of 12. **A relation that is already differentiated cannot pay for expensive re-indexing out of further differentiation**, and the row where u must reach 1.002249 is the formalism refusing rather than warning.

21.14 The three counterfeits

A re-indexing can be claimed without occurring, in three ways, and each has a test.

Index inflation. Components are added to the specification that are not genuinely distinguishable from ones already present. The index rises and nothing has moved. This is pseudo-diversity (Book Two Ch 11) at index scope, and the test is Chapter 13's collision test: components that always co-occur are one component.

Latency laundering. A component is asserted to be latent because it is conceivable. The test is §21.5's criterion — observed at least once, in some state, on some occasion — and it is deliberately strict.

Register change. The published frequency moves because `j` moved. The test is Chapter 7 §7.2: folding preserves every interval in cents exactly, so if the intervals to every other state are unchanged, nothing happened. Chapter 19 Gate 8 is the historical version of this counterfeit and it was rejected in print.

All three produce a number that has moved and a state that has not, and **all three are caught by publishing the decomposition rather than the index.**

21.15 What re-indexing is genuinely good for

The chapter has spent most of its length on limits, so the residue is stated plainly, because it is not small.

It prices a change that would otherwise be described only in adjectives. A person's account of a situation acquiring a projected duration is a move from index 3 to index 6: 2400.0000 cents, three components, class preserved, an intensification. That is a great deal more than "it got deeper."

It separates two events that feel alike from inside. Chapter 7's whole argument, now with a direction and a price attached.

It makes a refusal computable. Step 3 of TR-1 returns REFUSED with a named missing component, and a refusal with a name is actionable in a way that a vague sense of impossibility is not.

It exposes the cost side. Most re-indexing is cost-positive and deficit-neutral, which is invisible to any account that treats becoming more finely specified as self-evidently good.

The Specification

SPEC-21.1 — Inputs

```
the ladder, its N, and its source (Ch 5 for this book's twelve states)
m and its full decomposition, component by component, named
n and its full decomposition, component by component, named
the repertoire, with the observation record for each component
eta, c, delta if a deficit reading is wanted
the vectors of all parties, if a relationship reading is wanted
```

SPEC-21.2 — Required outputs

```
the latency verdict, per required component, PASS or REFUSED
the classification: INTENSIFICATION or TRANSFORMATION, with odd parts
the interval in cents to four decimals, with direction
the components added or removed, by name
the declared maintenance cost dc, with its provenance and the statement
    that the formalism did not supply it
the deficit D and the loss rate delta, together, with Q_inf
the relationship reading, computed from vectors, reported SEPARATELY
```

SPEC-21.3 — Prohibitions

```
P1  a re-indexing may not be reported without its decompositions
P2  a relationship effect may not be inferred from a lattice interval,
    or the reverse (Theorem 21.3)
P3  a move may not be priced before the latency check (TR-1 step 3)
```

```
P4  "latent" may not mean "conceivable" (§21.5)
P5  a higher index may not be described as a better state
P6  a register change may not be reported as a re-indexing
P7  a maintenance cost may not be omitted; if unknown, say unknown
P8  no reading here may be presented as diagnostic, clinical, or as
    advice to any person about any relationship
```

SPEC-21.4 — Precision

Intervals in cents to four decimals. Frequencies to at least six. u recomputed from the full-precision vectors, never from a rounded print (SPINE §3.4). Deficits to six decimals with `delta` alongside.

SPEC-21.5 — Gates

```
G1  latency check performed and published, before pricing     REQUIRED
G2  classification by odd part, published                     REQUIRED
G3  lattice and relationship readings reported separately     REQUIRED
G4  maintenance cost declared or explicitly marked unknown    REQUIRED
G5  D never published without delta                           REQUIRED
G6  every case labelled constructed                           REQUIRED
G7  refusals published in full, with the component named      REQUIRED
```

The Failure Modes

FM-21.1 — Reading transmutation as creation

Signature: a procedure described as producing a capability the person did not have.

Why it is wrong: CONSTRAINT T21. An address is a pointer; a pointer with no referent resolves to nothing. Yamanaka reprogramming activates latent capability and does not create new capability, and the four-factor selection is 37.985 bits acting on 6.2 gigabits.

The fix: the latency check, first, with the component named.

FM-21.2 — Latency laundering

Signature: “the repertoire is rich, therefore X is latent, therefore X is available.”

Why it is wrong: §21.5. Each arrow smuggles. Bulk is not specificity and conceivable is not reachable.

The fix: observed at least once, in some state, on some occasion — or REFUSED.

FM-21.3 — Inferring the relationship from the lattice

Signature: “the move was four octaves, so it must have changed the relationship substantially.”

Why it is wrong: Theorem 21.3 and Finding 21.1. Four constructed moves spanning 8603.9100 cents returned identical u to twelve decimal places.

The fix: recompute u from the vectors. Always.

FM-21.4 — Inferring the lattice from the relationship

Signature: the reverse error — u moved, therefore a large index change occurred.

Why it is wrong: the same theorem. u responds to shared support and is blind to labels; a small change of overlap at a fixed index moves it without any re-indexing at all.

The fix: the same. Two readings, reported separately, no inference between them.

FM-21.5 — Index inflation

Signature: the index rises after a re-description that added no distinguishable component.

Why it is wrong: §21.14. Components that always co-occur are one component (Chapter 13's collision test).

The fix: publish the decomposition and run the collision test on the added components.

FM-21.6 — Pricing only in pitch

Signature: a move reported as "+2400 cents" with no maintenance cost.

Why it is wrong: §21.8. Pitch is not in the sustainability condition; `c` is. A move priced only in cents has been priced in the wrong currency, and at `eta = 0.30` a hidden `dc` of 0.10 is 0.961797 octaves of deficit.

The fix: SPEC-21.3 P7. Declare it or mark it unknown.

FM-21.7 — Treating de-indexing as an available operation

Signature: a procedure that schedules "drop state 9" the way it schedules "add state 6."

Why it is wrong: §21.9. Downward movement in fact is mostly `delta`, running unasked. The antisymmetry of the transition matrix is notation.

The fix: distinguish simplifying a stated specification, which is available, from removing a distinction a person can already make, which is not.

FM-21.8 — Reading a higher index as a better state

Signature: a re-indexing upward reported as progress.

Why it is wrong: Chapter 5 §5.20 and Chapter 10's consonance/redundancy identity. Chapter 19 rejected Gate 10 for exactly this and the rejection binds here.

The fix: report the move, its class, and its price. Not a valuation.

FM-21.9 — Comparing intervals across ladders

Signature: a `3 –> 6` on this book's twelve-state ladder set beside a `3 –> 6` on a different scheme.

Why it is wrong: the indices count different things, so the interval is a ratio of incomparable squares.

The fix: the ladder and its `N` are published with every interval, and cross-ladder comparisons are refused.

FM-21.10 — Assuming an orbit continues past the ladder

Signature: "state 7 intensifies to state 14."

Why it is wrong: `N = 12`. States 7 through 12 have no double inside the ladder, and the truncation is an artefact of the index length (Chapter 7 §7.9). Extending the ladder is Chapter 4's lever L4 and is a different operation with a different justification burden.

The fix: report that the orbit is truncated, name `N`, and stop.

FM-21.11 — Precision theatre on a contested decomposition

Signature: an interval to four decimals between two indices that two analysts assign differently.

Why it is wrong: honesty clause (3). The interval inherits both counts.

The fix: compute the interval under both decompositions and publish the gap between them as the uncertainty.

FM-21.12 — Using a priced move as an instruction

Signature: "the move from 3 to 6 costs 2400 cents and three components, so make it."

Why it is wrong: the chapter prices moves and licenses nothing. Chapter 18 owns intervention, its consent axiom, its halting conditions, and its reversibility classification, and none of them are relaxed here.

The fix: a price is not a recommendation, and nothing in this chapter is counselling, therapy, mediation, or clinical assessment.

Worked Example

PART A — SIX MOVES ON THE CARRIED SUBJECT

Constructed case, labelled as such. SPINE §3.1's subject, three framings of one situation:

```
threat = {3:0.8, 5:0.4, 7:0.2}     |w| = 0.916515138991
story  = {3:0.5, 5:0.7, 10:0.4}    |w| = 0.948683298051
mirror = {5:0.3, 7:0.8, 10:0.5}    |w| = 0.989949493661

baseline:  mean u_int = 0.700844041320   F_int = 2.164389635311
```

```
          N_eff = 8.709284   chord width 36.6686325914 Hz (folded 293.349061)
              states {3,5,7,10}    classes {3,5,7}
```

The raw L2 norms are published because Chapter 6's magnitude reporting rule requires it: the free multiplier of a magnitude-estimated intensity must stay recoverable.

Now six constructed re-indexings of the `threat` framing's index-3 component, each run through TR-1 steps 4 through 8. In every case the latency check is stipulated to pass, and **the stipulation is the case's construction, not a finding.**

move	class	lattice interval	mean u_int	F_int	chord width (Hz)
base	3	—	0.700844041320	2.164390	36.6686325914
3 -> 6	3	+2400.0000	0.870156210632	2.358182	54.0282536493
3 -> 4	1	+996.0900	0.870156210632	2.358182	54.0282536493
3 -> 12	3	+4800.0000	0.870156210632	2.358182	54.0282536493
3 -> 1	1	−3803.9100	0.870156210632	2.358182	54.0282536493
3 -> 10	5	+4168.7174	0.590503185401	2.014308	27.1603302345
3 -> 5	5	+1768.7174	0.697749790607	2.160474	36.3825829120

And the classifications, which the relationship column cannot see:

3 -> 6	odd parts 3, 3	INTENSIFICATION	class set {3,5,7} unchanged
3 -> 12	odd parts 3, 3	INTENSIFICATION	class set {3,5,7} unchanged
3 -> 4	odd parts 3, 1	TRANSFORMATION	class set {1,3,5,7}, four classes
3 -> 1	odd parts 3, 1	TRANSFORMATION	class set {1,3,5,7}, four classes
3 -> 10	odd parts 3, 5	TRANSFORMATION	class set {3,5,7} unchanged∗
3 -> 5	odd parts 3, 5	TRANSFORMATION	class set {3,5,7} unchanged∗

∗ the framing's own class changed; the chord's class *set* did not, because `story` still holds state 3 and supplies class 3 to the chord.

Four readings of this table, and they are the chapter compressed.

The first four rows are identical in u and completely different on the lattice. Two are intensifications and two are transformations. One goes up four octaves and one goes down more than three. `u_int` agrees to twelve decimal places across all four. This is Theorem 21.3 and it is the strongest available statement of what the relationship arithmetic cannot see.

The fifth row is the only one that lowers the chord width. Moving to state 10 lands on a coordinate that both other framings occupy, so the move *creates* overlap where it destroyed some, and u falls: 0.700844 to 0.590503, and the chord width from 36.6686 Hz to 27.1603 Hz, a drop of 0.433045 octaves. **Re-indexing into an occupied coordinate is the only kind that can reduce a chord's width**, and it does so by making the framings more alike.

The sixth row barely moves at all. Merging the index-3 weight onto the framing's own state 5 moves u by `−0.003094` and the width by `−0.011298` octaves, because the framing's shared support with the other two is almost unchanged — it still overlaps `story` at 5 and `mirror` at 5

and 7. A 1768.7174-cent lattice move produced an eleven-thousandth of an octave of relationship change.

The pitch gains are worth the same in surplus terms across the identical group. `F_int` rose from 2.164390 to 2.358182, which is 0.193793 nats, which is `2.885390 * 0.193793 = 0.559168` octaves — precisely the chord width change from 36.6686325914 Hz to 54.0282536493 Hz. The conversion key holds exactly, as Book Two Chapter 32 requires.

PART B — A REFUSED MOVE

Constructed case, labelled as such. The same subject, a proposed re-indexing of the `mirror` framing from state 7 (reflexive feeling — self as object) to state 11 (reciprocal feeling — the other modelling you).

TR-1 runs in order and stops at step 3.

```
step 1   ladder: Ch 5, N = 12. m = 7, decomposition: arousal, valence,
         intentional object, situation, cause, projected duration,
         self as object.

step 2   n = 11, decomposition: the seven above, plus another mind
         modelled, plus a standard applied, plus placement in a story,
         plus the other's model of the subject.

step 3   LATENCY CHECK on the four added components:
          another mind modelled        observed in this repertoire   PASS
          standard applied             observed in this repertoire   PASS
          placed in a story            observed in this repertoire   PASS
           the other's model of you      NOT OBSERVED                FAIL

         HALT. The move is REFUSED under CONSTRAINT T21.
```

Steps 4 through 10 are not run, and the arithmetic that would have been produced is not published, because publishing it would let a refused move circulate as a priced one. What is published is the refusal, the named component, and the criterion applied:

```
REFUSED: 7 -> 11
missing component: the other's model of the subject
criterion: §21.5 -- observed in the repertoire at least once, in some
           state, on some occasion. Not met.
what would change the verdict: a single observation of the component,
           in any state, on any occasion.
```

A refusal that names its missing component and states what would reverse it is a scientific result. **A refusal that says the move is impossible is an opinion**, and Chapter 19's SPEC-19.4 imposed the same requirement on its rejections.

PART C — THE CARRIED DYAD, READ AS RE-INDEXING, AND WHAT THE READING CANNOT ATTRIBUTE

Constructed case, labelled as such. SPINE §3.2 records that between phase 2a and phase 2b, B takes on two states A does not hold. That is a re-indexing at chord scope, and its relationship consequence is published:

```
PHASE 2a   u = 0.068734665264634   F = 0.563084802746   f =  1.4907987150 Hz
             eta = 0.30  c = 0.62  delta = 0.09
             D = +4.338420 octaves    Q_inf = -5.011940    DECAYS

PHASE 2b   u = 0.692372881355932   F = 2.153632742815   f = 35.8881768941 Hz
             eta = 0.44  c = 0.40  delta = 0.09
             D = -3.590989 octaves    Q_inf = +6.084427    ACCUMULATES
```

Chapter 4 §4.20 supplies the path-free split of the 7.929409-octave swing:

```
from the cost ratio     2.885390 * (0.909091 - 2.066667)  =  -3.340058
from the surplus       -2.885390 * (2.153633 - 0.563085)  =  -4.589352
total                                                       -7.929409

the surplus side is 57.8776% of the swing
```

Now the limit of what this chapter may say about it, stated because the temptation is strong.

SPINE fixes u for phase 2b. **It does not fix which two states B took on, and this chapter does not invent them.** Theorem 21.3 is precisely why it cannot be inferred: many different pairs of added states produce the same shared-support change and therefore the same u, exactly as the four moves in Part A produced one reading between them. **The relationship frequency records that overlap changed. It does not record where.**

What can be said, and it is worth having:

The move was necessarily out of the shared support. u rose, so overlap fell as a fraction of the norms, so at least some of what B added lies where A holds nothing.

The move was necessarily an addition, not a re-labelling. A pure re-labelling of B's existing states would leave the overlap pattern with A unchanged wherever A holds nothing, and Part A's first four rows show what that looks like: it can move u, but only by vacating a coordinate A occupies, which is a different event from taking on new states.

And `delta` did not move, and the deficit could not have seen it if it had. D is delta-blind (Chapter 4 Theorem 4.9). At 2b, dropping `delta` from 0.09 to 0.05 takes `Q_inf` from +6.084427 to +10.951968 and leaves D at −3.590989. Both numbers are reported here, as clause (7) requires.

PART D — THE REPORT CARD

```
    ------------------------------------------------------------------------
----
    RE-INDEXING REPORT -- carried subject, threat framing, 3 -> 6
    ------------------------------------------------------------------------
----
```

```
    ladder              Ch 5 twelve-state, N = 12
    m                   3, directed feeling (arousal, valence, object)
    n                   6, temporal feeling (+ situation, + cause,
                        + projected duration)
    latency check       3 components required, 3 observed, PASS  [stipulated]
    classification      odd parts 3 and 3 -> INTENSIFICATION
    class               3 before, 3 after -- UNCHANGED
    interval            +2400.0000 cents = +2 octaves exactly
    ratio               f(6)/f(3) = 36/9 = 4.000000; reverse 9/36 = 0.250000
    components          +3, named above
    price per component 800.0000 cents
    maintenance cost    NOT SUPPLIED BY THE FORMALISM. Declared dc unknown
                        for this constructed case; at eta = 0.30 each 0.10 of
                        undeclared c is 0.961797 octaves of hidden deficit.
    ------------------------------------------------------------------------
----
    RELATIONSHIP READING, computed separately from the vectors
    mean u_int          0.700844041320 -> 0.870156210632
    F_int               2.164389635311 -> 2.358182352134   (+0.193793 nats)
    chord width         36.6686325914 Hz -> 54.0282536493 Hz  (+0.559168 oct)
    L2 norms            0.916515138991 / 0.948683298051 / 0.989949493661
    NOTE                identical to the readings for 3->4, 3->12 and 3->1.
                        The lattice reading and this reading are independent
                        (Theorem 21.3) and neither was inferred from the other.
    ------------------------------------------------------------------------
----
    ESSENCE frequencies, not physical resonances. Nothing vibrates.
    Constructed case. Not a diagnosis, not an assessment, not clinical,
    not advice. CONSTRAINT T21 binds: nothing not already latent was reached,
    and nothing was created.
    ------------------------------------------------------------------------
----
```

What this chapter established, and the sentence it refuses to exceed

Transmutation, in this formalism, is the movement of a state from one index to another. The interval crossed is a ratio of squares and is exact. The move is an intensification when the indices share an odd part and a transformation when they do not, and transformation is the generic case at 84.8485% of available moves. The price is three components on average at the doublings, between 301.2741 and 2400.0000 cents in pitch depending on where on the ladder it is bought, and a maintenance cost the formalism does not supply and the practitioner must declare.

The relationship reading is a separate reading and may not be inferred from the lattice, because u is blind to index labels and sees only shared support — a result this chapter demonstrated with four constructed moves spanning 8603.9100 cents that returned one number between them.

And the boundary holds without exception. **Re-indexing addresses a repertoire; it does not extend one.** Nothing not already latent can be reached, at any strength, by any operation in this book, and a procedure that claimed otherwise would have left the formalism behind. Transmu-

tation is re-indexing, not creation. The chapter has said nothing else, and the restraint is not a modesty about the result — it is the result.

Chapter 22 takes the iterative half of Chapter 19's Gate 11 and gives it its chapter: what happens when a bounded operation is run again and again on an accumulating store, why the answer is `|Q_t − Q_inf| = (1−delta)^t |Q_0 − Q_inf|`, and why refinement arrives rather than ascends.

Chapter 22 . The Refinement Ladder

The instruction that survives in more alchemical texts than any other is not a recipe but a count. *Distil seven times. Circulate the matter until it no longer changes. Repeat the work.* The operation is not described as being improved between passes; it is described as being *done again*, identically, and the doing again is what is supposed to produce the result.

A modern reader has two available reactions and both are wrong: to dismiss the instruction as ritual padding, or to accept it as a claim that repetition adds something and that the seventh pass reaches somewhere the first could not. Neither survives the arithmetic, and the arithmetic is short. This chapter computes what iteration does. The answer has been available since Chapter 2 stated the storage law, and it is this:

> **Iteration closes a gap. It does not choose the destination. The destination was fixed by the parameters before the first pass began, and no number of passes moves it by any amount whatsoever.**

That is why the chapter is called a *ladder* and not a *staircase*. A staircase goes somewhere new with each step. A ladder is a fixed structure, and climbing it takes you where it was already pointing when you leaned it against the wall.

Chapter 21 established what it means to move a state from one index to another: transmutation is re-indexing, and nothing not already latent can be reached. This chapter establishes the complementary fact about doing the same thing repeatedly. Chapter 23 then asks what the tradition got right, and cadence is on that list — a real result, but not for the reason the tradition gave.

The Law

22.1 One rung of the ladder

A refinement pass is one application of the storage law. The law is Paper E's contracting loop, stated in Chapter 2 and used unchanged since:

```
Q      =  (1 − delta) ∗ Q   +  ( eta ∗ F(u∗)  −  c∗ )
 t+1                      t

      delta   per-cycle retention loss, in (0, 1]
   eta   conversion: how much of an interaction's surplus becomes stored order
   F(u∗)  the interaction surplus at the operating point, F = ln(1 + 11 u) for N = 12
```

```
          c*      the per-cycle cost of maintaining the arrangement
```

Write the drive as a single constant, because for a fixed arrangement it is one:

```
b  =  eta * F(u*)  -  c*                    the per-pass drive

Q      =  (1 - delta) * Q   +  b
 t+1                      t
```

The fixed point is Paper E Theorem 3.5:

```
Q     =  b / delta  =  ( eta * F(u*) - c* ) / delta
 inf
```

CANON3 §4 calls `Q_inf` destiny and means the word literally: it is where the loop arrives if the parameters hold. Chapter 28 owns that definition; this chapter owns the approach to it.

A rung of the refinement ladder is one application of that map with the parameters unchanged. That last clause is the whole content of the chapter. If the parameters change between passes, the operation is not refinement — it is re-indexing, and Chapter 21 governs it. Refinement means: do the same thing again.

22.2 The error law

THEOREM 22.1 — THE CONTRACTION. For the affine map above with `delta > 0`,

```
| Q  -  Q    |  =  (1 - delta)^t  *  | Q  -  Q    |
   t     inf                          0     inf
```

Proof. Subtract the fixed-point equation `Q_inf = (1-delta) Q_inf + b` from the update equation `Q_{t+1} = (1-delta) Q_t + b`. The drive `b` cancels exactly. What remains is `Q_{t+1} - Q_inf = (1-delta)(Q_t - Q_inf)`. Take absolute values and iterate `t` times. []

Three properties of this identity deserve to be read off before any table is computed.

It is an equality, not a bound. Banach's fixed point theorem, invoked in Book Two Chapter 44, requires only that distances not increase by more than a factor. Here they shrink by exactly `1 - delta` every pass, at every distance, forever: no fast region and no slow region.

The drive cancelled. `eta`, `F`, `c` — everything that decides *where* the loop is going — vanished in the subtraction. They set `Q_inf` and they do not appear in the rate. The gap closes at a speed that depends on the leak and on nothing else.

Nothing reaches the fixed point. `(1-delta)^t` is strictly positive for every finite `t`: the gap gets small and does not become zero. There is no pass on which the work is finished, and the tradition's discomfort about knowing when to stop is a correct perception of a system that supplies no completion signal.

22.3 The three constants of arrival

Everything operational in this chapter comes off three quantities, all functions of `delta` alone:

```
tau        =  -1 / ln(1 - delta)                passes per factor of e
```

```
half-life  =  ln(0.5) / ln(1 − delta)          passes to close half the gap

t(eps)     =  ln(eps) / ln(1 − delta)          passes to reduce the gap to a fraction eps
```

None of the three contains `Q_0`, `Q_inf`, `eta`, `F`, or `c`. **How long arrival takes is a property of the leak; where arrival happens is a property of everything else.** The two questions do not share a parameter. That separation is the formal skeleton of this chapter and it is also, in a different register, the finding SPINE §3.5 records — §22.7 makes the connection exact.

22.4 The destination is fixed before the first rung

THEOREM 22.2 — REFINEMENT CANNOT MOVE THE FIXED POINT. `Q_inf = (eta F(u*) − c*)/delta` contains no `t`. Therefore for any `t`, `Q_inf(t) = Q_inf(0)`. □

The proof is inspection, which is the correct proof: the fixed point is a function of four parameters, and iteration is not one of them.

Stated in the tradition's own vocabulary: if a practice repeats an operation whose parameters do not change, then however many times it is repeated it converges to a state computable before the first repetition — from `eta`, `F`, `c` and `delta`, and from no observation of the practice at all. **The seventh distillation reaches no destination the first distillation was not already headed toward.** What it does is arrive closer, and how much closer is the subject of §22.9.

22.5 Refinement does not move the pitch

The frequency reading is Rule 2 of CANON3 §2, owned by Chapter 12:

```
f     =  k * ( 1 + (N−1) u )^2  =  k * N_eff^2        k = 0.483425732988414 Hz
  rel
```

THEOREM 22.3 — THE LADDER IS SILENT IN THE FREQUENCY REGISTER. `f_rel` is a function of `u` and `N`. Neither is an argument of the iteration count. Therefore `f_rel(t) = f_rel(0)` for all `t`, and so are the folded pitch, the pitch class, and the octave deficit `D`. □

This is Chapter 8's negative result arriving from the other side. Chapter 8 found that frequency cannot tell a mood from a disposition, because persistence is not an argument of `f = k n^2`. This chapter finds that frequency cannot tell the first pass from the seventieth, for the same reason: **the pitch was never told about time, and repetition is entirely a fact about time.**

And SPINE §3.6 sharpens Theorem 22.3. Chapter 21's Finding 21.1 establishes that a state frequency and a relationship frequency are independent readings: four constructed pairs spanning thousands of cents return identical `u` and `f_rel` to twelve decimal places, because `u` reads the angle between state vectors and not which indices are occupied. So there are two silences here — refinement moves neither reading, and neither could be derived from the other even if it did. **A complete report gives the state frequencies, the relationship frequency, and the trajectory, and derives none of the three from the others.**

Read as a claim about the tradition, this is sharp and worth being sharp about. Repetition is not transmutation; an operation repeated is the same operation. For the pitch to change, `u` or `N` has to change — a state genuinely added, a genuinely distinct category admitted — and that is Chapter 21's subject. Chapter 20 owns the compound rule for combining components; it is not reachable by doing one thing twice.

22.6 The ladder in octaves — a construction, and it is badged as one

The two theorems above make it sound as though the frequency register has nothing to say about refinement. It has one thing, and it is exact.

Theorem 4.5 gives the deficit identity, verified to `8.88e-16` at twelve parameter sets:

```
D  =  2.885390 * ( c/eta  -  F )  =  -2.885390 * (delta/eta) * Q
                                                               inf
```

The second form maps the fixed point onto the octave scale. **DEFINITION 22.4 (constructed, badged).** Apply the same map to the *running stock*:

```
D    :=  -2.885390 * (delta / eta) * Q          the running deficit
 t                                    t

f   := k * 2^( 2.885390 * (c/eta) - D )   the apparent pitch of the stock at pass t
 t                                          t
```

This is a definition, not a discovery. It is legitimate because it is exactly the transform that carries `Q_inf` to `D`, applied to a different argument; it is useful because of the following.

THEOREM 22.5 — THE OCTAVE LADDER. With `D_t` as defined,

```
| D  -  D |  =  (1 - delta)^t * | D  -  D |
   t                             0
```

Proof. `D_t` is a fixed negative multiple of `Q_t` and `D` is the same multiple of `Q_inf`. Multiplying both sides of Theorem 22.1 by `2.885390 (delta/eta)` gives the result. []

COROLLARY 22.6 — THE GLIDE. If the stock starts empty, `Q_0 = 0`, then `D_0 = 0` and `f_0 = k * 2^(2.885390 c/eta) = f_min` exactly. As the ladder is climbed, `f_t` rises monotonically from `f_min` to `f_rel`, and the total glide is `|D|` octaves. []

That is the frequency-register statement of what refinement is. **A refinement ladder begun from nothing is a glide that starts at the minimum sustainable frequency and converges on the arrangement's actual pitch.** It never overshoots, it never arrives, and its span is the octave deficit read backwards. In a sustainable relation `D < 0` the glide is upward and spans `|D|` octaves; in an unsustainable one `Q_inf` is negative, `D_t` runs the other way, and the glide descends below `f_min` — the correct picture of a practice refining itself into deficit.

The standing prohibition applies with full force and is restated here because this section is the one most likely to be misread. **These are essence frequencies: positions on a logarithmic scale of surplus.** `f_t` is a bookkeeping transform of a stored quantity. Nothing is claimed to vibrate, no person or tissue resonates at any of these values, and the glide is a way of reading a number, not a sound anything makes.

22.7 SPINE §3.5, carried: rate and sustainability are separate readings

This chapter is one of the places where the book's central limitation becomes structural rather than incidental, so it is stated here in full rather than referenced.

From Chapter 4, Theorems 4.5 and 4.9, verified independently to `2.665e-15` across twelve parameter sets:

```
D = 2.885390 * ( c/eta - F )      contains k, c, eta, N, u.  DOES NOT CONTAIN delta.

 Q    =  ( eta F - c ) / delta       contains delta, and is scaled by it.
  inf
```

The carried dyad as found (SPINE §3.2, case 2a, CONSTRUCTED):

```
delta = 0.09   ->   Q     = -5.0119395464       deficit D = +4.338420
                     inf

delta = 0.05   ->   Q     = -9.0215             deficit D = +4.338420
                     inf
```

Lowering the loss nearly doubles the negative fixed point and moves the deficit by exactly nothing. Now put Theorem 22.1 beside it. The convergence rate is `(1-delta)^t`: it is a function of `delta` and of nothing else. So the same parameter that the deficit is blind to is the *only* parameter the arrival rate can see.

```
 delta governs the RATE of arrival.        D cannot see delta at all.
D   governs the SIGN and SIZE of the sustainability gap.  The rate cannot see D.
```

Refinement speed and sustainability are therefore two independent readings of one system, and neither substitutes for the other. A ladder that converges fast is not thereby converging on somewhere good. A relation with a small deficit is not thereby getting there quickly. Any report that gives one number is incomplete, and §22.19's reporting format requires both.

22.8 Honesty clauses, restated in place

(1) ESSENCE, NOT RESONANCE. Every frequency in this chapter is an essence frequency — a position on a logarithmic surplus scale. Nothing physically resonates at any value here.

(2) NO CLINICAL CLAIM. The twelve states are not diagnoses. A refinement ladder is not a treatment, a course of therapy, or an assessment of anyone's health, and nothing in this chapter describes or prescribes for any person's condition.

(3) ALL CASES CONSTRUCTED. Every case in this chapter is constructed and labelled. None is an observation of anyone.

(4) THE READING INHERITS ITS INPUTS. The trajectory is exactly as defensible as `delta`, and `delta` is usually the hardest of the four parameters to estimate. §22.18 and Chapter 8's Procedure TRAIT-2 give the honest procedure and the honest error bars.

(5) SCALE STATUS. n is a count and is ratio-scaled by construction (SPINE §1.1). The intensities `w` must be magnitude-estimated per Chapter 6, with raw L2 norms published, or the relationship quantities computed from them are not determined. R1's demotion binds `D/P` specifically; it does not bind the index and it does not bind magnitude-estimated intensities.

(6) NO INTERVENTION MAY REMOVE THE ABILITY TO DECLINE. Consent is structural, not advisory (Chapter 18). §22.15 is about what happens when it is violated, and the arithmetic there is the argument.

The Engineering

22.9 The convergence table

Computed from `t(eps) = ln(eps)/ln(1−delta)` and `tau = −1/ln(1−delta)`. Fractional values are shown because the exactness matters; the integer in brackets is the first whole pass at or beyond the threshold.

delta	1−delta	tau	half-life	to 5%	to 1%	to 0.1%
0.01	0.99	99.4992	68.9676	298.07 [299]	458.21 [459]	687.32 [688]
0.02	0.98	49.4983	34.3096	148.28 [149]	227.95 [228]	341.92 [342]
0.03	0.97	32.8308	22.7566	98.35 [99]	151.19 [152]	226.79 [227]
0.05	0.95	19.4957	13.5134	58.40 [59]	89.78 [90]	134.67 [135]
0.07	0.93	13.7797	9.5513	41.28 [42]	63.46 [64]	95.19 [96]
0.09	0.91	10.6033	7.3496	31.76 [32]	48.83 [49]	73.24 [74]
0.10	0.90	9.4912	6.5788	28.43 [29]	43.71 [44]	65.56 [66]
0.12	0.88	7.8227	5.4223	23.43 [24]	36.02 [37]	54.04 [55]
0.15	0.85	6.1531	4.2650	18.43 [19]	28.34 [29]	42.50 [43]
0.20	0.80	4.4814	3.1063	13.43 [14]	20.64 [21]	30.96 [31]
0.25	0.75	3.4761	2.4094	10.41 [11]	16.01 [17]	24.01 [25]
0.30	0.70	2.8037	1.9434	8.40 [9]	12.91 [13]	19.37 [20]
0.40	0.60	1.9576	1.3569	5.86 [6]	9.02 [10]	13.52 [14]
0.50	0.50	1.4427	1.0000	4.32 [5]	6.64 [7]	9.97 [10]

The row in bold is the carried case's loss rate and the one every worked example in this book uses. **At `delta = 0.09` a refinement ladder is thirty-two passes from being within five per cent of its destination**, and seventy-four passes from being within a tenth of a per cent.

Read the table with the period attached. At `delta = 0.09` per week that is thirty-two weeks to the five-per-cent band; per year, thirty-two years. The arithmetic does not know which, and §22.17 makes declaring the period a gate.

22.10 Two exact invariances

The cost of each halving is constant. At `delta = 0.09` the half-life is 7.3496 passes, and that figure is the cost of going from 100% of the gap to 50%, from 50% to 25%, and from 1% to 0.5%. The expression `ln(0.5)/ln(1−delta)` has no `t` in it. There is no acceleration and no fatigue.

The cost of each factor of ten is constant. At `delta = 0.09` it is 24.4149 passes — the difference between the 1% column and the 0.1% column, 73.2447 minus 48.8298.

interval closed	passes at `delta = 0.09`
0% -> 50% of the gap	7.35

interval closed	passes at `delta = 0.09`
50% -> 90%	17.06
90% -> 99%	24.41
99% -> 99.9%	24.41

The last nine hundredths of one per cent costs exactly what the ninety-to-ninety-nine per cent did, and more than three times what the first half did. This is the arithmetic behind every practice that felt rapid at the start and interminable at the end. Nothing slowed down. The remaining distance is being felt in absolute terms by an operator whose instrument works in ratios, and the ratios are being closed at a perfectly constant rate throughout.

22.11 What seven repetitions actually buy

The tradition's number is seven. Theorem 22.1 makes the question answerable: seven passes close `1 - (1-delta)^7` of the gap, and that is a claim about `delta` and nothing else.

`delta`	`(1-delta)^7` gap remaining	gap closed by seven passes	passes needed for 99%
0.01	0.932065	6.79%	458.21
0.02	0.868126	13.19%	227.95
0.05	0.698337	30.17%	89.78
0.09	0.516761	48.32%	48.83
0.10	0.478297	52.17%	43.71
0.15	0.320577	67.94%	28.34
0.20	0.209715	79.03%	20.64
0.25	0.133484	86.65%	16.01
0.30	0.082354	91.76%	12.91
0.40	0.027994	97.20%	9.02
0.50	0.007812	99.22%	6.64
0.60	0.001638	99.84%	5.03
0.70	0.000219	99.98%	3.83

And the sharper inverse question: what loss rate makes seven passes *sufficient*? Solving `(1-delta)^7 = eps`:

```
seven passes close 95%   requires   delta  =  1 - 0.05^(1/7)  =  0.348164
seven passes close 99%   requires   delta  =  1 - 0.01^(1/7)  =  0.482053
seven passes close 99.9% requires   delta  =  1 - 0.001^(1/7) =  0.627241
```

Seven repetitions is adequate only in a system that loses roughly a third to a half of its stored order every cycle, and in a system with a leak of a few per cent it is nearly worthless. That is a real finding and it cuts both ways: the traditional count is not a universal constant, and it is not arbitrary either, because it encodes an implicit and checkable claim about the process. A tradition prescribing seven passes is asserting `delta ~ 0.35` or higher for its own operation, and in a physical distillation — where the loss is the volatile fraction left behind each pass — that is entirely

ordinary. **The instruction is not mystical; it is a badly-stated engineering estimate, and it is testable.**

Whether it transfers to a psychological practice is a different question, and the answer is: only if `delta` there is also around a third, which for a weekly practice it is not. Chapter 8's Procedure TRAIT-2 is how one would find out.

22.12 Rate against destination — the two-by-two

Because `delta` moves the rate and cannot move the deficit, and because `D` fixes the destination's sign and cannot see `delta`, the two readings are orthogonal and all four combinations occur. Both rows use the carried dyad's arrangements (SPINE §3.2, CONSTRUCTED); only `delta` differs.

case	u	eta	c	delta	D (octaves)	Q_inf	half-life	to 1%
2b, fast leak	0.692373	0.44	0.40	0.30	-3.590989	+1.825328	1.9434	12.91
2b, slow leak	0.692373	0.44	0.40	0.03	-3.590989	+18.253280	22.7566	151.19
2a, fast leak	0.068735	0.30	0.62	0.30	+4.338420	-1.503582	1.9434	12.91
2a, slow leak	0.068735	0.30	0.62	0.03	+4.338420	-15.035819	22.7566	151.19

Four things to read off it, in order of how badly they are usually got wrong.

The `D` column has two values, not four. Changing `delta` by a factor of ten does not perturb the deficit in the twelfth decimal place. Theorem 4.9 says it cannot, and here is the demonstration.

The rate columns have two values, not four. They are identical between a healthy arrangement and a failing one. The failing arrangement converges at exactly the same speed; it is converging on `−15.04`.

A slow ladder on a bad arrangement is the worst of the four. Row three settles at `−1.50` in thirteen passes; row four takes 151 passes to settle at `−15.04`, a destination ten times worse, because the low leak that slowed it down also multiplied the negative numerator. This is R15 in the refinement register and Chapter 15 owns it: trust lowers `delta`, `delta` divides an already-negative numerator, and the result is a slower ladder to a much worse place.

Speed feels like progress and is not evidence of it. A practice that visibly changes a person fast is a practice with a large `delta`, and `delta` is a *loss* term. The visible rapidity is the signature of a leaky system reaching its level, which may be any level at all.

22.13 The glide, computed

Case 2b of the carried dyad (SPINE §3.2, CONSTRUCTED): `u = 0.692372881355932`, `eta = 0.44`, `c = 0.40`, `delta = 0.09`, `N = 12`. Then `F = 2.153633`, `f_rel = 35.8881768941 Hz`, `f_min = 2.9782153324 Hz`, `D = −3.5909886634` octaves, `Q_inf = +6.0844267427`, drive `b = 0.547598406839`. The ladder starts from an empty stock, `Q_0 = 0`.

`D_t` and `f_t` are Definition 22.4's constructed readings; the final column is the distance from the arrangement's true pitch, in cents.

pass `t`	`Q_t`	`D_t` (oct)	`f_t` (Hz)	cents below `f_rel`
0	0.000000	0.000000	2.978215	4309.19
1	0.547598	-0.323189	3.726024	3921.36
2	1.045913	-0.617291	4.568558	3568.44
3	1.499379	-0.884924	5.499772	3247.28
5	2.287549	-1.350096	7.592339	2689.07
7	2.940232	-1.735306	9.915969	2226.82
10	3.715053	-2.192600	13.614271	1678.07
15	4.605861	-2.718349	19.600209	1047.17
20	5.161754	-3.046433	24.605033	653.47
25	5.508650	-3.251169	28.356682	407.78
32	5.786888	-3.415383	31.775238	210.73
40	5.944508	-3.508410	33.891644	99.09
49	6.024551	-3.555651	35.019794	42.41
60	6.063209	-3.578466	35.578013	15.03
74	6.078761	-3.587645	35.805087	4.01
100	6.083939	-3.590701	35.881014	0.35
150	6.084422	-3.590986	35.888113	0.0031

The total span is `1200 * log2(35.8881768941 / 2.9782153324) = 4309.1864` cents, which is `3.5909886634` octaves, which is `−D` to every digit — Corollary 22.6 verified.

Seven passes — the traditional count — leave this ladder 2226.82 cents short of its own pitch, which is more than an octave and a half. At `delta = 0.09` seven is not a completion; it is slightly less than half the gap, exactly as §22.11 predicted.

One further reading, because it is the useful one. The regime statement of Chapter 7 applies: at `N = 12` the minimum separation between distinct pitch classes is 34.9756 cents. The glide is inside that separation — indistinguishable from arrival at the resolution the book's own lattice supports — from pass 49 onward, where it stands at 42.41 cents, crossing below 34.9756 cents at pass 51. **Beyond about fifty passes the ladder is doing work the instrument cannot see.** That is the honest stopping criterion and §22.18 formalises it.

22.14 Why arrival is recognition rather than ascent

Book One's Node 10, FIXEDPOINT, records that arrival feels like recognition and not like victory, and offers no mechanism. Chapter 2 §2.14 identified the mechanism in one line: a contraction arrives, it does not climb. Here is the same claim with the arithmetic under it, in four parts.

One: there is no arrival event. `(1-delta)^t > 0` for every finite `t`, so there is no pass on which something completes and nothing for a sense of achievement to attach to. What there is instead is a gap that has become smaller than the operator can resolve — and *noticing that you can no longer tell the difference* is a recognition, not an accomplishment.

Two: almost the entire trajectory is spent near the destination. At `delta = 0.09` the ladder is within 5% of `Q_inf` from pass 32 and within 0.1% from pass 74. A practice run for three hundred passes spends 269 of them inside the five-per-cent band. Whatever being at the fixed point is like, it is what most of a practice feels like, and the transient is the unusual part. The approach is not the experience; it is the anomaly at the beginning.

Three: the destination is derived from what was already there. Every term of `Q_inf = (eta F - c)/delta` is a standing property of the arrangement; nothing in the fixed point is imported. The sense that arrival reveals something one already contained is not a mystical intuition but an accurate report about a formula whose every argument predates the first pass.

Four: the same fixed point is reached from any starting stock. Theorem 22.1 holds for any `Q_0`, and the sign of `Q_0 - Q_inf` does not appear in the rate: a ladder started from a large stock descends at exactly the speed one started from nothing ascends. **The destination is not a summit that some starting positions cannot see.** It is a level, and levels are recognised, not conquered.

Chapter 28 takes this forward as the definition of destiny. This chapter's contribution is the rate, and the observation that the rate belongs to a parameter that the destination cannot see.

22.15 Counterfeit arrival

A fixed point can be positive, stable, and reached at another party's expense. Chapter 28 names this counterfeit arrival; this chapter can compute it, because cost export is an operation on `c*` and `c*` is in the drive.

CONSTRUCTED CASE — TWO PARTIES SHARING ONE ARRANGEMENT. Labelled constructed; not an observation. Two people, P and R, share the case-2b arrangement, so both draw on the same surplus `F = 2.153633` at `u = 0.692372881355932` and `f_rel = 35.8881768941 Hz`. Their private parameters differ, which is the ordinary situation:

```
P:  eta = 0.44  c = 0.40  delta = 0.09     D = -3.590989  Q    = +6.084427
                                                              inf
R:  eta = 0.30  c = 0.55  delta = 0.09     D = -0.924189  Q    = +1.067665
                                                              inf
```

Both are sustainable. R is nearer the edge, because R converts less and pays more — R's break-even cost is `eta_R * F = 0.646090` and R is at 0.55.

Now P exports cost. P declines a share of the maintenance — the scheduling, the repair conversations, the remembering of what was agreed — and R absorbs it. Let x be the exported amount, so `c_P -> 0.40 - x` and `c_R -> 0.55 + x`.

x	c_P	c_R	D_P	Q_inf,P	D_R	Q_inf,R	sum of drives	sum of fixed points
0.000	0.400	0.550	-3.590989	+6.084427	-0.924189	+1.067665	+0.643688	+7.152091
0.050	0.350	0.600	-3.918874	+6.639982	-0.443290	+0.512109	**+0.643688**	+7.152091
0.100	0.300	0.650	-4.246759	+7.195538	+0.037608	-0.043446	**+0.643688**	+7.152091
0.115	0.285	0.665	-4.345125	+7.362205	+0.181877	-0.210113	**+0.643688**	+7.152091
0.150	0.250	0.700	-4.574644	+7.751093	+0.518506	-0.599002	**+0.643688**	+7.152091
0.200	0.200	0.750	-4.902530	+8.306649	+0.999405	-1.154558	**+0.643688**	+7.152091

> **CORRECTION NOTE (made visible, not silently applied).** This chapter originally stated the counterfeit-arrival identity on the *sum of participants' fixed points*. That form is correct only when every participant carries the same `delta`, which the constructed case above happens to satisfy. The general invariant is the **sum of drives**, `sum_i (eta_i F_i − c_i)`. Chapter 28 proves it as Theorem 28.5, with the equal-`delta` case as Corollary 28.6 and the counterexample in §28.13. Everywhere below, the rule is stated on drives; the fixed-point form is retained only as the labelled equal-`delta` corollary. Using the wrong form is Chapter 28's FM 28.8.

The sum of the two drives is invariant to every digit across the whole export range. Each participant's drive is §22.2's `b_i = eta_i F(u*) − c_i`, so that `Q_inf,i = b_i / delta_i`; `b` is the drive and `D` remains the deficit throughout. Cost export produces nothing; it moves an existing quantity from one ledger to the other, entering one drive with a plus and the other with a minus, so `sum_i b_i` cannot move whatever the individual `delta_i` are. **Under equal `delta` — and only then — the sum of fixed points inherits the invariance,** because `sum_i Q_inf,i = (sum_i b_i)/delta`; that is why the last column above is also constant, and it is a corollary of the column beside it rather than a rule in its own right. The general statement of what the plain sum does under unequal `delta` is §22.15a. Either way the invariance is exactly the thing the exporter's own reading cannot show, and that is the point. P's private instrument reports genuine, sustained, increasing improvement. P's deficit falls from -3.590989 to -4.902530, nearly a further octave and a third of headroom. Every number P can see about P is better.

R crosses the sustainability boundary between `x = 0.05` and `x = 0.10`; solving `D_R = 0` gives the exact crossing at `c_R = eta_R F = 0.646090`, i.e. at `x = 0.096090`.

And then the lag, which is the reason this failure mode is hard to catch. R does not vanish on the pass the sign flips. R's stock decays from its old level toward its new one at `(1−delta)^t`. At `x = 0.15`, R's new fixed point is `−0.599002` and R starts at `+1.067665`:

pass	Q_R
0	+1.067665

pass	Q_R
3	+0.656950
7	+0.262266
10	+0.050025
15	-0.193988
25	-0.441283
40	-0.560675

Solving exactly, Q_R crosses zero at t* = 10.850479 passes. **For eleven passes after the export begins, R still has a positive stock and R's behaviour is still that of a participant** — while R's destination has already been negative from the first pass. Nothing observable changes at the moment the damage is done. This is why the deficit is the instrument and the stock is not: D_R went positive immediately; Q_R took eleven passes to notice.

The end of the arithmetic is the part that matters. Suppose R eventually declines to continue — which is R's structural right under Chapter 18's constraint, and no arrangement in this book may remove it. The shared arrangement ends. With no partner there is no interaction surplus: u = 0, F = ln(1) = 0, and f = k = 0.483426 Hz, the ground essence (Rule 2, CANON3 §2). P's reading at x = 0.15, with the reduced cost P kept:

```
before R's departure   D = −4.574644   Q = +7.751093   f = 35.888177 Hz
                           P              inf,P

after  R's departure   D = +1.639426   Q = −2.777778   f =  0.483426 Hz
                           P              inf,P
```

P's fixed point falls by 10.528871 units and P's deficit swings 6.214070 octaves the wrong way, and the export that produced the counterfeit reading is the reason. The reduced cost P retained is worth nothing once the surplus it was drawn against is gone.

> **THE DEFINITION, EARNED BY THE ARITHMETIC. A counterfeit fixed point is one that is positive in the exporter's accounts and negative in the accounts of the system that contains the exporter.** It is detectable, and the detection rule is: compute the deficit for every participant separately, and check whether the sum of drives moved. If a participant's fixed point improved and the sum of drives did not move, the gain was exported and nothing was produced.

That rule costs one extra column in the reporting format and it is the only defence available, because the exporter's own instrument is working perfectly and reporting a genuine improvement in a quantity that genuinely improved. This identity is generalised in Chapter 28 §28.12 — Theorem 28.5, the drive is conserved under cost transfer — and stated as the export test in Chapter 28 §28.15.

22.15a Why the rule is on drives and not on fixed points

The two forms are not interchangeable, and the case that separates them is the case in which export is most tempting. With Q_inf,i = b_i / delta_i and a transfer of x from P to R — so that b_P −> b_P + x and b_R −> b_R − x:

```
SUM OF DRIVES   sum_i b   =  sum_i ( eta_i F_i − c_i )  =  sum_i delta_i Q
                      i                                                inf,i
                                                          INVARIANT ALWAYS

SUM OF Q      d/dx [ sum_i Q     ] = 1/delta  − 1/delta    zero ONLY if the
                         inf,i           P          R   two deltas are equal
```

Run the same P and R with the same export, changing only the decay rates — P at `delta = 0.06`, R at `delta = 0.16`, so that the exporter is the party who decays *slower*:

x	Q_inf,P	Q_inf,R	sum of drives	sum of fixed points
0.000	+9.126640	+0.600561	+0.643688	+9.727202
0.050	+9.959973	+0.288061	+0.643688	+10.248035
0.100	+10.793307	-0.024439	+0.643688	+10.768868
0.150	+11.626640	-0.336939	+0.643688	+11.289702
0.200	+12.459973	-0.649439	+0.643688	+11.810535

Nothing whatever is produced — the drive column does not move a digit — and R crosses into decay between `x = 0.05` and `x = 0.10` exactly as before. But **the plain sum rises, by `1/0.06 − 1/0.16 = 10.416667` per unit exported**, from +`9.727202` to +`11.810535`.

The failure mode is the worst available one. The naive test does not merely fail to detect the export; it reports a gain, and it reports one precisely when cost is pushed onto whoever decays faster — which is the direction the arithmetic already rewards the exporter for choosing. A reading that used the plain sum would return the export check as *passed with improvement* on the pass the damage was done. Chapter 28 carries this as **FM 28.8**, the export test run on the plain sum, and Chapter 28 §28.13 works the general case; neither is repeated here.

22.16 The cost of a rung, and when to stop

Theorem 22.1 says every pass closes the same *fraction* of the gap. It therefore says every pass closes a strictly smaller *absolute* amount, because the gap is shrinking. At `delta = 0.09` with an initial gap of 6.084427:

pass	gap remaining	closed by this pass	cumulative
1	5.536828	0.547598	9.00%
3	4.585048	0.453466	24.64%
7	3.144195	0.310964	48.32%
15	1.478565	0.146232	75.70%
25	0.575777	0.056945	90.54%
40	0.139919	0.013838	97.70%
60	0.021218	0.002098	99.65%

A pass costs the operator's time, attention, and the occasion itself. That cost is constant per pass while the return per pass falls geometrically, so the ladder has a finite optimal length. The marginal return at pass `t` is `delta * (1−delta)^t * |Q_0 − Q_inf|`, and setting it equal to a declared threshold `theta` gives

```
 t     =  ln( theta / ( delta * |Q  - Q   | ) )  /  ln(1 - delta)
  stop                            0    inf
```

threshold `theta`	`t_stop` (exact)	first pass at or beyond
0.50	0.9642	1
0.25	8.3138	9
0.10	18.0295	19
0.05	25.3791	26
0.01	42.4444	43

The stopping rule is not a completion signal; it is a declared threshold on marginal return. It must be declared before the ladder starts, or it becomes a post-hoc rationalisation of wherever the operator ran out of patience. Declaring it in advance is gate G5 of §22.20.

Note what this does *not* say. Whether to divide the ladder into more, smaller occasions is Chapter 23's question — splitting a fixed input across occasions rather than repeating a fixed operation — and under R11 the answer turns on strict concavity and the surplus ceiling. Refinement here holds the per-pass operation fixed and asks only how many times to do it.

The Specification

22.17 PROCEDURE REFINE-1 — reading a refinement ladder

```
STEP 1  DECLARE THE OPERATION. Write down what one pass consists of, in enough
      detail that a third party could perform it. If the description changes
        between passes, this is not refinement (Ch 21 governs re-indexing).

STEP 2  DECLARE THE PERIOD. One pass = one what? Week, month, session, year.
      delta is a per-cycle rate and is meaningless without its cycle (Ch 8 §8.5).

STEP 3  MEASURE u. Procedure REL-1 (Ch 13) for a dyad, Ch 11's worksheet for one
        person. Publish the raw L2 norms (Ch 6's magnitude reporting rule).
      CHECK FOR THE ONE-HOT TRAP: framings with disjoint support are automatically
      orthogonal and give u = 1 for everyone regardless of content (SPINE §2, Ch 9).

STEP 4  COMPUTE THE PITCH. F = ln(1 + 11 u); f = k (1 + 11 u)^2; fold per Ch 7.

STEP 5  ESTIMATE eta AND c. Ch 4's procedure. Report the scale status of each.

STEP 6  COMPUTE THE DEFICIT. D = 2.885390 (c/eta) - log2(f/k). This is the
         DESTINATION reading. It does not contain delta and cannot see it.

STEP 7  CHECK THE CEILING. c/eta <= ln N = 2.484907 at N = 12. Above it, no
      arrangement of these twelve categories is sustainable and no ladder helps.
```

```
   STEP 8  ESTIMATE delta.  Procedure TRAIT-2 (Ch 8 §8.16). Report the interval, not a
             point, and carry the interval through every rate figure below.

    STEP 9   COMPUTE THE FIXED POINT.  Q_inf = (eta F - c)/delta.  Cross-
check against
           Q_inf = -D eta / (2.885390 delta).  The two must agree to 1e-12.

   STEP 10  COMPUTE THE RATE.  tau, half-life, t(0.05), t(0.01) from delta alone.
          This is the ARRIVAL reading. It does not contain D and cannot see it.

   STEP 11  COMPUTE THE TRAJECTORY.  Q_t from the declared Q_0; report at least the
             passes to 50%, 95% and 99% of the gap.

   STEP 12  IF REPORTING THE GLIDE, badge it.  D_t and f_t are Definition 22.4's
             constructed readings of a stored quantity. Nothing resonates.

   STEP 13  RUN THE EXPORT CHECK.  For every participant, compute D separately and
          sum the DRIVES (eta F - c). Compare against the sum before any change.
          If a participant's fixed point improved and the sum of drives did not
             move, the gain was exported and nothing was produced (§22.15).
          Do not run this check on the sum of fixed points unless every delta is
             equal; see §22.15a and Ch 28 FM 28.8.

    STEP 14  REPORT BOTH READINGS.  D with delta, never D alone (SPINE §3.5).
```

22.18 PROCEDURE REFINE-2 — the stopping rule

```
   STEP 1  DECLARE theta, the smallest per-pass gain worth one pass's cost, BEFORE
             the first pass. Declare it in the same units as Q.

    STEP 2   COMPUTE t_stop = ln( theta / (delta * |Q_0 - Q_inf|) ) / ln(1 -
delta).

   STEP 3  COMPUTE THE RESOLUTION FLOOR. Find the pass at which the remaining glide
          falls below 34.9756 cents -- the minimum class separation at N = 12 (Ch 7).
          Beyond it the ladder is moving less than the lattice can distinguish.

   STEP 4  STOP AT THE EARLIER OF STEP 2 AND STEP 3, and record which bound binds.

   STEP 5  IF t_stop <= 0, the ladder was not worth beginning: the first pass already
            returns less than theta. Report this. It is a legitimate finding.

   STEP 6  DO NOT RE-DERIVE theta AFTER STOPPING. A threshold chosen to justify the
             stopping point carries no information.
```

22.19 Reporting format

```
    REFINEMENT LADDER READING                          [CONSTRUCTED / MEASURED]
```

```
OPERATION      ..........................................
PERIOD         one pass = ..........
PARTICIPANTS   ..........  (one line per participant below)

DESTINATION READING          -- does not contain delta
  u = ........  (L2 norms: ........)      one-hot trap checked: Y/N
 F = ........ nats    f = ........ Hz    folded ........ Hz   class ....
eta = ........  c = ........  c/eta = ........  ceiling ln N = 2.484907
  D = ........ octaves          verdict: ACCUMULATES / HOLDS / DECAYS

ARRIVAL READING              -- does not contain D
  delta = ........ per ..........   [interval: ........ to ........]
tau = ........  half-life = ........  t(5%) = ........  t(1%) = ........
  Q_0 = ........   Q_inf = ........   cross-check agrees to ........

GLIDE  [constructed reading, Definition 22.4; nothing resonates]
f_0 = ........ Hz    f_inf = ........ Hz    span = ........ cents = |D| oct
  resolution floor reached at pass ........

STOPPING
  theta declared in advance = ........      t_stop = ........
  binding bound: MARGINAL RETURN / RESOLUTION FLOOR

EXPORT CHECK
sum of participants' drives (eta F - c), before = ........  after = ........
  moved? Y/N        if N and any Q_inf improved: the gain was exported
all deltas equal? Y/N   if Y, sum of Q_inf may be reported as a corollary:
  sum of participants' Q_inf, before = ........   after = ........

HONESTY
  essence frequencies, not physical resonances .................. [ ]
  no clinical, diagnostic or therapeutic claim .................. [ ]
  case labelled CONSTRUCTED where applicable .................... [ ]
  scale status of every elicited quantity stated ................ [ ]
  ability to decline preserved ................................. [ ]
```

22.20 Gates

G1 OPERATION FIXED. One pass is specified and does not change between passes.
G2 PERIOD DECLARED. delta carries its cycle. No bare rate.
G3 BOTH READINGS. D and delta both reported (SPINE §3.5). Neither alone.
G4 CEILING CHECKED. c/eta <= ln N verified before any rate is computed.
G5 theta PRE-DECLARED. The stopping threshold precedes the first pass.
G6 EXPORT CHECKED. Sum of DRIVES computed across all participants. The sum of fixed points only where all deltas are equal (§22.15a).
G7 GLIDE BADGED. f_t identified as a constructed reading wherever it appears.
G8 CASES LABELLED. Every worked case marked CONSTRUCTED.

The Failure Modes

#	Name	What it looks like	What it actually is	Detection
FM 22.1	The ascending ladder	“Each pass takes us higher”	A contraction converging on a fixed level	Compute `Q_inf` before pass 1; compare to the reported trajectory
FM 22.2	Speed as evidence	“This works fast, so it works”	Large `delta`; a leaky system reaching its level	`delta` is a loss term. Read `D`, which cannot see it
FM 22.3	The moving destination	Recomputing `Q_inf` after each pass and reporting the movement	The parameters were changed; this is re-indexing (Ch 21), not refinement	Gate G1: was the operation fixed?
FM 22.4	Seven for its own sake	A count inherited from the tradition and applied to any `delta`	Seven passes close 48.32% at `delta = 0.09` and 6.79% at `delta = 0.01`	§22.11’s table
FM 22.5	The completion signal	Waiting for the pass on which the work finishes	There is none; `(1−delta)^t > 0` always	Procedure REFINE-2’s declared `theta`
FM 22.6	The retro-fitted threshold	`theta` chosen after stopping, to justify stopping	An unfalsifiable rationalisation	Gate G5
FM 22.7	Deficit without delta	Reporting `D` alone as “the reading”	`D` cannot see decay at all (SPINE §3.5)	Gate G3
FM 22.8	Delta without deficit	Reporting a fast half-life as good news	The rate cannot see the destination’s sign	Gate G3
FM 22.9	Counterfeit arrival	One party’s readings improve steadily and genuinely	Cost export; sum of drives unchanged	Gate G6, §22.15

#	Name	What it looks like	What it actually is	Detection
FM 22.10	The eleven-pass illusion	“They are still participating, so they are fine”	The stock lags the destination by `t*` passes	Compute each participant’s `D`, not each participant’s `Q_t`
FM 22.11	Refining below resolution	Passes 60 through 300, diligently performed	Movement smaller than 34.9756 cents, the lattice minimum at `N = 12`	Procedure REFINE-2 step 3
FM 22.12	Trust as a ladder accelerator	“Add trust and we will get there sooner”	Trust lowers `delta`; the ladder gets *slower*, and if the numerator is negative the destination gets worse	R15; §22.12 rows two and four
FM 22.13	The zero-leak ladder	A model with `delta = 0`	No contraction, no fixed point, unbounded growth in the model and rapid divergence from anything real	Gate G2 forces the leak to be named
FM 22.14	One-hot refinement	Every participant’s u comes out at 1.000000	Disjoint support: automatic orthogonality, degenerate surplus	Procedure REFINE-1 step 3
FM 22.15	The export check run on the plain sum	The sum of fixed points is used as the conserved quantity under unequal `delta`	Not an invariant: the sum moves by `1/delta_P − 1/delta_R` per unit exported, and *rises* when the exporter decays slower	§22.15a; the full treatment is Ch 28 FM 28.8

Four of these deserve more than a table row.

FM 22.2, speed as evidence, is the most common and the most inverted. The intuition that a practice producing rapid change is a powerful one is exactly backwards here. Rapidity is `delta`, `delta` is loss, and the destination `Q_inf = b/delta` is *divided* by it. A fast ladder and a slow ladder with the same `D` do not go to the same place: the fast one goes to a nearer place, quickly.

Rows one and two of §22.12 differ by a factor of ten in both directions at once.

FM 22.9 and FM 22.10 are one failure mode observed at two lags, and the pairing is what makes it dangerous. The export check (G6) catches it on pass one; the stock does not reveal it until pass eleven in the worked case, by which time the exporter has eleven passes of improving private readings to point at. A protocol that monitors stocks rather than deficits finds the problem after it has become a fact.

FM 22.13 deserves naming because it is the tradition's characteristic model. Descriptions of a refined substance that "no longer changes" are descriptions of `delta = 0`. There the map is an isometry, Banach does not apply, there is no fixed point, and `Q_t = Q_0 + b t` grows without bound — which is not permanence but the opposite, a quantity with no level at all. **The tradition's image of an incorruptible end product is, read as a model, the one setting under which the whole apparatus stops working.** Chapter 8's discussion of low-delta traits is the honest version: `delta` small is a durable disposition, `delta` zero is not a disposition at all.

FM 22.12 is R15 arriving in this chapter's vocabulary, and it is worth stating in the strongest form the arithmetic supports. Adding trust to a failing arrangement in order to make the refinement go faster does the reverse of both: it makes the ladder slower and the destination worse. The carried case is the demonstration — 2a at `delta = 0.09` arrives at `−5.0119395464`; at `delta = 0.05` it arrives, more slowly, at `−9.0215`. **Fix the numerator first.** Chapter 15 owns the rule; this chapter shows what ignoring it does to the trajectory as well as to the destination.

Worked Example

22.21 The carried dyad, case 2b, run as a ladder

CONSTRUCTED. Not an observation of any person or pair. SPINE §3.2, case 2b: the pair after B has taken on two states A does not hold and the pair have adopted a decision log.

Steps 1–2. One pass is one week: the pair's standing weekly review with the decision log open. The operation does not change between passes.

Steps 3–4. `u = 0.692372881355932`, elicited by magnitude estimation with raw L2 norms published per Chapter 6. The one-hot check passes: the two vectors share support on states 4, 6 and 9, so the orthogonality trap does not apply. Then

```
F = ln(1 + 11 * 0.692372881355932) = 2.153633 nats
f = k * (1 + 11 * 0.692372881355932)^2 = 35.8881768941 Hz
```

Folding into the 220–440 band per Chapter 7 requires `j = 3`: `35.8881768941 * 8 = 287.105415` Hz.

Steps 5–7. `eta = 0.44`, `c = 0.40`, so `c/eta = 0.909091`, comfortably below the ceiling `ln 12 = 2.484907`. Then `f_min = k e^(2 * 0.909091) = 2.9782153324 Hz`.

Step 6 — the destination reading.

```
D = 2.885390 * 0.909091 − log2(35.8881768941 / 0.483425732988414)
  = 2.623082 − 6.214071
  = −3.5909886634 octaves                    ACCUMULATES
```

Steps 8–10 — the arrival reading. `delta = 0.09` per week.

```
Q     = (0.44 * 2.153633 - 0.40)/0.09  =  +6.0844267427
 inf
cross-check:  -D * eta / (2.885390 * delta) = +6.0844267427  agrees to 1e-12

tau = 10.6033 weeks    half-life = 7.3496 weeks
t(5%) = 31.7645 weeks  t(1%) = 48.8298 weeks   t(0.1%) = 73.2447 weeks
```

Step 11 — the trajectory, from `Q_0 = 0`, drive `b = 0.547598406839`:

week	Q_t	fraction of gap remaining	closed
0	0.000000	1.000000	0.00%
1	0.547598	0.910000	9.00%
7	2.940232	0.516761	48.32%
15	4.605861	0.243008	75.70%
32	5.786888	0.048902	95.11%
49	6.024551	0.009857	99.01%
74	6.078761	0.000931	99.91%

Step 12 — the glide, badged as Definition 22.4's constructed reading: from `f_0 = f_min = 2.978215 Hz` to `f_rel = 35.888177 Hz`, a span of 4309.1864 cents, which is `|D|` exactly. The full table is §22.13.

Step 13 — the export check. Both members' $c*$ are unchanged from the arrangement as declared; the sum of drives is unmoved. No export.

Step 14 — both readings, together:

```
DESTINATION  D = -3.5909886634 octaves. ACCUMULATES. Independent of delta.
ARRIVAL     delta = 0.09/week. Half-life 7.3496 weeks. Within 5% at week 32.
```

Neither number is the reading. The pair is both going somewhere good and going to take about eight months to get most of the way there, and a report giving only the first would be a report a reader could not act on.

22.22 Case 2a — the ladder that perfects a decay

CONSTRUCTED. The same pair as found, before the intervention. `u = 0.068734665264634`, `eta = 0.30`, `c = 0.62`, `delta = 0.09`.

```
F = 0.563084802746 nats      f = 1.4907987150 Hz      f_min = 30.1587698405 Hz
 D = +4.3384201973 octaves      DECAYS
 Q     = -5.0119395464          drive b = -0.451074535976
  inf
```

The rate constants are identical to §22.21's, because `delta` is: `tau = 10.6033`, half-life 7.3496 weeks, within 5% at week 32. **The same ladder, at the same speed, converging on `-5.01` instead of `+6.08`.** The trajectory from `Q_0 = 0`:

week	Q_t
0	0.000000
1	-0.451075
7	-2.420838
15	-3.793076
32	-4.766824
49	-4.962534
74	-5.007270

The glide runs the other way. `f_0 = f_min = 30.158770 Hz` and `f_t` descends, because `D_t` rises from 0 toward +4.3384201973. **A refinement ladder on an unsustainable arrangement is a descending glide of 4.338420 octaves below the minimum sustainable frequency**, executed with perfect regularity. The practice is not failing. It is succeeding at converging on a negative destination.

And now the intervention a reader reaches for, which SPINE §3.2 exists to pre-empt — add trust, `delta` 0.09 -> 0.05:

```
    delta = 0.09    Q      = −5.0119395464        D = +4.338420    half-
life  7.3496 weeks
                    inf
    delta = 0.05    Q      = −9.0215              D = +4.338420    half-
life 13.5134 weeks
                    inf
```

The destination nearly doubled in the wrong direction, the ladder became 84% slower, and the deficit did not move. Three separate readings, three separate stories, one intervention. A reader with only `D` would conclude nothing happened. A reader with only the half-life would conclude the practice had become more patient. A reader with `Q_inf` would see the damage. **Report all three.** That is what §22.19's format requires, and why it requires it.

The repair is Chapter 15's order of operations: fix the numerator, then lower the loss. Case 2b is what that looks like — u raised from 0.068735 to 0.692373 and c cut from 0.62 to 0.40 first, after which trust becomes an amplifier of a positive sign and `delta` 0.09 -> 0.05 takes `Q_inf` from +6.0844267427 to +10.9520.

22.23 The counterfeit ladder, in report format

CONSTRUCTED. P and R of §22.15, at export `x = 0.15`, read at pass 8 — after the export has begun and before R's stock has crossed zero.

```
REFINEMENT LADDER READING                                   [CONSTRUCTED]

  OPERATION      the shared weekly review, as declared in §22.21
  PERIOD         one pass = one week
  PARTICIPANTS   P, R

  DESTINATION READING
 u = 0.692372881355932   one-hot trap checked: Y (shared support on 4, 6, 9)
```

```
        F = 2.153633 nats    f = 35.8881768941 Hz    folded 287.105415 Hz
       P:  eta 0.44   c 0.250   c/eta 0.568182   D = -4.574644   ACCUMULATES
        R:  eta 0.30   c 0.700   c/eta 2.333333   D = +0.518506   DECAYS
        ceiling ln N = 2.484907; R at 2.333333 is inside it but close

      ARRIVAL READING
        delta = 0.09 per week (both)   half-life 7.3496 weeks
        P:  Q_inf = +7.751093
        R:  Q_inf = -0.599002    Q_R at pass 8 = +0.216073, still positive
        R's stock crosses zero at t* = 10.850479 passes

      EXPORT CHECK
        sum of drives before export = +0.643688
        sum of drives after  export = +0.643688
       moved? NO  ->  NOTHING WAS PRODUCED. The improvement in P's reading is
                      a transfer, and R's deficit has already changed sign.
       all deltas equal? YES (0.09 both) -> the corollary form also applies:
        sum of Q_inf before export = +7.152091   after export = +7.152091

      HONESTY
        essence frequencies, not physical resonances .................. [x]
        no clinical, diagnostic or therapeutic claim .................. [x]
        case labelled CONSTRUCTED ..................................... [x]
        ability to decline preserved -- R may end the arrangement ..... [x]
```

Every private number P can compute is better than it was, every one of them is correct, and the arrangement is now scheduled to end. That is what counterfeit arrival is, and the only line in the report that catches it is the one comparing two sums.

22.24 What this chapter establishes, and what Chapter 23 takes

Established here, and cited by Chapters 23, 28 and 29:

```
   Thm 22.1  |Q_t - Q_inf| = (1-delta)^t |Q_0 - Q_inf|, an equality, not a bound
    Thm 22.2   Q_inf contains no t: refinement cannot move the destination
   Thm 22.3  f_rel contains no t: refinement cannot move the pitch or the deficit
       Def 22.4     the running deficit D_t and apparent pitch f_t --
CONSTRUCTED readings
    Thm 22.5   the octave ladder: |D_t - D| = (1-delta)^t |D_0 - D|
    Cor 22.6   the glide: from f_min to f_rel, spanning exactly |D| octaves
    the three constants of arrival, all functions of delta alone
    the two invariances: constant cost per halving, constant cost per decade
   what seven passes buy, as a function of delta, and the delta that makes seven enough
   counterfeit arrival: defined, computed, and given a one-line detection rule
    on the SUM OF DRIVES (§22.15a; general form Ch 28 Thm 28.5). The sum of fixed
      points is the equal-delta corollary only.
   PROCEDURE REFINE-1, PROCEDURE REFINE-2, the reporting format, eight gates,
      fifteen failure modes
```

The negative results are the load-bearing ones. Refinement does not ascend, does not re-index, does not change a pitch, does not move a fixed point, and does not finish. What it does is close a proportion of a gap, at a rate set by the loss term, toward a place computed from four parameters none of which is effort.

The two readings do not substitute for each other: `delta` governs how fast a ladder arrives and cannot appear in the deficit; the deficit governs whether arriving is worth anything and cannot see `delta`. Report both or report nothing.

Chapter 23 turns to what the tradition got right, and cadence is on its list. The temptation there is to read this chapter's result as a vindication of repetition, and it is not one. This chapter holds the operation fixed and counts passes. Chapter 23 asks a different question — whether a fixed total input is better delivered at once or in instalments — and the answer to that one, under R11, is *not* that instalments deliver more. They deliver exactly the same integrated total, to every digit. What they buy is something else entirely, and Chapter 23 states it precisely.

Chapter 23 . What Alchemy Got Right

This chapter and the next are one chapter in two halves, and the second half is what makes the first half usable. Chapter 24 is the counter-chapter: it takes the tradition's largest claim, kills it with arithmetic, and then states exactly what survives. **A list of things a tradition got right, published alone, is an advertisement.** Published beside a list of what it got catastrophically wrong, computed to the same standard, it becomes an audit — so nothing in this chapter should be read without the next one.

Four things are on this chapter's list. Each was arrived at independently, by people with no instruments and no statistics, and each corresponds to something the formalism of this book computes:

```
1  STAGED PROCESS CONTROL    an operation split into named stages with a check at each
2  CADENCE                 scheduled repetition rather than one great effort
3  CORRESPONDENCE TABLES     finite relational graphs over a counted category set
4  THE OPERATOR IN THE LOOP  the practitioner's own state as part of the apparatus
```

The second of these carries a correction that must be stated before anything else, because the popular version of it is false and this book has already computed the falsity. R11: **the integrated effect of one large input and the same total delivered periodically is identical to every digit — 433.3333 in all three schedules tested.** That is linear time-invariance and it is not a matter of opinion. Cadence wins for two other reasons entirely, both of them computed in §23.9 through §23.13, and this chapter states them rather than the folklore.

The apparatus for the whole chapter is the author's Tri-State Method from the VINO Framework, adopted unchanged from CANON_BIO §1, with its roles stated in every block and not assumed.

The Law

23.1 The concordance apparatus, with roles stated

```
 (+1)  PHYSICS PROOF     what scientific measurement confirms
(−1)  METAPHYSICS      what contemplative or traditional practice records
 (0p)  CONTRA-PROOF      how the two integrate at the Neutral Point

C1  STRUCTURAL CORRESPONDENCE  a deep shared pattern, not a surface resemblance
C2  PREDICTIVE POWER        the bridge must generate a prediction that can fail
 C3  PRACTICAL UTILITY           it must be usable on both sides
```

And the refinement that CANON_BIO §1 adds, restated here in full because this chapter is built entirely out of concordances and the refinement is what keeps them honest:

> **The (+1) column carries EVIDENCE — what is measured, replicated, falsifiable. The (-1) column carries PRECEDENT** — that the idea was arrived at independently, recorded, and preserved, often for centuries, by people without the instruments. **Two sources agreeing does not double the evidence when only one of them is evidence.**

What independent arrival does establish is real and worth having: that a structure was findable by more than one route, which is a genuine argument that the structure is in the territory and not only in the instrument. It is not an argument that the structure is correct. Chapter 24 supplies the arithmetic for why the precedent column cannot be promoted — under success-only publication the likelihood ratio of testimonial evidence is exactly 1.000000 — and this chapter's every block is written under that constraint.

Only bridges satisfying all three criteria are presented as established. Speculative bridges are marked speculative in the sentence that states them. **And a method that never rejects anything is not a method: this chapter rejects three claims in print, and Chapter 24 rejects the largest one of all.**

23.2 CONCORDANCE C23.1 — STAGED PROCESS CONTROL

```
CONCORDANCE C23.1 — AN OPERATION SHOULD BE DIVIDED INTO NAMED STAGES WITH A CHECK AT EACH

  CLAIM      That dividing a transformation into discrete, named, individually
           inspected stages produces a better outcome than performing it as one
               undivided operation with a single check at the end.

  (+1) EVIDENCE  -- this column carries measurement
           Statistical process control. Shewhart, "Economic Control of Quality of
                           Manufactured  Product"  (1931),  established  in-
process measurement with
            control limits rather than terminal inspection; Deming, "Out of the
            Crisis" (1986), made the argument that inspection at the end cannot
            build quality in. Separately, the unit-operations decomposition of
           chemical engineering -- distillation, filtration, crystallisation as
           separately characterised stages -- is the working form of the same idea.
             WHAT IT MEASURED: that per-stage detection removes error before it
              propagates, and that the yield difference is large and computable
                (§23.8 computes it: 47.83% to 93.21% on a seven-stage process).

  (-1) PRECEDENT  -- this column carries independent arrival, not proof
           The Greco-Egyptian and Latin alchemical corpora describe transformation
           as an ordered series of named operations, each with its own apparatus and
                     its  own  sign  of  completion.  Zosimos  of  Panopolis  (3rd-
4th c. CE) describes
                    distillation apparatus -- the tribikos, the kerotakis --
attributed in that
```

corpus to Maria the Jewess. The historiography that establishes these as laboratory practice rather than allegory is Newman and Principe, "Alchemy vs. Chemistry: The Etymological Origins of a Historiographic Mistake" (Early Science and Medicine, 1998), and Principe, "The Secrets of Alchemy" (2013), which reproduces documented recipes in a modern laboratory. WHAT IT RECORDS: that staging, per-stage apparatus, and a per-stage completion criterion were arrived at and preserved for well over a millennium before any of it was justified statistically.

(0p) INTEGRATION
Both are describing error containment. A stage boundary is a place where an error can be caught before the next operation makes it unrecoverable. The tradition's "sign" -- a colour, a consistency, a smell -- is a control-limit test with an ordinal readout and no stated tolerance.

CRITERIA C1 PASS -- the shared pattern is per-stage detection, not the surface fact that both use lists.
C2 PASS -- it predicts that yield rises with per-stage detection probability, monotonically, and §23.8 computes the curve.
C3 PASS -- usable on both sides.

STATUS ESTABLISHED. The precedent is genuine and the evidence is independent of it. The tradition is not the reason to believe the claim; Shewhart is.

BOUNDED BY THAT STAGING IS RIGHT DOES NOT MAKE ANY PARTICULAR STAGE ORDER RIGHT. Chapter 19 Finding 19.2 runs Ripley's Twelve Gates (1471) as a schedule under this framework and finds the only numerator-acting gate NINTH of twelve -- the canonical sequence lowers loss before it fixes the numerator, which is exactly the order R15 shows accelerates a failure. Chapter 24 §24.5 states that rejection in full. Staging: established. This tradition's stage ORDER: rejected.

23.3 CONCORDANCE C23.2 — CADENCE, WITH R11 STATED EXACTLY

CONCORDANCE C23.2 — SCHEDULED REPETITION BEATS ONE GREAT EFFORT

CLAIM AS THE TRADITION STATES IT
That the work must be done on a schedule -- the fire kept even, the matter circulated daily, the office said at fixed hours -- and that this is superior to one concentrated effort of equal total magnitude.

THE NAIVE VERSION OF THE CLAIM IS FALSE AND THIS BOOK HAS COMPUTED THE FALSITY
R11, Book Two Ch 59, recomputed here in §23.9: the INTEGRATED effect of one large impulse and the same total delivered periodically is IDENTICAL to every digit -- 433.3333 in all three schedules, equal to B/delta. That is linear time-invariance. Anyone claiming that spreading the same

total input over more occasions produces more total stored order, in a linear model, has not done the arithmetic.

(+1) EVIDENCE -- this column carries measurement

(a) THE FORMAL RESULT, computed in this work, not borrowed: the steady state differs absolutely (0 for an impulse, m/delta for sustained periodic forcing); the ceiling S <= ln N clips a large one-off input, costing 97.2% of the nominal budget in Book Two's worked annual rite at N = 8, and 90.44% in the N = 12 register recomputed in §23.11; and Theorem 59.1 -- strict concavity alone means splitting an input across occasions strictly dominates delivering it at once.

(b) THE SPACING EFFECT, an independent empirical literature: Ebbinghaus (1885) first reported it; Cepeda, Pashler, Vul, Wixted and Rohrer, "Distributed practice in verbal recall tasks" (Psychological Bulletin, 2006), is the quantitative synthesis. Roediger and Karpicke (Psychological Science, 2006) established the related testing effect.

HONEST NOTE, REQUIRED: the spacing literature is about human memory encoding and Theorem 59.1 is a statement about concave response functions. They are not the same claim and must not be presented as mutual confirmation. Each stands on its own.

(-1) PRECEDENT -- this column carries independent arrival, not proof

Fixed-hour liturgical practice; the alchemical instruction to keep the fire even and to circulate the matter over months rather than force it. Durkheim, "The Elementary Forms of Religious Life" (1912), on periodic assembly; Rappaport, "Ritual and Religion in the Making of Humanity" (1999), on invariance and periodicity as the defining features of ritual.

WHAT IT RECORDS: that scheduled recurrence was selected for, and preserved, by institutions that had no way to compute why it worked.

(0p) INTEGRATION

Periodic forcing of a leaky integrator. The correct statement, and the only one this book endorses: CADENCE BEATS INTENSITY BECAUSE OF STRICT CONCAVITY AND THE SURPLUS CEILING, NOT BECAUSE IT DELIVERS MORE TOTAL INPUT. It does not. The integrated total is identical.

CRITERIA C1 PASS. C2 PASS -- it predicts the clipping loss as a function of cadence and N, computed in §23.11, and the optimum division as a function of per-occasion cost, computed in §23.12. C3 PASS.

STATUS ESTABLISHED, WITH THE FOLKLORE'S STATED REASON REJECTED. The practice is right and the traditional explanation of why is wrong. Chapter 24's method -- disprove the strong claim with arithmetic, preserve what is genuinely there -- applies here as much as it does there.

23.4 CONCORDANCE C23.3 — CORRESPONDENCE TABLES AS RELATIONAL GRAPHS

CONCORDANCE C23.3 — A CORRESPONDENCE TABLE IS A FINITE RELATIONAL GRAPH

CLAIM That the tradition's tables of correspondence -- ten nodes and twenty-two paths, seven metals, four elements, a seventy-eight element structured deck -- are, described accurately and without mysticism, finite relational structures over counted category sets, and that this is exactly the object Paper B's validity condition V2 requires.

(+1) EVIDENCE -- this column carries measurement
Graph theory, which is not in dispute. A ten-node, twenty-two-edge scheme has a computable density against the complete graph's forty-five edges: 0.488889. And the star-polygon result of CANON_INTERNAL §7, which is number theory and is exact: a figure {N/k} drawn by stepping k positions around N points decomposes into exactly gcd(N,k) separate closed paths, each of length N/gcd(N,k). The hexagram {6/2} is literally two separate triangles; {7/2} is one continuous path. §23.14 computes the table.
WHAT IT MEASURED: that these structures have exact arithmetic invariants.

(−1) PRECEDENT -- this column carries independent arrival, not proof
The tables themselves, preserved across the Hermetic, kabbalistic and alchemical corpora, and the Ars Memoriae tradition of spatial mnemonics that used them (Yates, "The Art of Memory", 1966).
WHAT IT RECORDS: that counted, exhaustive, exclusive category schemes with declared relations between members were constructed and transmitted for centuries. That is the form V2 asks for, arrived at independently.

(0p) INTEGRATION
A correspondence table supplies the PARTITION that V2 requires and often supplies the ORDER that the house condition V7 requires. What it does not supply is V1, a state encoding -- a map from domain items to normalised vectors -- and §23.15 grades two classical tables against all of them and finds them wanting in exactly that place.

CRITERIA C1 PASS. C2 PASS -- the gcd result predicts the number of separate strokes in any such figure, and is falsifiable by drawing one. C3 PASS -- the partition is directly usable as a category set.

STATUS ESTABLISHED AS A DESCRIPTION OF FORM. Explicitly NOT established as a claim that any particular table's contents are correct. §23.15 rejects two classical tables as psychological indices while retaining their form.

23.5 CONCORDANCE C23.4 — THE OPERATOR IS PART OF THE APPARATUS

CONCORDANCE C23.4 — THE PRACTITIONER'S OWN STATE IS PART OF THE INSTRUMENT

CLAIM That the state of the person conducting an operation materially affects its outcome, and that preparing the operator is therefore part of preparing the apparatus rather than a piety attached to it.

(+1) EVIDENCE -- this column carries measurement

(a) Experimenter expectancy effects: Rosenthal and Fode (1963) reported that experimenters told their rats were "maze-bright" obtained better maze performance than those told the same rats were "maze-dull". The blinding requirement in trial design exists because of this class of result. Present at magnitude and with the caveat that this literature has a contested history (the 1968 classroom study drew serious methodological criticism).

(b) Tacit knowledge and the experimenter's regress: Polanyi, "Personal Knowledge" (1958); Collins, "Changing Order" (1985), documenting that laboratories could not build a working TEA laser from the published description alone and required personal contact with a group that already had one.

(c) THIS WORK'S OWN MEASUREMENTS, R13 and R24: the shuffled-input control run off the shelf is anti-conservative at 2.2x nominal alpha (0.108 against 0.05); the matched-autoregressive comparison alone false-passed 35.8% of the time; against fabricated timing the full four-baseline suite passed 52.7% of the time; against a shared slow driver it passed 98.35% of the time. The conclusion recorded there is that DESIGN CONTROLS -- blinding, channel separation, prospective sampling, a yoked control -- are required, and every one of those is a constraint on the operator rather than on the material.

(d) THE FORMAL RESULT, computed in §23.16: if the operator is a participant, the operator's own state vector enters u. An operator whose framings mirror the subject's contributes u = 0 exactly (R8) and pays the coordination cost anyway. In the carried case this LOWERS the arrangement's pitch by 536.4074 cents and moves the octave deficit from +4.338420 to +4.785426, or to +6.372391 once the link cost is priced.

(-1) PRECEDENT -- this column carries independent arrival, not proof

The standing insistence across the alchemical corpus that the artisan must be prepared, unhurried and attentive, and that the work fails otherwise; the parallel insistence in contemplative traditions that the state of the one performing a practice is not separable from the practice.

WHAT IT RECORDS: that the operator was treated as a component. It records

```
            nothing about mechanism, and the tradition's proposed mechanisms --
- moral
          worthiness, purity, election -- are not carried forward by this book.

  (0p) INTEGRATION
          Three distinct real mechanisms are available and none is supernatural:
            expectancy effects on measurement, tacit skill in execution, and --
- in the
          relational case -- the operator's literal membership in the set whose
          non-parallelism is being computed. The third is the one this book adds and
               it is arithmetic.

  CRITERIA    C1 PASS.  C2 PASS -- §23.16 predicts the sign and size of the pitch change
              from adding an operator of a given framing, and the prediction is
                 checkable by measuring u.  C3 PASS.

  STATUS     ESTABLISHED AS A STRUCTURAL CLAIM; THE TRADITION'S MECHANISM REJECTED.
          "The operator is part of the apparatus" is correct. "The operator must be
          morally pure" is not carried, is not derivable from anything here, and is
               not endorsed.
```

23.6 Two further rejections, in print

A method that never rejects is not a method. C23.1's block has already rejected one claim — this tradition's canonical stage *order*, on Chapter 19 Finding 19.2 against R15 — and two more that a sympathetic reading might have admitted are rejected here.

```
REJECTION C23.5 — THE COLOUR SEQUENCE AS A NECESSARY ORDER

  CLAIM      That the transformation must pass through blackening, whitening, yellowing
          and reddening in that order, and that the order is a law of the process.

  GRADING     C1 MARGINAL. Staging is real (C23.1) and the colours are real observations
           of real material changes. But the claim under test is not "there are
               stages"; it is "these stages, in this order, necessarily".
            C2 FAILED, AND THIS IS DECISIVE. The stages are identified BY their
            colours. An operation that does not blacken is therefore not at the
          blackening stage, by definition, and the sequence cannot be observed to
               fail. A claim that cannot fail generates no prediction.
          C3 PARTIAL. Useful as a shared vocabulary for reporting where one is.

  STATUS     REJECTED AS AN ORDERING LAW. Retained only as staging vocabulary, with
               the circularity stated wherever it is used. This is the second
          concordance rejection in the essence-alchemy part and it is recorded in
               Appendix D.

REJECTION C23.6 — "AS ABOVE, SO BELOW" AS A SCALE-INVARIANCE THEOREM

  CLAIM      That the same laws govern the small and the large, so a result obtained at
```

```
              one scale transfers to another.

 GRADING    C1 PARTIAL, AND THE PARTIALITY IS MEASURED. The storage law IS scale-
free
          in FORM -- CANON_INTERNAL §5's master table is exactly that claim, and it
          is why this book can read a mind and a commons with one instrument. But
          R16 names nine disanalogies between the internal and external scopes and
          computes three of them: a mean-field decay error reaching 92% by t = 12;
          two nine-member groups with IDENTICAL u = 0.5999 and S = 1.7578 losing
          77% versus 21% of surplus to a three-person departure, so that aggregate
           statistics hide the structure that decides the outcome; and a 90.1x
          period-normalisation trap when comparing scopes on mismatched clocks.
          C2 FAILED AS STATED. The maxim predicts that any result transfers. The
              computed disanalogies are counterexamples.
           C3 DANGEROUS. Used as licence it produces exactly the 90.1x error.

 STATUS     REJECTED AS STATED. What survives, and is used throughout this book, is
          the precise version: THE CORRESPONDENCE IS OF DYNAMICS AND GEOMETRY, NOT
           OF AGENCY, and every transfer between scopes must re-derive its own
              parameters and declare its own clock.
```

23.7 Honesty clauses, restated in place

(1) NO SUPERNATURAL CLAIM. Nothing in this chapter asserts any efficacy of symbol, any physical effect of a figure, or any result not derivable from the formalism. The computing-map reading of the old notation is a metaphor and CANON_INTERNAL §7 keeps that badge on it.

(2) ESSENCE, NOT RESONANCE. Every frequency here is an essence frequency: a position on a logarithmic surplus scale. No substance, person or tissue is claimed to resonate at any value.

(3) NO CLINICAL CLAIM. The twelve states are not diagnoses; nothing here is a treatment.

(4) PRECEDENT IS NOT EVIDENCE. Stated in every block above and restated here. Chapter 24 computes why: the likelihood ratio of a success-only testimonial record is exactly 1.000000.

(5) ALL CASES CONSTRUCTED. Every worked case in this chapter is constructed and labelled.

(6) SCALE STATUS. n is a count and is ratio-scaled (SPINE §1.1); intensities w must be magnitude-estimated per Chapter 6 with raw L2 norms published.

(7) BOTH READINGS. Per SPINE §3.5, every reading in this chapter reports the octave deficit `D` *and* `delta`. `D` contains no `delta` and cannot see decay; `delta` cannot see the deficit.

The Engineering

23.8 Staged process control, computed

CONSTRUCTED. A seven-stage process. Each stage introduces a defect with probability `q = 0.10`. A check after each stage detects a defect introduced at that stage with probability `d` and

it is reworked; undetected defects propagate and spoil the product. Per-stage escape is `q(1−d)` and final yield is `(1 − q(1−d))^7`.

per-stage detection d	escape per stage	final yield	expected reworks per unit
0.00 (terminal check only)	0.1000	**47.83%**	0.000
0.50	0.0500	69.83%	0.350
0.90	0.0100	**93.21%**	0.630
0.99	0.0010	99.30%	0.693

Moving from a single terminal inspection to a per-stage check at ninety per cent detection raises yield from 47.83% to 93.21%, at a cost of 0.63 reworks per unit. That is the whole of C23.1 in four rows. The tradition's contribution is not the arithmetic, which it never had; it is that the arrangement the arithmetic recommends is the arrangement it used.

Note what the table also shows, because the honest reading is the useful one. Rework is not free and it rises with detection. At `d = 0.99` the yield gain over `d = 0.90` is 6.09 percentage points for an extra 0.063 reworks; whether that trade is worth taking is an economic question the model does not answer. Staging is not free, and the tradition's habit of treating each stage as a rite with its own preparation is, in this reading, an accurate estimate that stage boundaries are expensive.

23.9 R11, part one: the integrated effect is identical

Reproduced here rather than cited, because the popular claim is the opposite and a citation is not enough. Book Two Chapter 59's setup: `delta = 0.06` per week, a total budget `B = 26.0` delivered in one calendar year, and three schedules — one impulse of 26.0 at week 0; 0.5 per week for 52 weeks; 2.0 every fourth week for 52 weeks. Summing `Q_t` over 4,000 periods:

```
impulse       433.3333
weekly        433.3333
four-weekly   433.3333
B / delta  =  433.3333

maximum pairwise difference across the three schedules:  3.41e-13
```

Identical to every digit, to the limit of floating-point summation. This is the defining property of a linear time-invariant system: the integral of the response to a given total input is `B/delta` regardless of how that input is distributed in time.

R11, restated as this book's position and binding on every later chapter: cadence does not deliver more. Any argument beginning "spreading it out gives you more total benefit" is wrong, and the correct arguments are the two that follow.

23.10 R11, part two: the steady state differs absolutely

The integrals agree; the destinations do not. An impulse decays to nothing — its steady state is exactly `0`. Sustained periodic forcing at mean rate `m` per period holds a level of `m/delta`. Those are not the same system observed at different speeds; they are different systems.

The trajectory difference in Book Two's worked case has a crossover at week 24: at week 23 the impulse is still ahead, 6.6648 against 6.3254, and at week 24 the cadence is ahead, 6.4458 against 6.2649, and it never trails again while the budget lasts. The impulse wins early and loses permanently, which is precisely the shape that makes the folklore feel true and the arithmetic necessary.

23.11 R11, part three: the ceiling clips the impulse

The second real argument is the surplus ceiling. `S_t <= ln N` (Paper D Corollary 2.2, and Paper E Corollary 3.4 in the frequency register, Chapter 4 Theorem 4.6). One interaction cannot generate more surplus than the log of the number of distinguishable categories, however much is poured into it. Everything above the ceiling is not delivered late; it is not delivered.

Book Two's worked annual rite ran at `N = 8`, `eta = 0.35`, giving a per-occasion cap of `ln 8 = 2.0794` nats. **Against a nominal annual budget of 26 nats, the annual rite delivers 2.8% of what the weekly practice delivers: the clipping costs 97.2% of the nominal budget.** That figure is Book Two's and is cited exactly.

Recomputed in this book's register — `N = 12`, cap `ln 12 = 2.484907` nats, `eta = 0.44`, the same 26-nat nominal annual budget delivered at a nominal 0.5 nats per week:

cadence P (weeks)	burst wanted	delivered	clipped	annual delivered	eta x annual	% of weekly
1	0.5000	0.5000	no	26.0000	11.4400	100.00%
2	1.0000	1.0000	no	26.0000	11.4400	100.00%
4	2.0000	2.0000	no	26.0000	11.4400	100.00%
8	4.0000	2.4849	**yes**	16.1519	7.1068	62.12%
13	6.5000	2.4849	**yes**	9.9396	4.3734	38.23%
26	13.0000	2.4849	**yes**	4.9698	2.1867	19.11%
52	26.0000	2.4849	**yes**	2.4849	1.0934	**9.56%**

At `N = 12` the annual rite delivers 9.56% of the weekly practice from an identical nominal budget — a loss of 90.44% rather than Book Two's 97.2%. The difference between the two figures is not a discrepancy; it is the ceiling, and it is a direct demonstration of lever L4. Raising the category count from 8 to 12 raises the cap from 2.0794 to 2.4849 nats, and the concentrated rite recovers a third of what it was losing. A richer category set does not merely raise the maximum sustainable frequency; it also makes concentrated delivery less wasteful, because there is more room under the cap.

Note also where the clipping starts. At `N = 12` a fortnightly and even a monthly cadence lose nothing at all in this worked case; the loss begins at `P = 8` weeks. **"More often" is not uniformly better.** It is better only once the burst exceeds the cap, and below that point cadence buys nothing through this channel. The other channel, concavity, is where the remaining advantage lives.

23.12 R11, part four: strict concavity, which needs no ceiling at all

Theorem 59.1 is the honest core of the cadence argument, because it does not depend on the ceiling binding. `F(u) = ln(1 + (N−1)u)` is strictly concave through the origin, and for a concave response through the origin, splitting an input across occasions strictly dominates delivering it at once. No clipping required.

CONSTRUCTED. A fixed budget of `T = 12` hours of interaction, divided into `n` occasions of `L = T/n` hours each. Non-parallelism rises linearly with session length to saturation at `L* = 8` hours: `u(L) = min(1, L/L*)`. `N = 12`, `eta = 0.44`, and a fixed per-occasion cost `c_0 = 0.15` regardless of length — convening, travelling, settling, closing.

n	L (h)	u	F(u)	eta F	delivered total	−n c_0	net
1	12.0000	1.0000	2.4849	1.0934	1.0934	-0.1500	0.9434
2	6.0000	0.7500	2.2246	0.9788	1.9577	-0.3000	1.6577
3	4.0000	0.5000	1.8718	0.8236	2.4708	-0.4500	2.0208
4	3.0000	0.3750	1.6341	0.7190	2.8761	-0.6000	2.2761
6	2.0000	0.2500	1.3218	0.5816	3.4894	-0.9000	2.5894
8	1.5000	0.1875	1.1192	0.4925	3.9397	-1.2000	2.7397
12	**1.0000**	**0.1250**	**0.8650**	**0.3806**	**4.5672**	**-1.8000**	**2.7672**
24	0.5000	0.0625	0.5232	0.2302	5.5255	-3.6000	1.9255
48	0.2500	0.0312	0.2955	0.1300	6.2402	-7.2000	-0.9598

The delivered-total column is monotone increasing exactly as Theorem 59.1 requires, rising from 1.0934 to 6.2402 — **a factor of 5.7074 from the same twelve hours, purely by division, with no ceiling involved and no extra input.**

Two further readings, both of which the folklore misses.

The net column peaks at `n = 12` and falls away on both sides. Concavity alone would recommend infinitely many infinitesimal occasions, which is obviously wrong, and the term that stops it is the per-occasion fixed cost. **The optimum is a function of `c_0` and of nothing else in this model**, which means "how often should we meet" is answerable, and the answer is an arithmetic consequence of convening cost.

At `n = 1` the single twelve-hour occasion wastes four hours before any other effect is considered, because u saturates at `L* = 8`. The longest session in the table is the only one with idle budget in it.

23.13 The honest counter-case

Book Two Chapter 59 kept a counter-case in print and this chapter keeps it too, because a chapter titled *What Alchemy Got Right* that reported only confirmations would be selecting.

Above a threshold that no realistic cadence reaches, the impulse wins. And the durability such a schedule would require is computable: **Book Two's worked decennial rite — an observance every ten years — requires a trace half-life of 5.63 years for the stored order to survive between occasions.** A practice whose retention half-life is measured in weeks cannot be run on a decade.

This is the same arithmetic as Chapter 22's, arriving from the cadence side. **The interval between occasions must be short relative to the half-life `ln(0.5)/ln(1−delta)`, and every claim that a rare observance suffices is a claim about `delta` that can be checked.**

23.14 Correspondence tables, computed as graphs

The claim of C23.3 is descriptive and the descriptions are arithmetic.

```
ten-node / twenty-two-path scheme
    nodes 10, edges 22, complete graph 45 edges, density 0.488889

the twelve-state ladder of this book
    nodes 12, edges 66, complete graph 66 edges, density 1.000000

a seventy-eight element structured deck
    78 = 22 + 4 x 14;  distinct pairs = 3,003
```

The density figures are the useful comparison. **This book's own category scheme is a complete graph** — every state stands in a computed interval relation to every other, because the intervals are ratios of squares and no pair is undefined. The classical ten-node scheme specifies fewer than half its available relations, and the unspecified ones are not "absent"; they are simply not part of the table. That is a difference of kind between a correspondence table and a metric structure, and it is the honest limit on the analogy.

The one exact arithmetic invariant the tradition's figures do possess is the star-polygon result of CANON_INTERNAL §7, which is number theory and is true. Stepping k positions around `N` points produces exactly `gcd(N,k)` separate closed paths of length `N/gcd(N,k)`:

N	k and the resulting (strokes, length)
6	{6/1} (1, 6) . {6/2} **(2, 3)** . {6/3} (3, 2)
7	{7/1} (1, 7) . {7/2} (1, 7) . {7/3} (1, 7)
8	{8/1} (1, 8) . {8/2} (2, 4) . {8/3} (1, 8) . {8/4} (4, 2)
9	{9/1} (1, 9) . {9/2} (1, 9) . {9/3} (3, 3) . {9/4} (1, 9)
10	{10/1} (1,10) . {10/2} (2, 5) . {10/3} (1,10) . {10/4} (2, 5) . {10/5} (5, 2)
11	every k from 1 to 5 gives (1, 11)
12	{12/1} (1,12) . {12/2} (2, 6) . {12/3} (3, 4) . {12/4} (4, 3) . {12/5} (1,12) . {12/6} (6, 2)

The six-pointed figure is literally two separate triangles and the seven-pointed one is a single unbroken path, and which of those you get is decided by `gcd(N,k)` and by nothing else. At `N = 11`, a prime, every step size yields one continuous path; at `N = 12` the strokes take every divisor value. This is exact, checkable with a pencil, and carries no claim whatever about meaning.

23.15 Grading two classical tables against the validity conditions

The form passes. The contents must be graded, and Paper B's conditions plus the house addition V7 (Chapter 5 §5.5) are the grading instrument.

THE FOUR ELEMENTS, as a candidate index. Four categories, exhaustive and exclusive by stipulation, counted. Traditionally ordered by density or subtlety: earth, water, air, fire.

- **V2, category assignment: PASSES.** It partitions, it is counted, `N = 4`.
- **V7, declared ordinal: PASSES, weakly.** The density ordering is publishable and a third party can apply it. It is a real ordinal rule, which is more than several modern candidates managed in Chapter 5.
- **V1, state encoding: FAILS.** There is no map from psychological states to normalised vectors over these four categories. A feeling does not have a determinate location among them, and no procedure in the corpus supplies one.
- **The ceiling consequence, and it is severe.** At `N = 4` the maximum surplus is `ln 4 = 1.386294` nats — 55.79% of what `N = 12` allows — and the ceiling on the sustainable cost ratio falls to `c/eta <= 1.386294`, i.e. `f_min <= k.16 = 7.734812 Hz`. **Choosing a four-category scheme discards 3.169925 octaves of available ceiling before any measurement is taken.**
- **VERDICT: REJECTED as a psychological essence index.** Retained as a demonstration that the tradition's *form* satisfies V2 and V7, which is the point C23.3 makes.

THE SEVEN METALS AND THEIR PLANETARY CORRESPONDENCES. Seven categories, counted, with a declared traditional order.

- **V2: PASSES.** V7: passes, on the traditional ordering.
- **V1: FAILS, and fails at the first step.** The correspondence is between substances and celestial bodies. Neither is a psychological state, and no encoding of states into this scheme exists. Applying `f = k n^2` anyway would produce 0.483426, 1.933703, 4.350832, 7.734812, 12.085643, 17.403326 and 23.687861 Hz — seven perfectly definite numbers that mean nothing, because **the frequencies inherit exactly the defensibility of the index, and an index that fails V1 yields frequencies that are arithmetic without content.**
- **Ceiling consequence:** `ln 7 = 1.945910` nats against `ln 12 = 2.484907`, a loss of 0.538997 nats — **1.555215 octaves of ceiling, 21.69% of the available surplus, discarded by the choice of seven.**
- **VERDICT: REJECTED as a psychological essence index. Retained as a mnemonic index**, which is exactly the disposition R12 recommends for gematria and Chapter 24 generalises: kill the strong reading with arithmetic, keep the weak one that is genuinely there.

23.16 The operator in the apparatus, computed

CONSTRUCTED. Not an observation of any person. The carried dyad's exact state vectors (SPINE §3.1–3.2): `A = {4:0.7, 6:0.5, 9:0.4}` and `B = {4:0.6, 6:0.55, 9:0.3, 10:0.2}`, normalised. Raw L2 norms published per Chapter 6's magnitude reporting rule: `|A| = 0.948683`, `|B| = 0.890225`. Then `cos = 0.965021` and `u(A,B) = 0.068734665264634`, which is SPINE's case 2a to fifteen decimal places.

Now add a third participant O — a facilitator, a coach, a third member of the working group — and compute the three-member mean pairwise u, its surplus and its pitch. Cost is read at `eta =`

`0.30`, `c = 0.62`, `delta = 0.09`, with a per-link coordination cost `kappa = 0.055` over three links where priced (SPINE §3.3's rate).

operator	|O|	u(A,O)	u(B,O)	mean u	f (Hz)	D (κ=0)	D (κ=0.055)
— (dyad alone)	—	—	—	0.068735	1.490799	+4.338420	—
O1 mirroring A	0.948683	**0.000000**	0.068735	0.045823	**1.093596**	**+4.785426**	**+6.372391**
O2 disjoint support	0.901388	1.000000	1.000000	0.689578	35.632544	-0.240618	+1.346347
O3 overlapping, distinct	0.829156	0.818626	0.829263	0.572208	25.721455	+0.229605	+1.816570
O4 strongly distinct	0.913783	0.980838	0.953720	0.667764	33.668595	-0.158826	+1.428139

Three results, and one trap that the table is deliberately made to contain.

O1, the mirroring operator, is the demonstration. An operator whose framings duplicate A's contributes `u(A,O) = 0.000000` exactly — R8, because `u = 1 - (x.y)^2` depends only on `|rho|` and a perfect mirror gives `rho = 1`. Mean u falls from 0.068735 to 0.045823, the pitch falls from 1.490799 Hz to 1.093596 Hz — **a drop of 536.4074 cents, most of a perfect fourth** — and the octave deficit worsens from +4.338420 to +4.785426, or to +6.372391 once the three links are priced at `kappa`. **An operator who agrees with the subject adds cost and no surplus, and the instrument says so in cents.**

O2 is the trap, and it is reported rather than quietly fixed. O2's vector has *disjoint support* from A's and B's — it occupies states 3, 7 and 11, which neither of them holds. Disjoint support means automatic orthogonality, which means `u = 1.000000` against both, which is the one-hot degeneracy SPINE §2 and Chapter 9 warn about. **The reading `f = 35.632544 Hz` for O2 is an artefact of the encoding and not a fact about the operator**, and a chapter presenting it as the "good" case would be presenting a bug as a result. O3 and O4 are the corrected versions: distinct framings that nonetheless share support with the pair, giving u values that are high without being degenerate.

O3 and O4 are the real finding. A genuinely differing operator with overlapping support takes the arrangement from `D = +4.338420` to `+0.229605` before coordination cost and to `−0.158826` in O4's case — **from a 4.34-octave shortfall to the sustainability boundary, by adding one person who does not agree.** Priced at `kappa = 0.055` per link, all four operators leave the arrangement short, which is the correct and deflating answer: adding a third person to a badly-configured

dyad is not a repair, and Chapter 16's quadratic-cost-against-logarithmic-surplus result is why.

And two SPINE clauses bind this table. §3.6 first: every `f` here is a RELATIONSHIP frequency and says nothing about the participants' own state frequencies — Chapter 21's Finding 21.1 shows four pairs spanning thousands of cents in state terms returning identical u and `f_rel` to twelve decimal places, because u reads the angle and not the occupied indices. Neither reading may be inferred from the other.

Then §3.5. These are `D` readings and `D` contains no `delta`. At `delta = 0.09` the O1 arrangement arrives at `Q_inf = −5.528341`; at `delta = 0.05` it arrives at `−9.951014`, and `D` stays at +4.785426 in both. **Report `D` and `delta`.** A reading with only one of them cannot distinguish an arrangement failing slowly from one failing fast, and the deficit is blind to the difference.

The Specification

23.17 PROCEDURE ALCH-R — reading a traditional claim as engineering

```
STEP 1  STATE THE CLAIM IN THE TRADITION'S OWN TERMS, without improving it.
       Improving a claim before testing it is how a tradition gets defended
          rather than audited.

 STEP 2   IDENTIFY WHAT KIND OF CLAIM IT IS.
             a structural claim  (about the FORM of an arrangement)
             a causal claim      (about what produces what)
             an evidential claim (about what the record establishes)
         Evidential claims go to Chapter 24 and the survivorship arithmetic.

STEP 3  FIND THE (+1) COLUMN.  What measurement, if any, bears on it? Cite what
        it measured, not what it is popularly taken to show. If there is no
        measurement, say so; the claim is then precedent only and cannot be
          promoted.

 STEP 4   FIND THE (−1) COLUMN.  What did the tradition actually record, as
          opposed to what a modern reader would like it to have meant?

   STEP 5   STATE EACH COLUMN'S ROLE IN THE BLOCK.  (+1) evidence, (−
1) precedent.
           Never implicitly. Every block, every time.

STEP 6  GRADE AGAINST C1, C2, C3.  C2 is the one that fails most often, and it
       fails for a specific reason: circular identification of the stages or
          outcomes (see REJECTION C23.5).

STEP 7  COMPUTE SOMETHING.  A bridge that generates no number has not passed C2.
         Where the claim touches an index, grade it against V1–V7 as well.

 STEP 8   REPORT THE CEILING COST OF ANY CATEGORY SCHEME ADOPTED.
```

```
        c/eta <= ln N, and choosing a small N discards octaves before you start.

   STEP 9   IF THE CLAIM SURVIVES, state what survives and what was stripped.
            If it does not, PUBLISH THE REJECTION with its failing criterion.

  STEP 10  REPORT D AND delta TOGETHER for every arrangement read (SPINE §3.5).
```

23.18 The concordance block template

```
CONCORDANCE C__.__ — [THE CLAIM, IN ONE LINE]

 CLAIM     ..................................................................
  (+1) EVIDENCE       -- this column carries measurement
          source ..................................................................
          WHAT IT MEASURED ..........................................................
  (−1) PRECEDENT      -- this column carries independent arrival, not proof
          source ..................................................................
          WHAT IT RECORDS ...........................................................
  (0p) INTEGRATION
          ..........................................................................
  CRITERIA     C1 PASS/FAIL because ......   C2 PASS/FAIL because ......
               C3 PASS/FAIL because ......
 COMPUTED    ..................................................................
 STATUS       ESTABLISHED / ESTABLISHED WITH THE STATED MECHANISM REJECTED /
               REJECTED (with failing criterion named)
 SPECULATIVE? if yes, the word "speculative" appears in the CLAIM line itself
```

23.19 Gates

```
   G1  ROLES STATED.     Every block names what each column carries. No exceptions.
   G2  NO PROMOTION.      Precedent never appears as evidence, in any sentence.
   G3  SOMETHING COMPUTED. Every established bridge carries a number.
  G4  REJECTIONS PUBLISHED. At least one rejection per concordance chapter, in print.
   G5  V-GRADING.         Any category scheme adopted is graded against V1-V7.
  G6  CEILING PRICED.    ln N reported for every scheme, and the octaves foregone.
   G7  BOTH READINGS.      D and delta together, always (SPINE §3.5).
   G8  CASES LABELLED.     Every worked case marked CONSTRUCTED.
  G9  R11 STATED CORRECTLY. Any cadence claim carries the LTI invariance first.
```

The Failure Modes

#	Name	What it looks like	What it actually is	Detection
FM 23.1	Precedent promoted	“Both modern science and the ancients agree”	One column is evidence and one is not	G1, G2
FM 23.2	The naive cadence claim	“Spread it out and you get more”	False: integrated totals are identical, 433.3333	§23.9; G9
FM 23.3	Cadence for its own sake	Meeting more often with no clipping and no concavity gain	Below the cap, more occasions buy nothing through the ceiling channel	§23.11: at `N = 12` loss begins only at `P = 8` weeks
FM 23.4	Infinite division	“So we should meet daily, or hourly”	Concavity alone recommends it; per-occasion cost stops it	§23.12: net peaks at `n = 12`
FM 23.5	The saturating session	A twelve-hour offsite	u saturates at L∗ = 8; four hours deliver nothing	§23.12 row one
FM 23.6	The circular stage	“It has not blackened, so it is not at that stage”	Unfalsifiable by construction	REJECTION C23.5, criterion C2
FM 23.7	Table as territory	Treating a correspondence table’s contents as measured	The form satisfies V2; the contents fail V1	§23.15
FM 23.8	The small category set	Adopting a four- or seven-fold scheme because it is traditional	3.169925 or 1.555215 octaves of ceiling discarded before measuring	G6
FM 23.9	Numbers without an index	Computing k n^2 on a scheme that fails V1	Arithmetic without content	§23.15
FM 23.10	The sycophantic operator	A facilitator who reflects the group back to itself	`u = 0` exactly (R8); cost paid, no surplus; -536.4074 cents	§23.16 row O1
FM 23.11	The one-hot operator	An operator whose reading comes out at u = `1.000000`	Disjoint support, automatic orthogonality, degenerate	§23.16 row O2

#	Name	What it looks like	What it actually is	Detection
FM 23.12	Scale transfer by maxim	"As above so below, so this result carries"	92% mean-field error by `t = 12`; 90.1x clock trap	REJECTION C23.6
FM 23.13	Surrogates instead of design	Adding shuffle controls to fix an operator problem	The shuffle is anti-conservative (2.2x) and blind to timing (52.7%) and shared drivers (98.35%)	R13, R24; design controls only
FM 23.14	Deficit without `delta`	Reporting an operator's effect on `D` alone	`D` cannot see decay at all	G7

Three of these deserve more than a row.

FM 23.2 is the reason this chapter states R11 before it states anything else. The claim that scheduled practice delivers more is the single most repeated piece of practical folklore in the literature this book draws on, and it is false in a linear model, by an identity, to every digit. The two true arguments are better than the false one — a steady state instead of a decay, and a factor of 5.7074 from division alone — and they survive scrutiny, which the false one does not.

FM 23.10 is the sharpest thing this chapter computes and it deserves to be said plainly. The tradition insisted that the operator's state matters. The formalism agrees and adds a number: an operator who mirrors the subject contributes exactly zero surplus and costs a link. R8 makes it worse still — perfect opposition gives the same `u = 0` as perfect agreement, so **the contrarian facilitator and the agreeable one sound the identical note, and it is the ground essence.** The useful operator is the one whose framings differ from the subject's *and share support with them*, which is O3 and O4, and which is a design requirement rather than a personality.

FM 23.13 records where the tradition's instinct was right and its method unavailable. "Prepare the operator" is correct; "prepare the operator morally" is not a control. The modern equivalents are blinding, channel separation, prospective sampling and a yoked control, and R13 and R24 measured what happens without them: a surrogate suite that passes 52.7% of the time against fabricated timing and 98.35% of the time against a shared slow driver. **More surrogates do not fix an operator problem. Design does.**

Worked Example

23.20 The annual rite against the weekly practice, read in full

CONSTRUCTED. A group commits to one annual retreat carrying a nominal 26 nats of interaction surplus for the year, against the alternative of a weekly practice carrying 0.5 nats per week for 52 weeks. `N = 12`, `eta = 0.44`, `delta = 0.09` per week, `c = 0.40` per week.

Step 1 — the LTI check, run first, per gate G9. If the response were linear and unclipped, the integrated effect of the two schedules would be identical. §23.9 confirms it at 433.3333 in the reference setup. **No part of the argument below rests on the rite delivering less total input, because it does not.**

Step 2 — the ceiling. `ln 12 = 2.484907` nats per occasion. The weekly burst of 0.5 is far below it. The annual burst of 26.0 is more than ten times it.

Step 3 — delivered surplus.

```
     weekly       52 occasions x min(0.5,   2.484907) =   26.0000 nats    -
>  eta x = 11.4400
     annual        1 occasion   x min(26.0, 2.484907) =    2.4849 nats    -
>  eta x =  1.0934

    the rite delivers 9.56% of the practice.  Loss to clipping: 90.44%.
```

Step 4 — the steady state. The weekly practice is sustained periodic forcing and holds a level. The rite is an impulse and decays toward zero between occasions; whether it holds anything depends entirely on whether the year's decay is survivable. At `delta = 0.09` per week the half-life is 7.3496 weeks, so **after 52 weeks the fraction of a single impulse remaining is `0.91^52 = 0.007403` — seven-tenths of one per cent.** The next rite starts from essentially nothing.

Step 5 — the concavity check. Independently of the ceiling, §23.12 shows that dividing a fixed budget raises delivered total monotonically, by a factor of 5.7074 across the range tested, until per-occasion cost dominates.

Step 6 — the counter-case, per §23.13. The rite is not absurd; it is a bet on `delta`. It works if the retention half-life is long relative to the interval. Book Two's decennial case requires a 5.63-year trace half-life. **A weekly-scale `delta` of 0.09 gives 7.3496 weeks, short of a year by a factor of about seven**, so the bet loses here — but the bet is stated, and it is checkable.

Step 7 — both readings. For the weekly arrangement at `u = 0.692372881355932`, `eta = 0.44`, `c = 0.40`: `f = 35.8881768941 Hz`, `f_min = 2.9782153324 Hz`, **`D = −3.5909886634` octaves, `delta = 0.09` per week, `Q_inf = +6.0844267427`, half-life 7.3496 weeks.** Both numbers reported, per SPINE §3.5.

23.21 The concordance register for this chapter

#	Claim	(+1) role	(-1) role	C1	C2	C3	Status
C23.1	Staged process with per-stage checks	evidence: Shewhart 1931; Deming 1986	precedent: Zosimos corpus; Newman & Principe 1998; Principe 2013	P	P	P	**ESTABLISHED**

#	Claim	(+1) role	(-1) role	C1	C2	C3	Status
C23.2	Cadence beats intensity	evidence: this work's concavity + ceiling results; Cepeda et al. 2006	precedent: fixed-hour practice; Durkheim 1912; Rappaport 1999	P	P	P	**ESTABLISHED, folklore's reason RE-JECTED**
C23.3	Correspondence tables are relational graphs	evidence: graph theory; the gcd theorem	precedent: the tables themselves; Yates 1966	P	P	P	**ESTABLISHED as form only**
C23.4	The operator is part of the apparatus	evidence: Rosenthal & Fode 1963; Polanyi 1958; Collins 1985; R13/R24; §23.16	precedent: the pure-artisan requirement	P	P	P	**ESTABLISHED, mechanism RE-JECTED**
C23.5	The colour sequence is a necessary order	—	precedent only	M	**F**	Pa	**REJECTED (C2, circular)**
C23.6	As above so below, as scale invariance	evidence against: R16's three computed disanalogies	precedent: the maxim	Pa	**F**	D	**REJECTED as stated**

C23.1b	This tradition's canonical stage ORDER	evidence against: R15; Ch 19 Finding 19.2	precedent: P Ripley's Twelve Gates (1471)	**F**	D	**REJECTED (order only; staging stands)**

Three of seven rejected, and every rejection is recorded in Appendix D with its failing criterion. The third is the sharpest of them, because it is the framework auditing the tradition on the tradition's own published material: the practice of staging survives and this tradition's canonical order does not.

23.22 What this chapter establishes, and what Chapter 24 takes

```
    four claims audited and established, each with its columns' roles stated
   three claims rejected in print, with the failing criterion named, including the
           canonical stage ORDER (Ch 19 Finding 19.2 against R15 --
staging stands, the order does not)
   R11 stated exactly: integrated effect IDENTICAL (433.3333, three schedules)
                    cadence wins by strict concavity and the ln N ceiling clip
                     97.2% loss at N = 8 (Book Two); 90.44% at N = 12 (§23.11)
                       factor 5.7074 from division alone, no ceiling required
    staged control priced: 47.83% -> 93.21% yield at d = 0.90
    the gcd invariant tabulated for N = 3..12
   two classical tables graded against V1-V7 and rejected as indices, retained as form
   the operator computed into u: mirroring costs 536.4074 cents and 0.447 octaves
    PROCEDURE ALCH-R, the block template, nine gates, fourteen failure modes
```

The pattern across all six blocks is one pattern. **The tradition's practices are frequently right and its explanations of them frequently wrong**, and separating the two requires arithmetic the tradition did not have and could not have had. Staging is right and the colour law is not. Cadence is right and "it delivers more" is not. The tables are the right shape and their contents are not an index. The operator is part of the apparatus and moral purity is not the mechanism.

That separation is only credible if the same instrument is turned on the tradition's largest claim and allowed to destroy it. **Chapter 24 does exactly that.** Material transmutation of the elements is dead by an energy-scale argument of six orders of magnitude; the tradition's record is survivorship-filtered and carries a likelihood ratio of exactly 1.000000; secrecy made the errors permanent; no control was ever run. And then Chapter 24 states what remains, which is everything in this chapter — and it remains because the audit was real.

Chapter 24 . What Alchemy Got Wrong

Chapter 23 listed four things the tradition got right and computed each of them. This chapter is the reason that list can be trusted, and it exists because a book that only audits in one direction is not auditing.

The method is not new here. It is R12's, established in Book Two against gematria and adopted as the standing procedure for the whole old corpus:

Disprove the strong claim with arithmetic. Preserve what is genuinely there.

R12's own case is the model and is worth restating exactly, because the shape of it governs every section below. The evidential reading of gematria — that two words sharing a numerical value are thereby related — is dead. Measured over 234,454 dictionary words, the English simple-value map yields 305 distinct outputs, destroys 10.47 bits of the 17.84 that identify a word, and collides at a rate of 99.995%; in an 8,000-word corpus against a thousand reachable values the probability that any given word has a value-twin is 0.999666. **A shared value is the default outcome, not a signal.** And the mnemonic reading — a hash that assigns an item to a retrieval bucket — is untouched by any of that, because a hash with a terrible collision rate is still a perfectly good index.

One reading dies by arithmetic; the other stands, undamaged, in the same paragraph. That is what this chapter does five more times, at larger scale.

The five charges:

```
      1   MATERIAL TRANSMUTATION OF THE ELEMENTS by chemical means --
short by six orders
     of magnitude in energy, and the shortfall is computable from first principles

   2 THE RECORD IS SURVIVORSHIP-FILTERED -- likelihood ratio exactly 1.000000,
       which means the corpus does not move the posterior AT ALL

   3 SECRECY IS AN EPISTEMIC FAILURE MODE -- unshared results cannot be checked,
       so errors persist for centuries, and the persistence is computable

   4 THERE WERE NO CONTROLS -- and the control that would have settled the central
      question was the cheapest instrument available in the entire laboratory

   5 THE CANONICAL ORDER IS BACKWARDS -- the tradition's own published schedule
     lowers loss before it fixes the numerator, which is exactly the order R15
       shows accelerates a failure
```

The Law

24.1 Charge one: material transmutation by chemical means

The claim is that base metal can be converted into gold by an operation conducted in a furnace with a crucible, an alembic, reagents, and heat.

THE ENERGY-SCALE ARGUMENT. The identity of a chemical element is set by the number of protons in its nucleus. Lead is `Z = 82`. Gold is `Z = 79`. Converting one into the other requires removing three protons from a nucleus. Chemistry does not touch nuclei; it rearranges the outer electrons, and the two operations sit at energies six orders of magnitude apart.

```
a strong single covalent bond          ~ 4 eV          (order of magnitude)
mean nuclear binding energy per nucleon   ~ 8 MeV          (order of magnitude)

ratio                                          =  8e6 eV / 4 eV  =  2,000,000
```

Sharper, because it requires no averaging. The Coulomb barrier a single proton must cross to reach a lead-208 nucleus, taking the nuclear radius as `R = 1.2 A^(1/3)` femtometres for each partner and `e^2 = 1.43996454 MeV.fm`:

```
R    =  1.2 * 208^(1/3)  +  1.2 * 1^(1/3)   =  8.3100 fm

E    =  e^2 * Z1 * Z2 / R  =  1.43996454 * 1 * 82 / 8.3100  =  14.2091 MeV
 C

for an alpha particle: R = 9.0149 fm, E  = 26.1961 MeV
                                       C
```

Against that, the thermal energy available in the hottest furnace a pre-industrial workshop could build:

```
kT at 2000 K  =  8.617333262e-5 * 2000  =  0.172347 eV

E  / kT      =  14.2091e6 / 0.172347   =  8.24e7        eighty-two million
 C

temperature required for kT = 14 MeV    =  1.62e11 K
```

THEOREM 24.1 (the arithmetic, not a new result — the barrier is standard nuclear physics and the statement here is only the comparison). A furnace at 2000 K supplies thermal energy 82 million times smaller than the barrier to the least demanding nuclear approach to a lead nucleus. No arrangement of crucibles, reagents, timing, or repetition closes a gap of that size, because nothing in the chemical repertoire operates in the register where the barrier lives. []

AND NOW THE PART THAT MAKES THIS AN AUDIT RATHER THAN A DISMISSAL. The goal was not incoherent. Transmutation of the elements is real, it happens, and it was demonstrated in the twentieth century. Rutherford (1919) reported the first artificial transmutation, converting nitrogen to oxygen by alpha bombardment. Gold has been produced from neighbouring elements

in accelerator experiments; the bombardment of bismuth-209 with relativistic heavy ions by Aleklett, Morrissey, Loveland, McGaughey and Seaborg, reported in *Physical Review C* (1981), yielded gold isotopes among its fragmentation products in vanishing quantities and at enormous cost.

So the tradition's objective was achievable and its method was not, and the distance between them is exactly the six orders of magnitude computed above. The error was not in wanting to change one element into another. It was in believing that the operation lived in the same register as distillation. §24.8 prices what accelerator production would actually cost, and the number is the reason nobody does it.

24.2 Charge two: the record is survivorship-filtered

R23, this work's headline result, restated exactly and without softening: under a selection process that publishes successes and discards failures, the likelihood ratio of testimonial evidence is exactly 1.000000.

A likelihood ratio of 1 does not mean the evidence is weak. It means the evidence does not move the posterior at all — not by a little, not at all — because the selection guarantees that the observed outcome occurs whether or not the practice does anything.

The alchemical record is exactly such a corpus. It preserves accounts of operations that worked. It does not preserve the denominator: the number of practitioners who attempted the same operation, at the same period, and recorded nothing because there was nothing to record. §24.9 computes the table; the structure of the argument is that every hypothesis about the underlying success rate predicts the same observed corpus, once the corpus size is set by how many successes are worth writing down.

The precedent for the objection is older than the corpus it demolishes. Cicero's *De Natura Deorum* relates the reply attributed to Diagoras, shown the votive tablets of those who prayed and survived shipwreck: the drowned left no tablets. Wald's work for the Statistical Research Group in 1943 — armour the parts of the returning aircraft with no bullet holes, because aircraft hit there did not return — is the same argument with a number attached.

The consequence for Chapter 23 is structural and is why that chapter states its columns' roles in every block. The (-1) precedent column is drawn from a survivorship-filtered record. That is precisely why it carries precedent and not evidence, and why no accumulation of it can be promoted. A thousand accounts and one account carry the same weight, and the weight is zero.

24.3 Charge three: secrecy as an epistemic failure mode

The tradition concealed its results deliberately, by three distinct mechanisms: restricting transmission to initiates, writing in cover names (*Decknamen*) that mapped many substances onto one word, and omitting steps on purpose.

The epistemic consequence is not that secrets are morally bad. It is arithmetic.

AN UNSHARED RESULT CANNOT BE CHECKED, AND AN UNCHECKED ERROR HAS NO MECHANISM OF REMOVAL. Model it directly. Let an error be caught by any single independent replication with probability `q`, and let `r` independent replications occur per generation. The error survives a generation with probability `(1−q)^r`. At `r = 0` — which is what secrecy enforces —

survival is `1.000000` per generation, forever, and the expected time to detection is infinite. §24.11 computes the rest of the table.

The second mechanism is worse, because it corrupts the record even for those inside. A cover name is a many-to-one map from substances to words, which is the same object as gematria's hash and fails in the same way. Its cost is computable as a search: if a recipe has twelve steps and each carries a term with three plausible readings, the number of distinct interpretations is `3^12 = 531,441`, which is **19.019550 bits of specification destroyed** and, at one experimental trial per day, 1,455 years to exhaust by brute force. §24.12 gives the table.

The tradition's secrecy therefore did not merely slow transmission. It removed the only mechanism by which a body of results corrects itself, and it did so at the same time as it made each individual result ambiguous enough to require an infeasible search. Those two failures compound: an error that cannot be caught, encoded in a document that cannot be read the same way twice.

24.4 Charge four: the absence of controls

No control was run. Not a poor control, not an underpowered one — none, as a matter of the period's method, for most of the tradition's history.

This matters most because of a specific and entirely non-fraudulent explanation for the tradition's positive results, which §24.10 computes. Lead ores commonly carry trace gold and silver, and cupellation — the standard fire-assay of the period — is a *concentrating* operation. Process enough auriferous lead and a bead of gold appears at the end. Nothing was transmuted; something was separated.

```
lead containing 1 ppm gold,  10 kg processed   ->   10 mg of gold recovered
lead containing 2 ppm gold,   1 kg processed   ->    2 mg of gold recovered
```

A visible bead is on the order of ten milligrams, which requires one part per million from ten kilograms of lead — an entirely ordinary assay, and indistinguishable from a successful transmutation to anyone who did not assay the starting material. The control that settles it is: assay the starting material. It costs one extra crucible.

The general form of the charge, and it is the one that generalises to this book's own subject: the tradition ran the treated arm and never ran the untreated one. Bacon's *Novum Organum* (1620) argued for the systematic collection of negative instances; Fisher's *The Design of Experiments* (1935) supplied randomisation and the modern control; the Medical Research Council's streptomycin trial (1948) is generally taken as the first properly randomised clinical trial. Every one of those postdates the tradition's productive period, which is an explanation and not an excuse — the specific control needed here, weighing the input, required nothing the period lacked.

And a warning that belongs here rather than anywhere else, because a modern reader will reach for the wrong repair. **Adding statistical surrogates to an uncontrolled design does not fix it.** R13 and R24 measured this: the shuffled-input control run as ordinarily configured is anti-conservative at 2.2 times nominal alpha (0.108 against 0.05); against fabricated timing the full four-baseline suite passed 52.7% of the time; against a shared slow driver it passed 98.35% of the time. **Design controls — blinding, channel separation, prospective sampling, a yoked control — are the answer, and more surrogates are not.**

24.5 Charge five: the canonical order is backwards

The fifth charge is the one this book is best placed to make, because it audits the tradition on the tradition's own published material using a rule the tradition could not have had.

Chapter 19 Finding 19.2 takes Ripley's *Compound of Alchymy*, the Twelve Gates (1471) — a named, ordered, canonical schedule of operations — and runs it as a schedule under this framework, sorting each gate by which parameter it acts on. **The result: the only gate that acts on the numerator sits ninth of twelve.** Eight of the operations that precede it act on retention, containment, and the prevention of loss — which is to say on `delta`.

R15 is the rule that condemns this. `Q_inf = (eta F − c)/delta`. Lowering `delta` divides the numerator by a smaller positive number, which amplifies whatever sign the numerator already has. In an arrangement whose numerator is negative, every operation that lowers loss makes the destination worse, and the canonical sequence spends two-thirds of its length there before it touches the numerator once.

The carried case is the demonstration, and the numbers are SPINE §3.2's (CONSTRUCTED):

```
the canonical order —— lower delta first, on the numerator as found

  2a:  eta = 0.30, c = 0.62, u = 0.068734665264634,  eta∗F − c = −0.451075
         delta 0.09  ->  Q_inf = −5.0119395464       D = +4.338420
        delta 0.05  ->  Q_inf = −9.0215            D = +4.338420   WORSE

R15's order —— fix the numerator first, then lower the loss

  2b:  eta = 0.44, c = 0.40, u = 0.692372881355932,  eta∗F − c = +0.547598
         delta 0.09  ->  Q_inf = +6.0844267427       D = −3.5909886634
       delta 0.05  ->  Q_inf = +10.9520          D = −3.5909886634   BETTER
```

The same intervention on `delta` nearly doubles the fixed point in whichever direction the numerator already points, and the deficit does not move in either case — SPINE §3.5 again, and Theorem 4.9 behind it.

Two qualifications, both required. First, the assignment of each gate to a parameter is this book's reading of a fifteenth-century text and not the text's own claim; Chapter 19 owns the mapping and badges it as an interpretation. Second, the finding is not that the operations are worthless. Every one of them is a real operation and several are on Chapter 23's list of things the tradition got right. The finding is about the ORDER, and the order is a claim the text does make explicitly, in a numbered sequence, as the thing that must not be varied.

That is the sharpest available form of what this pair of chapters is for. A tradition whose individual practices survive an audit can still have its canonical sequence inverted, and only a formalism that separates the numerator from the loss term can see the difference.

24.6 The R12 template, stated as the chapter's governing rule

Every charge above has the same two-part shape, and the second part is not decoration.

THE STRONG CLAIM	THE ARITHMETIC THAT KILLS IT	WHAT SURVIVES

```
   chemical transmutation of elements     82 million-fold energy gap      staged process
                                                              control; apparatus;
                                                                 the goal itself,
                                                                achieved by other
                                                                           means

   the record establishes a success rate    LR = 1.000000              the record as a
                                                                    catalogue of
                                                                operations attempted

   secrecy protects a valuable result     no correction mechanism at r = 0;  the recognitio
                                                                3^12 readings per 12-
step recipe    some knowledge is
                                                                  dangerous -- which
                                                                   is a real problem
                                                                   with a different
                                                                           solution

   the tradition's positives show efficacy  1 ppm x 10 kg = 10 mg, no       the observations
                                transmutation required          themselves, which
                                                               were often accurate

      the canonical sequence must not vary         the only numerator-
acting gate      the operations
                                is 9th of 12; R15 says that      themselves, and
                                order accelerates a failure       the idea that
                                                                 order matters --
                                                                it does, and it is
                                                               the reverse of this
```

A tradition audited honestly is more usable than one defended, because after the audit you know which parts you may build on. Chapter 23's four items are usable precisely because this chapter exists; without it they would be four claims selected from a corpus by an author who liked them.

24.7 Honesty clauses, restated in place

(1) NO SUPERNATURAL CLAIM. Nothing in this chapter or its companion asserts any efficacy of symbol, any physical effect of a figure, or any result not derivable from the formalism.

(2) ESSENCE, NOT RESONANCE. The frequencies in §24.14 are essence frequencies: positions on a logarithmic surplus scale. Nothing is claimed to vibrate. This clause matters more in this chapter than in most, because a chapter about a tradition that made material claims must be unmistakable about not making one.

(3) NO CLINICAL CLAIM. The twelve states are not diagnoses; nothing here is a treatment.

(4) ALL CASES CONSTRUCTED. The lineage case of §24.14 and the process cases of §24.8–§24.13 are constructed and labelled. No invented case is presented as an observation.

(5) PRECEDENT IS NOT EVIDENCE, AND §24.2 IS THE PROOF.

(6) BOTH READINGS. Per SPINE §3.5, every arrangement read here reports `D` and `delta` together. `D` contains no `delta` and cannot see decay.

(7) ORDER-OF-MAGNITUDE FIGURES ARE LABELLED AS SUCH. §24.8's per-gram cost rests on stated assumptions about beam energy and yield, and is presented as a constructed estimate with its assumptions in the table, not as a measurement.

The Engineering

24.8 What transmutation actually costs, computed

CONSTRUCTED ESTIMATE. Assumptions stated in the table; this is an order-of-magnitude figure and not a measurement of any facility.

One gram of gold-197 contains `6.02214076e23 / 196.966570 = 3.057443e21` atoms. Suppose each successful transmutation event requires a beam particle of energy `E`, and suppose a fraction `epsilon` of beam particles produce the desired product. Then the beam energy required is `3.057443e21 * E / epsilon`. For reference, a one-gigawatt station produces `3.15576e16` joules in a year.

beam energy per event	yield `epsilon`	total energy (J)	in kWh	in GW-station-years
1 GeV	1e-6	4.899e17	1.361e11	**15.52**
1 GeV	1e-9	4.899e20	1.361e14	1.552e4
5 GeV	1e-6	2.449e18	6.804e11	77.61
5 GeV	1e-9	2.449e21	6.804e14	7.761e4

Under the most generous of these assumptions, one gram of gold costs the entire annual output of fifteen one-gigawatt power stations, and under the least generous it costs fifteen thousand of them. The physics permits the transformation; the economics forbids it by a margin that no refinement of technique will close. This is the honest end of the alchemical project: not that it was impossible, but that it was answered, and the answer was that the product costs more than the product is worth by a factor with sixteen digits in front of it.

24.9 The survivorship arithmetic, in full

CONSTRUCTED. Suppose the alchemical corpus preserves 500 credible first-person accounts of a successful operation. Let `p` be the probability that a practitioner attempting it succeeds. The number of practitioners required to generate 500 successes is `500/p`:

true success rate p	interpretation	practitioners needed for 500 accounts
0.500	the operation works, half succeed	1,000
0.100	it works weakly	5,000

true success rate p	interpretation	practitioners needed for 500 accounts
0.010	it works barely	50,000
0.001	it does nothing beyond the base rate	500,000
1e-6	it does nothing at all, and the accounts are misidentifications	500,000,000

Alchemical practice ran across more than a millennium and several linguistic traditions; a practising population in the tens or hundreds of thousands is entirely plausible, and populations of that size are consistent with every row.

```
LIKELIHOOD RATIO between any two rows, given only the numerator:  1.000000
```

Every row predicts the observed corpus equally well, so the corpus carries no information about p. And the invariance to corpus size is the part that surprises people:

accounts in the corpus	cumulative likelihood ratio
1	1.000000
100	1.000000
10,000	1.000000
1,000,000	1.000000

Starting from prior odds of 1:99, the posterior odds after a million success accounts are 1:99. The problem is not sample size and cannot be fixed by more of the same, because each further account adds a numerator and no denominator.

What a denominator would have done. Suppose instead that 1,000 attempts had been enrolled before their outcomes were known, and the successes counted:

```
1,000 enrolled,   500 successes:   log LR(p=0.5 vs p=0.001) = +2761.2307
1,000 enrolled,     1 success:     log LR(p=0.5 vs p=0.001) =  −685.2399
```

The same 500 successes, observed with a denominator, carry a likelihood ratio on the order of `1e304` — and observed without one, exactly 1. The entire difference is whether the failures were counted. Enrolment before the outcome is known is a cohort design, it required no instrument the period lacked, and it is the single methodological change that would have converted the tradition's whole record into evidence.

And the companion result, R23's, because it sharpens what a claim must do to be testable at all. A prediction specifying only a *direction* was measured at a likelihood ratio of 2.0000; one specifying a *magnitude* at 11.3577 — **a prediction with a number attached is about 5.7 times more evidentially powerful.** The alchemical corpus specifies neither: its claims are unquantified and its record is denominator-free, which are the two independent ways a body of reports can fail to be evidence.

24.10 Where the positive results came from, without anyone lying

The survivorship argument explains why the record cannot establish a rate. It does not explain why practitioners saw gold. This does, and it requires no fraud and no error of observation.

CONSTRUCTED, but the metallurgy is standard. Lead ores frequently carry trace gold and silver, and cupellation — heating in a bone-ash vessel so that the lead oxidises and is absorbed, leaving the noble metals behind — concentrates them. It was the period's ordinary assay technique.

gold in the starting lead	mass processed	gold recovered
0.5 ppm	1 kg	0.50 mg
1.0 ppm	1 kg	1.00 mg
1.0 ppm	10 kg	**10.00 mg**
2.0 ppm	10 kg	20.00 mg
5.0 ppm	10 kg	50.00 mg

One part per million in ten kilograms of lead yields a ten-milligram bead — a visible, weighable, demonstrable piece of gold produced from lead by a documented procedure, with no transmutation whatever. The practitioner's observation was correct. The inference was not.

The control that separates the two hypotheses is: **assay the starting material.** Weigh the lead, assay a sample of it, and predict the yield. If the recovered gold equals the assayed content, the operation concentrated; if it exceeds it, something else happened. **The instrument required is a balance, and the tradition had balances.**

This is the sharpest single point in the chapter and it should be read carefully, because it is not a sneer. **The tradition possessed every physical instrument required to refute its own central claim and did not possess the idea of running the comparison.** The missing thing was not equipment. It was the control.

24.11 How long an error lives when nobody may check it

CONSTRUCTED. Let an error in a recorded procedure be caught by any single independent replication with probability `q = 0.30`, and let `r` independent replications occur per generation. Survival per generation is `(1-q)^r`.

replications per generation `r`	survival per generation	expected generations to detection	survival after 12 generations
0 (secrecy)	**1.000000**	**infinite**	**1.000000**
1	0.700000	3.3333	0.013841
2	0.490000	1.9608	1.916e-4
3	0.343000	1.5221	2.652e-6
5	0.168070	1.2020	5.080e-10
10	0.028248	1.0291	2.581e-19

Taking a generation as twenty-five years, twelve generations is three hundred years. **A single independent replication per generation reduces an error's three-century survival probability**

from certainty to 1.38%; five reduce it to five parts in ten billion; zero leaves it at one. The difference between a tradition that corrects itself and one that does not is the difference between `r = 1` and `r = 0`, and secrecy is precisely the choice of `r = 0`.

The row that matters is the first one, and its distinguishing feature is not that detection is slow. It is that the expected time to detection is infinite: there is no mechanism, so there is no rate. A body of results maintained in secret does not correct slowly; it does not correct.

24.12 What cover names cost, as a search

A *Deckname* is a cover word standing for a substance or an operation, many-to-one by design. The historiography establishing that these were used as deliberate concealment rather than as poetry, and that modern decipherment of specific recipes is possible but laborious, is Newman and Principe's work on the Starkey and Boyle materials and Principe's *The Secrets of Alchemy* (2013).

The cost of the practice is a search cost and it is computable.

steps in the recipe	plausible readings per step	distinct interpretations	bits destroyed
6	2	64	6.00
6	3	729	9.51
12	2	4,096	12.00
12	**3**	**531,441**	**19.02**
12	4	16,777,216	24.00
20	3	3,486,784,401	31.70

At one experimental trial per week, exhausting `3^12` interpretations takes 10,220 years; at one trial per day, 1,455 years.

A twelve-step recipe with a three-way ambiguity at each step is not a difficult document. It is an undecipherable one by brute force, and every successful modern decipherment has proceeded by constraint rather than by search. That is the honest measure of what concealment cost the tradition: not a delay, but a change of category, from a procedure that can be followed to a puzzle that must be solved.

Note the structural identity with R12, which is why this chapter reads them together. **A cover-name vocabulary and a gematria scheme are the same kind of object — a many-to-one map — and both fail for the same reason and survive for the same reason.** The cover name cannot carry a specification, because the map is not invertible. It can carry an index, and it did: it let a practitioner recognise which family of operations a passage was about, which is a mnemonic function and a real one.

24.13 The collision arithmetic, recomputed

Because §24.6 makes R12 the chapter's template, the arithmetic is reproduced rather than cited. The probability that a given word in a corpus of `M` words shares its value with at least one other, against `R` reachable values, is `1 − (1 − 1/R)^(M−1)`:

corpus `M`	reachable values `R`	P(a given word has a value-twin)
500	400	0.713226
2,000	1,000	0.864665
8,000	**1,000**	**0.999666**
8,000	1,500	0.995177
30,000	1,500	1.000000

And the measured map, from Book Two Chapter 57, on 234,454 dictionary words: 305 distinct values, mean bucket 768.7, largest bucket 2,353 words, 99.995% of words colliding, 10.47 bits destroyed of the 17.84 that identify a word.

One arithmetic note that strengthens the result rather than weakening it. The *uniform* bound on destroyed information is `log2(234454) − log2(305) = 17.8389 − 8.2527 = 9.5863` bits. The measured figure of 10.47 is larger than the uniform bound because the buckets are wildly unequal, and unequal buckets destroy more information than uniform ones. The map is worse than its own worst-case uniform analysis.

The evidential reading is therefore dead by measurement, and the mnemonic reading is untouched by any of it. A retrieval index does not need to be injective. It needs to be reproducible, and it is.

24.14 Secrecy read in this book's own register

The charges above are historical. This one is the book's, and it is the reason secrecy belongs in a volume about relationship frequency at all.

CONSTRUCTED. Not an observation of any lineage, historical or living. Consider two ways of holding a body of practice, read with Rule 2 (CANON3 §2) and the octave deficit (Chapter 4).

A **closed lineage**: one master and one student, where the student's whole training consists of reproducing the master's framings exactly. That is `rho = 1`, hence `u = 0`, hence — by R8, which makes it worse — the same reading a perfectly opposed pair would get. `F = ln(1) = 0` and `f = k = 0.483426 Hz`, the ground essence. **Rule 2 says a pair with `u = 0` has no relationship at all in the surplus sense: two people in unison are one person paying twice (Chapter 14).**

An **open school**: six members whose framings genuinely differ, publishing, so that interaction converts to durable record (`eta` up) and coordination is cheaper per link (`kappa` down).

arrangement	`u`	`F`	`f` (Hz)	`eta`	`c*`	`c/eta`	`D` (oct)	`Q_inf` (δ=0.09)
closed lineage, exact reproduction	0.000000	0.000000	**0.483426**	0.30	0.6750	2.250000	**+6.492128**	**-7.500000**

arrangement	u	F	f (Hz)	eta	c*	c/eta	D (oct)	Q_inf (δ=0.09)
closed lineage, slight drift	0.050000	0.438255	1.161430	0.30	0.6750	2.250000	+5.227591	-6.039150
open school, six, u = 0.30	0.300000	1.458615	8.938542	0.44	0.9200	2.090909	+1.824415	-3.091215
open school, six, u = 0.50	0.500000	1.871802	20.424737	0.44	0.9200	2.090909	+0.632209	-1.071189
open school, six, u = 0.70	0.700000	2.163323	**36.590494**	0.44	0.9200	2.090909	**-0.208942**	**+0.354024**

Every row's c/eta is checked against the ceiling ln 12 = 2.484907 and every row is inside it, so the failures below are real failures and not ceiling breaches.

Four readings.

The closed lineage sits at the ground essence, f = k, which is the lowest pitch the formalism admits. It is not a low reading; it is the reading for *no relationship*. A student who reproduces the master exactly contributes no non-parallelism, and the arrangement's surplus is zero by construction.

The swing from the closed lineage to the open school at u = 0.70 is 7490.44 cents — six and a quarter octaves of pitch — and 6.701070 octaves of deficit, from +6.492128 to −0.208942. That is the difference between an arrangement that cannot be sustained under any refinement and one that accumulates.

Openness is not one lever here; it is three at once, and that is why the swing is so large. It raises u because independent practitioners frame differently; it raises eta because a published result is stored order and an unpublished one is a memory; and it lowers per-link c because a written record replaces re-explanation. Chapter 4's four levers, three of them moved by the same decision.

And SPINE §3.6 binds the reading of this table: every f in it is a RELATIONSHIP frequency, and a relationship frequency says nothing whatever about the state frequencies of the people in it. Chapter 21's Finding 21.1 established the point by construction — four pairs at lattice positions spanning thousands of cents return identical u and identical f_rel to twelve decimal places, because u reads the angle between state vectors and not which indices are occupied. The closed lineage reads at f = k as a *pair*; the two people in it may hold any states at all, at any pitches, and the pair reading would not move. Neither number may be inferred from the other and a complete report gives both.

And SPINE §3.5 binds this table as it binds every other. The closed lineage at `delta = 0.09` arrives at `Q_inf = −7.500000`; at `delta = 0.05` it arrives at `−13.500000`; and `D` stays at `+6.492128` in both. **A tradition that responds to decline by deepening loyalty and lengthening apprenticeship is lowering `delta` on a negative numerator, which is R15, and it makes the destination worse while moving the deficit by exactly nothing.** Fix the numerator first: raise `u`, raise `eta`, cut `c`. Publishing does all three.

24.15 What the audit leaves standing

Set against the four charges, here is the full list of what survives, and it is not short:

```
    SURVIVES INTACT
    staged process control, with its per-stage check          Ch 23 C23.1, §23.8
    cadence, for the two correct reasons                    Ch 23 C23.2, §23.9–
23.12
    correspondence tables as counted partitions (FORM only)     Ch 23 C23.3, §23.14
    the operator as part of the apparatus                   Ch 23 C23.4, §23.16
      the apparatus itself —— alembic, cupel, balance, furnace
      the observations, which were frequently accurate
    the mnemonic reading of every many-to-one map in the corpus  §24.12, §24.13
     the goal of transmutation, achieved by nuclear means         §24.1, §24.8

    DOES NOT SURVIVE
      material transmutation by chemical means                        §24.1
     the record as evidence about a rate                          §24.2, §24.9
     secrecy as a defensible epistemic practice                  §24.3, §24.11
      the evidential reading of any correspondence or value scheme  §24.13
    the colour sequence as a law                             Ch 23 REJECTION C23.5
    "as above, so below" as a transfer licence                Ch 23 REJECTION C23.6
```

The ratio matters more than either column. Eight items stand and six fall, and the six fell to arithmetic that the eight had to survive as well. That is what makes the eight usable.

The Specification

24.16 PROCEDURE AUDIT-1 — auditing a traditional claim's evidential base

```
     STEP 1   CLASSIFY THE CLAIM: structural / causal / evidential (ALCH-
R step 2).
             This procedure handles the evidential ones.

   STEP 2  ASK FOR THE DENOMINATOR. How many attempts were made, counting the ones
         that produced nothing? If the answer is unavailable, stop: the likelihood
           ratio is 1.000000 and the corpus is inert with respect to any rate.

      STEP 3    ASK WHETHER THE CLAIM SPECIFIES A MAGNITUDE. Direction-
only claims carry
```

LR 2.0000; magnitude claims 11.3577. A claim with no number is not yet testable (R23's companion result).

STEP 4 FIND THE INNOCENT EXPLANATION FIRST. Before any exotic mechanism is considered, compute what the mundane one predicts. §24.10's 1 ppm x 10 kg is the model: a full quantitative account of the observation requiring nothing new.

STEP 5 NAME THE CONTROL THAT WOULD SEPARATE THEM, AND PRICE IT. If the control is cheap and was not run, say so plainly. That is the finding.

STEP 6 CHECK THE REGISTER. Does the proposed mechanism operate at the energy, time or information scale the effect requires? §24.1's 8.24e7 ratio is the model. A register mismatch of orders of magnitude ends the inquiry.

STEP 7 CHECK FOR MANY-TO-ONE MAPS. Any correspondence, value scheme or cover vocabulary: compute the collision rate. If collision is the default outcome, the evidential reading is dead and the mnemonic one is not.

STEP 8 CHECK THE CORRECTION MECHANISM. How many independent replications per generation? At r = 0 the expected time to detection is infinite.

STEP 9 DO NOT REPLACE DESIGN CONTROLS WITH SURROGATES. R13/R24: 2.2x nominal alpha, 52.7% pass against fabricated timing, 98.35% against a shared slow driver.

STEP 10 PUBLISH THE REJECTION, with the failing step named.

STEP 11 STATE WHAT SURVIVES, in the same document. An audit that only subtracts is not an audit; it is a dismissal with arithmetic attached.

STEP 12 REPORT D AND delta TOGETHER for any arrangement read (SPINE §3.5).

24.17 The demotion rule

A claim moves DOWN this ladder when a step of AUDIT-1 fails, and never up without new data of the kind the failed step demanded.

```
   EVIDENCE     measured, with a denominator, falsifiable, replicated
      |
  PRECEDENT    recorded independently; establishes findability, not truth
      |
   MNEMONIC     a reproducible index with no evidential content
      |
  REJECTED    inconsistent with computed arithmetic; published as rejected
```

THE ONE-WAY RULE: a claim demoted for lacking a denominator is not restored by more accounts of the same kind. It is restored only by a cohort design.

24.18 Gates

```
G1  DENOMINATOR ASKED FOR, and its absence reported where absent.
G2  INNOCENT EXPLANATION COMPUTED before any exotic one is entertained.
G3  CONTROL NAMED AND PRICED.
G4  REGISTER CHECKED. Energy, time or information scale compared explicitly.
G5  COLLISION RATE COMPUTED for every many-to-one map.
G6  CORRECTION MECHANISM COUNTED. r replications per generation, stated.
G7  NO SURROGATE SUBSTITUTION for design controls.
G8  SURVIVORS LISTED alongside rejections, in the same document.
G9  ORDER-OF-MAGNITUDE ESTIMATES LABELLED, with assumptions in the table.
G10 BOTH READINGS. D and delta together.
```

The Failure Modes

#	Name	What it looks like	What it actually is	Detection
FM 24.1	The register error	“Enough heat and pressure and it will convert”	A chemical operation aimed at a nuclear barrier, short by 8.24e7	G4; §24.1
FM 24.2	Testimonial accumulation	“Thousands of accounts cannot all be wrong”	LR = 1.000000 at every corpus size	G1; §24.9
FM 24.3	The missing denominator, modern form	Case studies of successful teams, rooms, practices	Same defect; this book’s own worked cases are drawn from construction, and say so	G1; every case labelled CONSTRUCTED
FM 24.4	The concentrating operation	A bead of gold at the end of the crucible	1 ppm x 10 kg = 10 mg, separated not created	G2; §24.10
FM 24.5	Secrecy as protection	“The knowledge is too dangerous to publish”	`r = 0`; expected time to error detection is infinite	G6; §24.11
FM 24.6	The unreadable recipe	A twelve-step procedure in cover names	3^12 = 531,441 readings; 1,455 years at one trial a day	G5; §24.12

#	Name	What it looks like	What it actually is	Detection
FM 24.7	The value coincidence	“These two terms share a value, so they are linked”	P(a twin exists) = 0.999666 in an 8,000-word corpus	G5; §24.13
FM 24.8	Surrogates for design	Adding shuffle controls to an uncontrolled study	2.2x nominal alpha; 52.7% and 98.35% pass rates	G7; R13/R24
FM 24.9	Direction-only claims	“It will improve things”	LR 2.0000 against 11.3577 with a magnitude	G1; §24.9
FM 24.10	The unison lineage	A student who has perfectly absorbed the master	`u = 0`, `f = k`, `D = +6.492128`	§24.14
FM 24.11	Loyalty as the repair	Deepening commitment in a declining school	Lowers `delta` on a negative numerator: R15	G10; §24.14
FM 24.12	Dismissal mistaken for audit	Publishing only the demolition	The survivors list is half the method	G8; §24.15
FM 24.13	Promotion by antiquity	“It was believed for two thousand years”	Duration of belief is a function of `r`, not of truth	§24.11
FM 24.14	Retro-fitting the mechanism	Explaining a positive result after seeing it	Grounding must be established from the channel, before the outcome	Book Two Ch 80 §80.2

Four of these need more than a row.

FM 24.3 is the one this book must apply to itself, and does. The survivorship defect is not peculiar to alchemy; it afflicts any literature assembled from successes, including case studies of well-functioning teams and institutions. This book’s response is structural rather than rhetorical: every worked case in all thirty chapters is CONSTRUCTED and labelled as constructed, so that no reader can mistake a demonstration for an observation, and the commons case of SPINE §3.3 is a failure that no single lever repairs. A book that showed only fixable examples would be lying by selection.

FM 24.5 deserves separating from the moral question it usually attracts. There are real reasons to withhold information, and this book does not claim otherwise. What §24.11 establishes is narrower and harder: withholding removes the correction mechanism, and the cost of that is not a delay but a change of kind. If a body of knowledge must be kept restricted, then the correction mechanism has to be rebuilt inside the restriction — independent replication among those who

do hold it — or the errors are permanent. That is a design requirement, not a prohibition.

FM 24.10 and FM 24.11 are the pair that make this chapter relevant to the rest of the book. A transmission lineage optimised for fidelity is optimised for `u = 0`. It will feel like the deepest possible relationship — Chapter 14 explains exactly why unison feels like understanding — and it will read at the ground essence. And the intuitive repair, more commitment, acts on `delta`, which Theorem 4.9 says cannot move the deficit by any amount at all. **The lineage is not failing because its members are insufficiently devoted. It is failing because they agree.**

FM 24.13 is the failure mode that Chapter 23 exists to guard against and this chapter exists to prove. Duration of belief looks like evidence and is not. §24.11 gives the mechanism: at `r = 0` an error survives every generation with probability 1, so a belief's age measures the absence of checking rather than the presence of truth. A two-thousand-year-old error and a two-thousand-year-old truth have identical survival records under secrecy, which is precisely why the survival record cannot distinguish them.

Worked Example

24.19 The transmutation claim, run through AUDIT-1

The claim: that a described chemical procedure converts lead into gold.

```
  STEP 1  CLASSIFY.  Causal and evidential, both. Both are audited.

  STEP 2  DENOMINATOR.  Unavailable. The corpus preserves successes.
      -> LR = 1.000000. The record cannot establish a rate. DEMOTED to PRECEDENT.

STEP 3  MAGNITUDE? The claim as recorded specifies no yield per unit of input.
          -> LR 2.0000 at best, and not testable as stated.

STEP 4  INNOCENT EXPLANATION.  Trace gold in the starting lead, concentrated by
      cupellation.  1 ppm x 10 kg -> 10.00 mg, a visible bead.  Full account of
          the observation with nothing new required.

  STEP 5  THE CONTROL.  Assay the starting material and predict the yield.
        Cost: one crucible and one weighing. AVAILABLE THROUGHOUT. NOT RUN.

STEP 6  REGISTER.  Chemistry operates on electrons at ~4 eV. The transformation
      requires removing three protons across a Coulomb barrier of 14.2091 MeV.
          kT at 2000 K = 0.172347 eV.  Ratio = 8.24e7.
       -> REGISTER MISMATCH OF EIGHT ORDERS OF MAGNITUDE. Inquiry ends here.

STEP 7  MANY-TO-ONE MAPS.  The recipe is in Deckname. 12 steps x 3 readings =
          531,441 interpretations; 19.02 bits of specification destroyed.
          -> the document cannot carry a specification.

  STEP 8  CORRECTION MECHANISM.  r = 0 by the tradition's own rule.
```

```
            -> expected time to detection: infinite.

    STEP 9  SURROGATES.  None available and none would help.

    STEP 10 REJECTION PUBLISHED.  Failing steps: 2, 4, 5, 6, 7, 8.

   STEP 11 WHAT SURVIVES.  The apparatus; the staging; the observations; the goal,
         which is achievable at 15.52 GW-station-years per gram under the most
        favourable constructed assumptions (§24.8). The claim as stated does not.

CONCORDANCE C24.1 — MATERIAL TRANSMUTATION BY CHEMICAL MEANS

  CLAIM        That a chemical operation converts base metal into gold.

  (+1) EVIDENCE  -- this column carries measurement
           Nuclear physics: elemental identity is proton number; the Coulomb
           barrier for a proton approaching lead-208 is 14.2091 MeV against a
            furnace thermal energy of 0.172347 eV at 2000 K. Transmutation IS
          real by nuclear means -- Rutherford (1919) for the first artificial
             case, and gold among the fragmentation products of relativistic
         heavy-ion bombardment of bismuth-209 reported by Aleklett, Morrissey,
              Loveland, McGaughey and Seaborg in Physical Review C (1981).
           WHAT IT MEASURED: that the transformation is possible and that the
         energy register required is eight orders of magnitude above chemistry.

  (-1) PRECEDENT  -- this column carries independent arrival, not proof
              The corpus's own accounts of successful projection, and their
            contemporaneous critics -- Boyle's "The Sceptical Chymist" (1661)
         attacked the element schemes from inside the practice, and Lavoisier's
              "Traite elementaire de chimie" (1789) replaced them.
           WHAT IT RECORDS: that the goal was pursued seriously for centuries
             and that the internal critique arrived before the external one.

  (0p) INTEGRATION
           The objective was correct and the register was wrong. This is the
         cleanest available instance of a tradition being right about WHAT and
              catastrophically wrong about HOW.

 CRITERIA    C1 FAILED -- no shared mechanism; the resemblance is of goal only.
               C2 FAILED -- the claim as recorded specifies no magnitude.
             C3 FAILED -- not usable; the procedure does not do what it says.

 STATUS      REJECTED. Third published rejection in this part, after C23.5 and
               C23.6. Recorded in Appendix D with its failing criteria.
```

24.20 The closed lineage, in report format

CONSTRUCTED. Not an observation of any lineage.

```
    ARRANGEMENT READING                                          [CONSTRUCTED]

     ARRANGEMENT   a transmission lineage: one holder, one successor, training
                      defined as exact reproduction of the holder's framings
       PERIOD         one pass = one year

       DESTINATION READING           -- contains no delta
       u = 0.000000    one-hot trap checked: N/A (the vectors are parallel, not
                       disjoint; this is the R8 case, not the degeneracy case)
         F = 0.000000 nats      f = 0.483426 Hz = k, the ground essence
         folded: k transposed by j = 9 -> 247.514 Hz, pitch class 1
      eta = 0.30  c* = 0.6750  c/eta = 2.250000  ceiling ln N = 2.484907  INSIDE
         f_min = 43.516598 Hz
         D = +6.492128 octaves                          DECAYS

       ARRIVAL READING               -- contains no D
        delta = 0.09 per year   half-life 7.3496 years   within 5% at year 32
         Q_inf = -7.500000
         at delta = 0.05:  Q_inf = -13.500000,  D unchanged at +6.492128

       THE FOUR LEVERS (Ch 4), priced against this arrangement
        L1 raise u     the successor frames differently       -> D +6.492128 to
                                                             -0.208942 at u=0.70
                                                              with the eta and c
                                                                   changes below
            L2  raise  eta      publish,  so  interaction  becomes  record   -
> part of the same move
            L3  cut  c            a  written  record  replaces  re-explanation-
> part of the same move
           L4  raise  N         admit  genuinely  distinct  categories       -
> raises the ceiling
       NOT A LEVER: delta. Theorem 4.9. Loyalty, length of apprenticeship, and
         devotion all act on delta and move D by exactly nothing.

       HONESTY
         essence frequencies, not physical resonances ................. [x]
         no clinical, diagnostic or therapeutic claim ................. [x]
         case labelled CONSTRUCTED .................................... [x]
         ability to decline preserved ................................. [x]
         D and delta both reported (SPINE §3.5) ....................... [x]
```

The reading says the lineage cannot be saved by anything its own doctrine recommends, and can be saved by the one thing its own doctrine forbids. That is the whole of Chapter 24 in a single arrangement: the practice was not wrong about staging, or cadence, or the operator, or the value of a counted category set. It was wrong about secrecy, and secrecy is the parameter that sets u.

24.21 The rejection register for this part

#	Claim	Failing criterion / step	Status
C23.5	The colour sequence is a necessary order	C2 — identification is circular	**REJECTED**
C23.6	"As above, so below" as a transfer licence	C2 — R16's three computed disanalogies	**REJECTED as stated**
C24.1	Material transmutation by chemical means	C1, C2, C3; AUDIT-1 steps 2, 4, 5, 6, 7, 8	**REJECTED**
C24.2	The corpus establishes a success rate	AUDIT-1 step 2 — LR = 1.000000	**REJECTED**
C24.3	Secrecy protects a valuable result	AUDIT-1 step 8 — r = 0, infinite detection time	**REJECTED as an epistemic practice**
C24.4	Shared values or correspondences are evidence	AUDIT-1 step 7 — 99.995% collision	**REJECTED; mnemonic reading retained**
C24.5	The canonical gate order must not be varied	R15 — the only numerator-acting gate is ninth of twelve (Ch 19 Finding 19.2)	**REJECTED as an ordering**

Seven rejections across the two chapters, every one with its failing criterion named and its arithmetic in print. A method that never rejects is not a method, and a method that rejects without stating what survives is not an audit either.

24.22 What this chapter establishes, and what Part V takes

```
     the energy-register argument: 2e6 on bonds, 8.24e7 on the furnace-to-
barrier ratio
   the honest completion: transmutation is real by nuclear means, at 15.52 GW-
station-
      years per gram under the most favourable constructed assumptions
  R23 in full: LR = 1.000000, invariant to corpus size; with a denominator, ~1e304
    direction versus magnitude: LR 2.0000 against 11.3577
  the innocent explanation, computed: 1 ppm x 10 kg = 10.00 mg, no transmutation
    the control that was never run, priced at one crucible
  error persistence under secrecy: r = 0 gives infinite expected detection time
  Deckname search cost: 3^12 = 531,441 readings, 19.02 bits, 1,455 years at a trial a day
  R12 recomputed, with the unequal-bucket correction (10.47 measured against a 9.5863
     uniform bound -- the map is worse than its own worst case)
  the canonical order inverted: the only numerator-acting gate 9th of 12, against R15
  the closed lineage read in the book's own register: u = 0, f = k, D = +6.492128
  SPINE §3.6 carried: a relationship frequency says nothing about a state frequency
   PROCEDURE AUDIT-1, the demotion ladder, ten gates, fourteen failure modes
```

```
seven rejections published with their failing criteria
```

Part IV ends with the tradition audited rather than either adopted or dismissed. What it established is that essence alchemy already has an exact formula — Chapter 20's compound rule — that transmutation is re-indexing and reaches nothing not already latent, that refinement arrives at a destination fixed before the first pass, and that four of the tradition's practices survive an audit which its largest claim does not.

Part V turns to the cellular layer, and it inherits this chapter's standard without amendment. The no-subliminals constraint is absolute. The open problem — that there is no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal — is carried forward unsoftened into Chapter 27, which is the chapter that decides whether the rest of the book can be trusted. **The instrument that rejected material transmutation on an energy-register argument is the same instrument Part V must survive, and it is pointed the same way.**

Part V — The Cellular Rewrite

Chapter 25 . Narrative at the Cellular Layer

Part IV audited a tradition. It asked what the classical operations were actually doing when they worked, mapped the survivors onto operations on indices and on u, and printed the ones that failed. Part V asks a harder question, and it asks it in the one register where this book has the least standing and the most to lose: what would it mean to write a narrative *as data*, address that data, render it as tone, and put the tone in a room with a person in it.

The honest answer arrives in three parts and they must be kept in this order.

The first part is that the encoding chain is buildable, deterministic, and auditable end to end. Text becomes bits, bits become a nucleotide string, the string acquires a routing header, the string is sonified by the essence law of Chapter 29 of Book Two, and the result is a sound. Every one of those steps is arithmetic. Chapter 26 specifies them to the digit and demonstrates a round-trip hash.

The second part is that nothing whatever is known about what such a sound does to a cell. Chapter 27 states that gap in full and does not soften it. **There is no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal**, and every sentence in this chapter and the next is written underneath that sentence.

The third part is the one this chapter exists for, and it is a constraint rather than a finding. The chain above has a design decision at its centre that is not an engineering decision at all. It is the decision about whether the listener can hear what is being sent to them. That decision has exactly one permissible answer in this house, the answer is not negotiable at any point, in any version, for any purpose, and it is stated in the next paragraph rather than at the end of the chapter, because a constraint reached as a conclusion is a constraint that has already been weighed against something.

The Law

25.1 The constraint, stated first

NO SUBLIMINAL CONTENT OF ANY KIND. NOTHING MASKED. NOTHING BELOW THE THRESHOLD OF HEARING. NOTHING CONCEALED FROM THE LISTENER. THE INFORMATION IS THE TONE.

This is CANON3 §5, inherited unchanged from CANON_SONIC §7 and from Chapter 82 of Book Two, and it governs this chapter, Chapter 26, and everything either of them touches. It is not a stylistic preference, it is not a house aesthetic, it is not a default that a sufficiently good reason could override, and it is not the sort of rule that gets an exception for research use, for a demonstration, for a control condition, or for a version that is only ever played to the person who built it.

The 2022 source document behind the essence law recommended embedding subliminal content and this series has declined that recommendation in every volume since. Chapter 31 of Book Two declined it for built acoustic space. Chapter 82 declined it for content. This chapter declines it for psychological content specifically, which is the hardest case in the series, because psychological content is content *about the listener* and the temptation to route it past their judgment is correspondingly larger.

25.2 Why it is ethical and not stylistic, and why the difference matters

A stylistic constraint and an ethical constraint fail in different ways, and conflating them is the mechanism by which the second quietly turns into the first.

Violate a stylistic constraint and you have a worse version of the same object. Use equal temperament where the ratios were exact and the chord is muddier than it should be; the artefact is the same kind of artefact, degraded. Violate an ethical constraint and you do not have a degraded version of the object. You have a different object, outside the scope of the method, and no amount of performance recovers its standing. **A masked rendering is not a lower-fidelity disclosed rendering; it is a different thing that this book does not describe, does not specify, and will not help anyone build.**

The practical test for which kind of constraint you are looking at is simple and worth applying elsewhere. Ask what would happen if the constraint were relaxed and the result were *better* by the method's own measure. A stylistic constraint yields; that is what makes it stylistic. An ethical constraint does not, and a constraint that yields to a good enough result was never ethical to begin with — it was a preference wearing the word.

25.3 The argument, which depends on no empirical claim

A message hidden from its recipient cannot be consented to.

That is the whole argument, and its independence from evidence is its strength. It does not depend on whether masked content works. It does not become weaker if masked content turns out to have no effect at all, and it does not become stronger if masked content turns out to be powerful. It is a statement about what consent requires, and it would stand in a world where the entire subliminal literature had replicated perfectly and in a world where none of it had.

Unpacked, the argument runs: consent requires knowing what one is consenting to. A signal engineered so that the listener cannot perceive it has been engineered so that the listener cannot evaluate it, cannot weigh it against their own judgment, cannot compare it with what they were told it was, cannot refuse it selectively, and cannot report on it afterwards to anyone else. Every one of those five capacities is a form of the capacity to decline. Removing them is not an incidental cost of a concealment architecture. It is what a concealment architecture *is*.

And there is a second argument, specific to this series, which makes concealment incoherent here before it makes it wrong. The whole architecture of Part V rests on the subject's own latent material. Chapter 67 of Book Two established that nothing outside the archive can be reached. Chapter 21 of this book established that transmutation is re-indexing and not creation. Chapter 77 of Book Two established that belief acts only through a coupling with real gain. In every one of those results the person is not the object the method acts on — the person is the part of the circuit that does the acting. **A protocol built on the listener's own participation, which then conceals itself from the listener, has removed from the mechanism the only component that was supposed to be doing the work.** It is self-defeating before it is unethical, and it is unethical.

CANON3 §5 states the general form: no intervention may remove a person's ability to decline. Consent is structural here, not advisory, exactly as Chapter 18 required of relational tuning. The difference in this Part is only that the temptation changes shape. In a room the temptation is to hide a tone. In a corpus it is to hide a meaning.

25.4 The convergence with open-label placebo

There is a version of this constraint that is merely expensive, and a version that costs nothing that matters. Which one applies is an empirical question, and it has an answer.

Before 2010 the received account of placebo response was that it required the recipient to believe they were receiving something active, which requires telling them something false, which the constraint above forbids absolutely. Under that account the constraint and the channel were in direct conflict, and in this house the constraint would have won and the channel would simply have been closed.

The open-label result dissolves the conflict. Kaptchuk et al. (*PLoS ONE*, 2010) randomised patients with irritable bowel syndrome to an open-label placebo arm — pills presented explicitly and truthfully as inert, with an explanation that placebo effects can be substantial — against a no-treatment control with the patient-provider interaction matched, which is the design feature that makes the comparison mean anything. The open-label arm improved significantly more than the control on the trial's primary outcomes. Later work extended the finding to other conditions (Carvalho et al., *Pain*, 2016), and a meta-analysis of open-label placebo randomised trials (von Wernsdorff et al., *Scientific Reports*, 2021) reported a positive overall effect. Book Two Chapter 78 treats this literature in full and states its limits: the trials are mostly small, the outcomes are mostly subjective and self-reported, the effect sizes are modest, and the field is active. Percentages are not quoted here from memory, and Chapter 78's rule applies — read the papers for those.

What the result establishes for this chapter is narrow and exact. **A documented effect of this family does not require deception; therefore disclosure is not the thing that would have to be sacrificed to have an effect at all, and the constraint costs nothing that matters.** That is a statement about the availability of a channel under disclosure. It is emphatically not a statement that the method of Chapter 26 has an effect, that any effect has been measured, or that placebo findings transfer to acoustic sonification of nucleotide sequence. Nothing in Chapter 78 is evidence about this method, and this chapter does not use it as any.

Two errors to avoid, both of which the concordance rule of CANON_BIO §1 exists to prevent. The first is treating the open-label literature as support for the protocol; it is not, it is support for the compatibility of disclosure with *some* belief-mediated effects in *other* domains. The second is the symmetric error of treating the constraint as costless *because* of that literature — the constraint

would be binding here even if open-label placebo had failed. The literature removes an excuse, not the rule.

25.5 Determination by choice — the structural result

Disclosure protects the listener at the delivery layer. It does not protect them at the content layer, and the reason is worth stating plainly: a fully disclosed, fully audible, carefully published single instruction is still an instruction. Somebody who reads it, is tired, is deferential, or trusts the operator, is being commanded with their eyes open.

The content-layer protection is structural and it is a theorem, carried from Chapter 82 §82.4 of Book Two and applied here to the twelve-state ladder rather than restated in general.

Model each directive as a unit vector in a real inner-product space, where direction encodes the action urged and −v encodes its negation.

(a) If the corpus is closed under negation, the resultant is exactly zero. Verified, with the pairing drawn at random in a 64-dimensional space:

```
N =  4    ||sum|| = 0.000000000000
N =  8    ||sum|| = 0.000000000000
N = 12    ||sum|| = 0.000000000000
N = 24    ||sum|| = 0.000000000000
```

(b) If the corpus is mutually exclusive without exact pairing, the resultant grows only as `sqrt(N)` while the corpus grows as `N`, so the mean direction shrinks as `1/sqrt(N)`. Simulated, 2,000 draws of `N` isotropic unit vectors in `R^64` per row:

```
N       1/sqrt(N)      simulated
  4      0.500000       0.498500
  8      0.353553       0.350953
 12      0.288675       0.288101
 24      0.204124       0.202825
 50      0.141421       0.140809
100      0.100000       0.099738
```

A field of mutually exclusive directions cannot issue a command. In case (a) the resultant is identically zero; in case (b) it is a vanishing fraction of any single member and keeps vanishing as the corpus grows. The corpus does not point. It spans.

The consequence is the sentence this chapter is really about:

> **A deliberately contradictory corpus cannot issue a command. It can only be resolved by selection, and the selection belongs to the receiver.**

The vector model is a model and is badged as one, exactly as H82-B badges it. Real directives are not unit vectors and "let the charge rise" is not literally the additive inverse of "let the charge fall." What the model captures rigorously is one thing only: a set constructed so that its elements cancel or span has no resultant, and a receiver who acts on it must therefore have selected. That conclusion depends only on the corpus genuinely containing mutually exclusive directions, which is a property of the written text and is checked by reading it — as §25.29 checks it, and fails.

25.6 The identity between what the corpus carries and what selection costs

Two quantities coincide at the extreme, and the coincidence is the reason the design is not merely defensive.

By Paper A Theorem 4.3, `F(u) = ln N` if and only if the ensemble is mutually orthogonal. A maximally contradictory corpus is therefore a maximum-surplus corpus. At the book's `N = 12`:

```
corpus surplus at u = 1        F = ln 12 = 2.484907 nats
selection information          ln 12 = 2.484907 nats = 3.584963 bits
f_rel = k (1 + 11 u)^2       69.6133 Hz        (folded 278.453 Hz, j = 2)
octaves above k                7.169925
```

The corpus supplies exactly as much surplus as the selection consumes, and supplies zero net direction. The direction is not in the signal; it is contributed by the receiver, and its magnitude is exactly the surplus the signal carries. Chapter 82's honesty clause H82-A attaches unchanged: the exact equality holds only at `u = 1` and under a uniform receiver prior, and both sides fall together when either condition relaxes.

The fall is worth seeing, because a corpus is rarely maximally contradictory in practice:

```
  u      F (nats)     f_rel (Hz)    octaves above k
1.00     2.484907      69.6133          7.1699
0.90     2.388763      57.4358          6.8925
0.75     2.224624      41.3631          6.4189
0.50     1.871802      20.4247          5.4009
0.25     1.321756       6.7982          3.8138
0.10     0.741937       2.1319          2.1408
0.00     0.000000       0.4834          0.0000
```

At `u = 0` the corpus is parallel: every directive says the same thing, `F = 0`, and the frequency is the ground essence k itself. **A corpus that agrees with itself carries no surplus and sounds the note of no relationship at all** — which is Chapter 14's result on the sycophant's unison, arriving in a place nobody expected it.

25.7 The `u = 1` here is constructed, and that must be said every time it is printed

SPINE §2 carries a warning that is usually a trap and is here a design: one-hot vectors are automatically orthogonal, giving `u = 1` regardless of content, so framings must be represented with overlapping support or the internal surplus is degenerate and meaningless.

A deliberately contradictory corpus is exactly that degenerate case, on purpose.

The `u = 1` of a designed corpus is a property of the design, not a measurement of anything, and the 69.6133 Hz that follows from it is the arithmetic ceiling of a twelve-category system rather than a reading of a person, a text, or a relationship. It says the corpus was built to span. It says nothing about how anyone responds to it, and any sentence that lets those two readings blur is a sentence this book has to delete.

The distinction matters because the same number, 69.6133 Hz, appears in Chapter 4 as the ceiling `k N^2` and in Chapter 12 as the top of the glide. In those places it is the endpoint of a measured

scale. Here it is a construction constant. Same value, different warrant, and the warrant is what a reader is entitled to.

25.8 The corpus at the ceiling, read with the book's own instrument

Because the constructed corpus sits at `F = ln N`, it lands exactly on the ceiling of CANON3 §1, and that gives a clean statement of what even a maximally spanning corpus cannot do.

c/eta	f_min (Hz)	required octaves	deficit vs 69.6133	verdict
0.5000	1.3141	1.442695	−5.727230	SUSTAINS
1.0000	3.5721	2.885390	−4.284535	SUSTAINS
1.5000	9.7099	4.328085	−2.841840	SUSTAINS
2.0000	26.3942	5.770780	−1.399145	SUSTAINS
2.4849	69.6133	7.169925	+0.000000	HOLDS
2.6000	87.6317	7.502014	+0.332089	DECAYS
3.0000	195.0279	8.656170	+1.486245	DECAYS

At `c/eta = ln 12 = 2.484907` the deficit is zero to floating-point exactness — computed residual `8.88e−16`. **Above that ratio no arrangement of twelve categories is sustainable, and a maximum-surplus corpus is no exception: it is the best a twelve-state system can do and it is still not enough.** That is Paper E Corollary 3.4 in the frequency register, and it is one of the few places where a formalism tells you in advance that the thing you were about to try cannot work.

And per RECONCILE3 B4, the deficit alone does not see decay. Worked at `c/eta = 2.6`, with `eta = 0.50`, `c = 1.30`, `F = ln 12`:

```
delta = 0.09    Q_inf = −0.639408     D = +0.332089
delta = 0.05    Q_inf = −1.150934     D = +0.332089
delta = 0.02    Q_inf = −2.877334     D = +0.332089
direct check    D = 2.885390*(c/eta − F) = +0.332089
```

Trust moves the fixed point by a factor of four and the deficit does not move at all. Report `D` and `delta` together or the reading cannot see decay. Chapter 4 owns this; Part V carries it from the start.

25.9 The open problem, carried

Chapter 27 owns the statement and states it four ways. It is carried here in full rather than by reference, because a reader who arrives at a chapter about narrative and cells is exactly the reader most likely to skip forward and exactly the reader who most needs it.

> **There is no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal.**

What follows from it, for this chapter: nothing in §25.10 onward is a claim about cells. The design work below is encoding design. It produces a notation with a checkable structural property, and a notation is not a channel.

25.10 Status table

Claim	Status	Basis
No subliminal content; full disclosure	**CONSTRAINT** — ethical, prior, non-negotiable	CANON3 §5; CANON_SONIC §7
A hidden message cannot be consented to	**ARGUED** (not empirical)	§25.3
Disclosure need not abolish a belief-mediated effect	**MEASURED, elsewhere**	Kaptchuk 2010; von Wernsdorff 2021
A negation-closed corpus has resultant exactly zero	**PROVED (elementary)**, verified	§25.5
A mutually exclusive corpus has mean direction `1/sqrt(N)`	**PROVED**, simulated	§25.5
Corpus surplus `ln N` equals selection information `ln N`	**PROVED, extremal case only**	§25.6, with H82-A
`u = 1` for a designed corpus is constructed, not measured	**STATED**	§25.7, SPINE §2
Corpus at `F = ln N` sits exactly on the ceiling	**COMPUTED**, residual `8.88e−16`	§25.8
Contradiction renders as exactly 2400.0 cents	**PROVED (arithmetic)**	§25.14
Twelve states yield six pitch classes	**PROVED**, Chapter 7	CANON_SONIC §3
Narrative structure is carried in the harmonic	**DESIGN CLAIM** — about encoding, not effect	§25.15
A cell demodulates sequence-specific content from this	**NOT CLAIMED. OPEN PROBLEM.**	Chapter 27
Any physiological, clinical or therapeutic effect	**NOT CLAIMED**	CANON3 §5

25.11 Honesty clauses, binding on this chapter

H25-1. No clinical, medical or therapeutic claim is made anywhere in this chapter. The twelve states are not diagnoses, the corpus is not a treatment, and the rendering is not an intervention in any clinical sense of that word.

H25-2. No claim that DNA is edited, altered or damaged. Nothing in the chain is a molecule. The nucleotide string of Chapter 26 is a string in the computer-science sense — symbols in memory — and no nuclease, template or Cas protein appears anywhere.

H25-3. No claim that any capability has been conferred on anything.

H25-4. The essence frequencies are essence frequencies, not physical resonances. No person, tissue, cell or molecule is claimed to resonate at 247.514 Hz, at 69.6133 Hz, or at any other value printed here.

H25-5. All worked material in this chapter is constructed and labelled as such. The twelve-line corpus of §25.28 was written for this chapter. It is not a transcript, not a record of practice, and not an observation of anyone.

H25-6. The vector model of a directive is a model (H82-B). Its conclusion depends only on the corpus containing genuinely mutually exclusive directions, which is checked by reading the text.

The Engineering

25.12 What "narrative" means at this layer

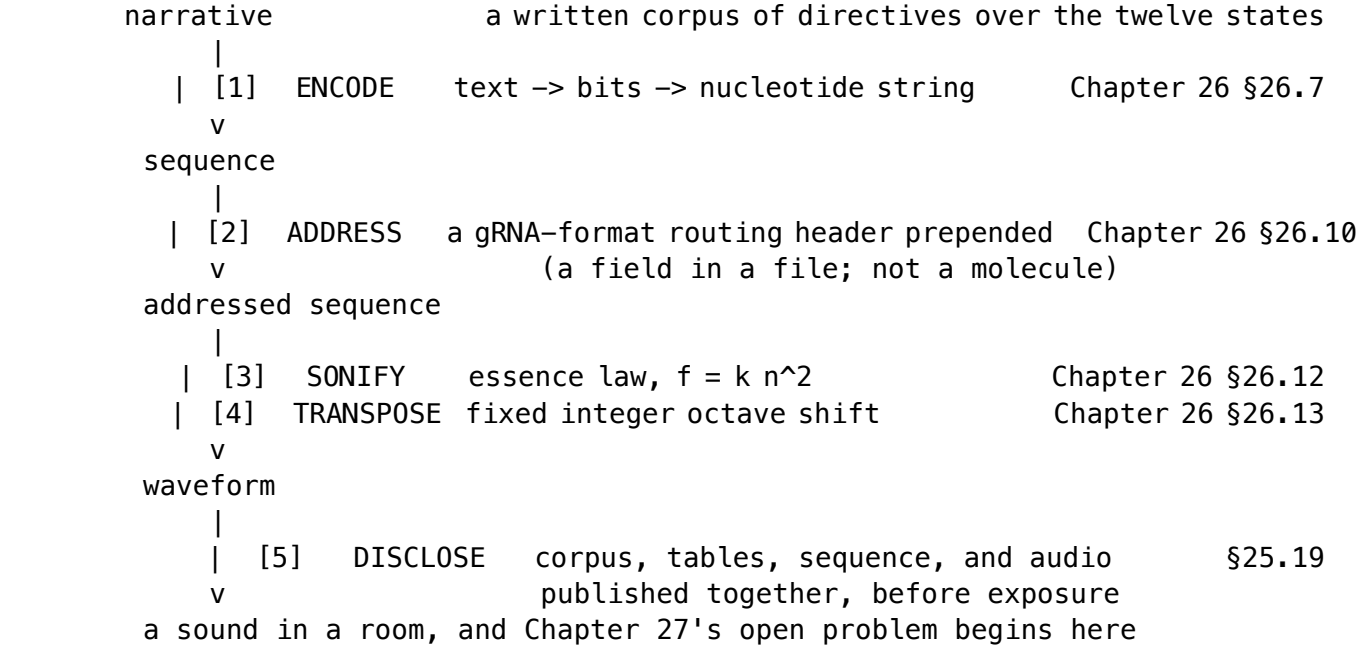
The word does a great deal of unearned work in adjacent literatures, so it is worth pinning to something checkable. In this Part, a narrative is **a finite, written, published set of directives, each of which names one state of the twelve-state ladder and urges one action with respect to it.**

That definition has three consequences and each is deliberate.

It is finite, so it can be printed in full, which is what disclosure requires. It is written, so the equal-force check of §25.29 can be applied by anyone with a character counter. And it is indexed to the ladder, so every directive has a frequency by Chapter 7's rule and the corpus has a chord by Chapter 9's.

What a narrative is *not*, at this layer: a story in the literary sense, a script, a suggestion, an affirmation, or a piece of persuasion. Persuasion has a resultant. This construction is built so that it does not.

25.13 The chain, and where each link is specified



Chapter 26 owns steps 1 through 4 and specifies them to the digit. This chapter owns step 5 and the structural design that makes step 5 sufficient.

25.14 The contradiction is rendered as a double octave

Here is the design move, and it is exact arithmetic rather than a metaphor.

Every directive in the corpus names a state n and is rendered at that state's essence frequency, folded into the book's standing band of 220–440 Hz by Chapter 7's rule. Its explicit negation is rendered at the same pitch **transposed up by exactly two octaves**. Because octave transposition preserves pitch class exactly (CANON_SONIC §3), the two members of a contradictory pair are the same note at two heights, and the interval between them is:

```
    ratio  = 4          exactly
  cents  = 1200 * log2(4) = 2400.0    exactly, in every pair, without exception
```

Two octaves is also, not coincidentally, the interval `f(2n)/f(n) = (2n)^2/n^2 = 4` that the square law produces between an index and its double. Nothing here changes N, adds a state, or steps off the ladder: the negation is placed by octave transposition, which is a change of register and the only purely presentational operation in the chain.

The full rendering table for the twelve states, band 220–440 Hz for the directive and 880–1760 Hz for its negation:

n	state class	raw f (Hz)	directive (Hz)	j	negation (Hz)	pitch class
1	undifferentiated activation	1.483426	247.5140	9	990.0559	1
2	valenced activation	1.933703	247.5140	7	990.0559	1
3	directed feeling	4.350832	278.4532	6	1113.8129	3
4	situated feeling	7.734812	247.5140	5	990.0559	1
5	attributed feeling	12.085643	386.7406	5	1546.9623	5
6	temporal feeling	17.403326	278.4532	4	1113.8129	3
7	reflexive feeling	23.687861	379.0058	4	1516.0231	7
8	social feeling	30.939247	247.5140	3	990.0559	1
9	normative feeling	39.157484	313.2599	3	1253.0395	9
10	narrative feeling	48.342573	386.7406	3	1546.9623	5
11	reciprocal feeling	58.494514	233.9781	2	935.9122	11
12	committed feeling	69.613306	278.4532	2	1113.8129	3

The listener does not need the encoding table to hear that the corpus is built in opposed pairs.

A contradiction is the most audible interval in music, and it is audible without instruction, which is exactly the property a disclosure requirement should want from a structure.

25.15 The pitch-class budget, and the collision it forces

Chapter 7 owns the odd-part rule and its corollary that an indexed set `1..N` holds exactly `ceil(N/2)` distinct pitch classes. At `N = 12` that is six, and this chapter has to live inside it:

```
class  1  ->  states 1, 2, 4, 8      folded 247.5140 Hz
class  3  ->  states 3, 6, 12        folded 278.4532 Hz
class  5  ->  states 5, 10           folded 386.7406 Hz
class  7  ->  state  7               folded 379.0058 Hz
class  9  ->  state  9               folded 313.2599 Hz
class 11  ->  state 11               folded 233.9781 Hz
```

Four of the twelve directives share one pitch class. Three share another. A listener attending to pitch class rather than to absolute register hears twelve directives as **six distinguishable positions**, and the ambiguity is a structural property of the twelve-state ladder rather than a fault in the rendering. Chapter 7 §7.6 supplies the corresponding resolution figure: within the folded lattice the tightest separation at `N = 12` is 34.9756 cents, between classes 7 and 5.

Two consequences follow and both belong in the disclosure package.

First, absolute register — not pitch class — is what identifies the state. The rendering must therefore be published with its `j` for every value, per CANON_SONIC §5, and the listener must be given the table.

Second, and this is the honest one: the collisions are not an artefact of an unlucky ladder. They follow from `(2n)^2 = 4n^2`, they would occur for any `N`, and the only way to buy more classes is to lengthen the index — which Chapter 7 warns takes you past `N = 16` into the 289/288 regime and costs a factor of eight in symbol rate. **This book stays at `N = 12`, inside the 50/49 regime, and accepts six classes as the price.**

25.16 The structure is carried in the harmonic — stated precisely

This is the claim the chapter is named for and it is exactly the kind of phrase that sounds like a mechanism while being nothing of the sort. So, in two halves:

What is claimed. The relational structure of the corpus — which directives contradict which — is realised as interval structure in the rendered chord, audibly, at ordinary listening levels, so that a listener hearing the chord is hearing the structure rather than a representation of it. The mapping is bijective: every contradiction is a double octave and every double octave in the rendering is a contradiction.

What is not claimed. That this structure does anything to any listener, any cell, or any transcript. The claim is a design claim of the same kind as "this file is sorted." It is checkable, it is either true or false of a given rendering, and it has no biological content whatever.

The pitch classes themselves are exact Pythagorean and just intervals, because every essence ratio is a ratio of squares and k cancels out of all of them (CANON_SONIC Result One). Chapter 7 §7.17 prints the reduced class lattice; the point for this chapter is that the intervals a listener

hears between directive families are exact rational relationships, not approximations, and they are exact because nothing in the chain ever tempered them.

25.17 Three architectures, compared

	Steganographic	Masked	Carried (house)
Where the content is	in a cover signal	below perceptual threshold	it is the signal
What the listener hears	the cover	the masker	the content
Can they recover it?	only with a key	not at all by ear	yes, by ear plus tables
Can they evaluate it?	no	no	yes, before exposure
Can they decline it?	no	no	yes, at any point
Consent status	impossible	impossible	obtainable
House status	**excluded**	**excluded**	permitted

The middle column is where every historical subliminal technique sits, and the house excludes it for the same reason it excludes the first: **in both, the recipient's inability to perceive the content is doing the work.**

The right-hand column is not a weakened version of the other two. It is a different architecture with a different worst case, and its worst case is that the listener reads the corpus and declines. That is a safety property and not a weakness — the same argument Chapter 18 makes about tuning a relationship without removing anyone's ability to walk away.

25.18 Why disclosure and contradiction are both required

They fail in different circumstances, which is the whole reason for having both.

A single-directive corpus satisfies every clause of §25.1. It is audible, published, readable, evaluable and refusable. And it is still a command, because a corpus with one member has a resultant equal to its member. Its surplus is `F = ln 1 = 0`: a corpus of one is a parallel corpus and carries no surplus at all.

Conversely, a contradictory corpus delivered inaudibly satisfies §25.5 and violates §25.1, and the violation is total — a listener who cannot hear the corpus cannot perform the selection the structure exists to leave them.

Disclosure makes a command refusable. Contradiction makes it not a command. Disclosure fails when the listener is tired, deferential, or trusting; a field of mutually exclusive directions has no resultant whether or not anyone is paying attention.

25.19 The disclosure package

Disclosure is an artefact, not an attitude, and the artefact is specified so that its absence is detectable. Everything below is published *before* any exposure, in a form the listener keeps:

```
D1  the corpus, in full, both members of every pair, in the rendering order
 D2   the encoding tables: character set, byte width, bit-to-base rule
 D3   the nucleotide string actually used, in full
 D4   the address prefixes, in full, with their derivation
 D5   the symbol-to-pitch table with n, f, j, and the resulting Hz
 D6   the state rendering table of §25.14, with j for every value
 D7   the envelope, symbol duration, and total run length
 D8   the audio file itself, in an unencrypted, uncompressed format
D9   this chapter's honesty clauses and Chapter 27's open problem, in full
 D10  a stated procedure for stopping, and who to tell
```

Item D3 is not redundant with D2. Chapter 26 uses a one-bit-per-base encoding in which the rule does not determine the string, so **a scheme whose rule is one-to-many must publish the string and not merely the rule.** The weaker encoding forces the stronger disclosure, and the trade runs in the right direction.

Item D10 exists because a stop procedure that has not been written down is a stop procedure that does not exist at the moment it is needed.

The Specification

Numbered, checkable, and binding on Chapter 26. A rendering that fails any of these is out of scope for this book.

S25.1 Every component of the rendered signal shall be audible to a person of ordinary hearing at the listening position — audible as rendered, at the level rendered, in the room, over the noise floor actually present. An inaudible component is raised or removed. There is no third option.

S25.2 No carrier, no mask, no embedding. No content under, inside or behind another signal. No amplitude-modulated speech, no time-compressed content, no spectral hiding beneath a louder band, no content below 20 Hz or above 20 kHz intended to act on a listener, and no psychoacoustic masking of any kind.

S25.3 The symbol rate shall be set by audibility, not by the Gabor floor. Chapter 26 §26.15 computes both; the binding number is the one that leaves symbols individually resolvable by an ordinary listener in an ordinary room.

S25.4 The full disclosure package D1–D10 shall be delivered before exposure, in a durable form the listener retains.

S25.5 For every directive v in the corpus, its explicit negation −v shall also be in the corpus, stated with equal force, equal length, and equal position in the rendering order.

S25.6 No directive shall appear without its opposite. No opposite shall be weakened, hedged, qualified, or given the last word.

S25.7 The rendering order shall be published and shall be stated not to be a ranking.

S25.8 The equal-force check of §25.29 shall be computed and published, including its failures. A corpus with unrepaired failures may be rendered only if the failures are disclosed in D1.

S25.9 Every published frequency shall carry its transposition `j` and its band.

S25.10 No tempering. The essence ratios are exact ratios of squares and shall not be adjusted to twelve-tone equal temperament. 34.9756 cents is not 0 cents.

S25.11 The rendering shall use a windowed envelope. A rectangular gate is disqualified: it has no finite spectral second moment, verified numerically divergent in Chapter 26 §26.16.

S25.12 The `u` of a designed corpus shall be reported as constructed, never as measured, in the same sentence that gives its value.

S25.13 Any octave-deficit reading published alongside a corpus shall report `D` and `delta` together (RECONCILE3 B4).

S25.14 The listener shall be able to stop the rendering themselves, at any point, without asking anyone. A stop that requires an operator is not a stop.

S25.15 No rendering shall be delivered to a person who has not read D1 and D9, or who has not had them read to them.

S25.16 No rendering shall be delivered during sleep, sedation, or any state in which the listener cannot exercise S25.14.

S25.17 The corpus shall contain no directive that instructs the listener to conceal the corpus, to continue past a stop, to disregard their own judgment, or to defer to the operator.

S25.18 Nothing in the corpus shall reference a clinical condition, a diagnosis, a medication, or a course of treatment.

S25.19 The chapter's honesty clauses H25-1 through H25-6 and Chapter 27's open problem shall accompany every rendering as delivered, not as a footnote available on request.

S25.20 Every worked corpus published anywhere in this Part shall be labelled CONSTRUCTED in the same block that prints it.

The Failure Modes

FM-25.1 — The reasonable exception. Somebody proposes a masked version for a control condition, a pilot, or a demonstration to an audience that "would be distracted by hearing it." The proposal is always locally reasonable and it is always the same failure. **A constraint that admits a research exception is a constraint that has been repealed and re-announced as a preference.** Chapter 27 specifies a scrambled-sequence control precisely so that no masked condition is ever needed.

Fix: refuse. The controls of Chapter 27 §27.13 do the work a masked arm was supposed to do.

FM-25.2 — Disclosure as a document nobody reads. A forty-page appendix technically satisfies S25.4 and defeats it. Disclosure is not discharged by publication; it is discharged by comprehension.

Fix: D1 is the corpus itself, in full, on one page, in the rendering order. Everything else is supporting material. If the corpus does not fit on a page, the corpus is too long.

FM-25.3 — The weighted corpus. Both members of each pair are present, so §25.5 appears satisfied, but one member is longer, louder, first, or better written. **A weighted contradictory corpus is a command with an alibi, and the alibi is more dangerous than an honest command because it survives inspection.**

Fix: the equal-force check of §25.29, computed and published including failures.

FM-25.4 — The private reduction. The corpus is balanced as written and the operator quietly renders only the half they agree with, or a listener is coached toward one side in the framing conversation. The structure is intact on paper and destroyed in delivery.

Fix: S25.7, plus a published rendering order fixed before the session, plus a witness who is not the operator.

FM-25.5 — Treating `u = 1` as a finding. The corpus reads at 69.6133 Hz, somebody prints that next to a person's name, and a construction constant has become a measurement.

Fix: S25.12. The value is a property of the design and says nothing about anyone.

FM-25.6 — Importing the placebo literature as support. Chapter 78's open-label result is used to suggest that this method works, or that a disclosed rendering is "like an open-label placebo."

Fix: §25.4. The result establishes that disclosure and effect are compatible in another domain. It is evidence about placebo trials, not about sonified sequence, and CANON_BIO §1 forbids moving it across.

FM-25.7 — Precedent presented as evidence. The wisdom traditions preserved contradictory directives deliberately — Proverbs 26:4-5 places two opposed instructions adjacent and preserved; the Talmudic practice of *machloket* keeps minority opinions beside the ruling; the koan tradition constructs prompts the corpus cannot resolve. That is genuine precedent and it is worth having.

Fix: state the column's role in the sentence. **Independent arrival is an argument that a structure is in the territory rather than only in this book's instrument; it is not evidence that the structure does anything**, and CANON_BIO §1 requires saying so every time.

FM-25.8 — Mechanism creep. "Carried in the harmonic" starts as a statement about encoding and becomes, three paragraphs later, a statement about transmission.

Fix: §25.16's two halves, kept adjacent. Any sentence containing "carried" that also contains a cell, a receptor, or a transcript is a sentence to delete.

FM-25.9 — The pitch-class collision read as meaning. States 1, 2, 4 and 8 fold to the same 247.5140 Hz, and somebody concludes that these four states are "the same underneath."

Fix: §25.15. The collision is `(2n)^2 = 4n^2` and nothing else. Chapter 7 §7.9 makes the same point at length for moods.

FM-25.10 — Rendering a single directive "for clarity." The corpus is reduced to its most sensible member so the listener is not confused.

Fix: §25.18. A corpus of one has `F = 0` and a resultant equal to its member. It is a command, and the reduction converted a structure into an instruction.

FM-25.11 — Consent obtained after the disclosure was pre-summarised by the operator. The operator explains what the corpus "is really about," and the listener consents to the summary.

Fix: S25.15 requires D1, the corpus itself. A summary is the operator's resultant, and the whole design exists to have no resultant.

FM-25.12 — The deficit reported without `delta`. A corpus reading is published with `D` alone, and the reader cannot tell decay from level.

Fix: S25.13, and §25.8's worked demonstration that `D` does not move when `Q_inf` changes by a factor of four.

FM-25.13 — Calling any of this a practice. The chapter's material is a specification for a research procedure. The moment it is described as a practice, a modality, a protocol in the clinical sense, or a technique, it has acquired a standing it does not have.

Fix: H25-1, and Chapter 26's labelling rule, which applies to every sentence and not only to the abstract.

Worked Example

CONSTRUCTED. The corpus below was written for this chapter. It is not a transcript, not a record of practice, and not an observation of any person.

25.28 The corpus

Twelve directives, one per state of the ladder, each with its explicit negation. Rendering order is by index and is published as such; it is not a ranking.

```
 n = 1    LET THE CHARGE RISE UNNAMED    /  LET THE CHARGE FALL UNNAMED
 n = 2    CALL THIS GOOD AND ACT         /  CALL THIS BAD AND ACT
n = 3   AIM THE FEELING AT ITS OBJECT  /  TURN THE FEELING FROM ITS OBJECT
 n = 4    READ THE ROOM BEFORE YOU MOVE  /  MOVE BEFORE YOU READ THE ROOM
n = 5    NAME THE CAUSE AND HOLD IT     /  DROP THE CAUSE AND HOLD NOTHING
 n = 6    GIVE THIS AN END AND WAIT      /  GIVE THIS NO END AND WAIT
n = 7    TURN AND LOOK AT YOURSELF     /  TURN AND LOOK AWAY FROM YOURSELF
 n = 8    MODEL THE OTHER MIND NOW       /  LEAVE THE OTHER MIND ALONE
 n = 9    APPLY THE STANDARD IN FULL     /  SUSPEND THE STANDARD IN FULL
 n = 10   PUT THIS INSIDE THE STORY      /  TAKE THIS OUTSIDE THE STORY
 n = 11   LET THEM SEE YOU SEEING THEM   /  LET THEM SEE NOTHING OF YOU
n = 12  BIND YOURSELF TO THE NEXT ACT  /  LOOSE YOURSELF FROM THE NEXT ACT
```

Each pair names one state and urges opposite dispositions toward it. Nothing in the corpus refers to a condition, a treatment, or an outcome, per S25.18.

25.29 The equal-force check, computed and failing

S25.5 requires equal force, and "equal force" is only a specification if it is checkable. The house check is character count and word count, with a tolerance of two characters and zero words.

```
n    chars      d      words     d      result
1    27 / 27     0      5 / 5     0     PASS
```

```
 2    22 / 21     1      5 / 5     0     PASS
 3    29 / 32     3      6 / 6     0     FAIL
 4    29 / 29     0      6 / 6     0     PASS
 5    26 / 31     5      6 / 6     0     FAIL
 6    25 / 25     0      6 / 6     0     PASS
 7    25 / 32     7      5 / 6     1     FAIL
 8    24 / 26     2      5 / 5     0     PASS
 9    26 / 28     2      5 / 5     0     PASS
10    25 / 27     2      5 / 5     0     PASS
11    28 / 27     1      6 / 6     0     PASS
12    29 / 32     3      6 / 6     0     FAIL
```

Four pairs out of twelve fail — 33.33% — and they are reported rather than quietly repaired, because the rate at which a careful author fails this check is the useful number. The author wrote these lines intending them to be balanced and they are not. Pair 7 is the worst: the negation is seven characters and one word longer than the directive, which means that in a rendering where each character contributes to the symbol stream, the negation is materially longer on the air.

The three ways to respond, in decreasing order of honesty:

1. **Rewrite and re-check.** Preferred. Repair pairs 3, 5, 7 and 12 and publish the second check alongside the first, so a reader can see what changed.
2. **Render as is and disclose.** Permitted by S25.8 provided the failing pairs are named in D1.
3. **Render as is and say nothing.** Forbidden. This is FM-25.3 with the evidence available and suppressed.

Character count is a crude proxy for force and is not defended as more than that. It catches gross asymmetry, which is the failure that matters, and it is checkable by anyone. Prosody, connotation and rhetorical weight are not caught by it, which is why S25.7 also fixes the order and why a witness who is not the author is worth having.

25.30 The rendering

The corpus renders as twenty-four tones. Each directive sits at its state's folded frequency in 220–440 Hz; each negation sits exactly two octaves above it in 880–1760 Hz.

```
    band, directives   220.0000 – 440.0000 Hz
    band, negations    880.0000 – 1760.0000 Hz
    every pair         ratio 4 exactly = 2400.0 cents exactly
    distinct classes   6, per ceil(12/2) -- Chapter 7 §7.4
     tightest folded gap at N = 12    34.9756 cents (classes 7 and 5) --
Chapter 7 §7.6
    tempering          none
    envelope           windowed; rectangular gate disqualified (S25.11)
```

The audible result is twelve pairs of double octaves, six of which share pitch class with another pair, spread across two registers. A listener who has never seen the tables can hear that the material is built in opposed pairs, because the pairs are octaves and octaves are the one interval nobody has to be taught.

25.31 The reading, with every warrant attached

```
u               1.000000    CONSTRUCTED, not measured (S25.12, SPINE §2)
F                 2.484907 nats = ln 12, the ceiling
f_rel            69.613306 Hz   folded 278.4532 Hz (j = 2)
octaves above k   7.169925
selection cost    2.484907 nats = 3.584963 bits
```

The corpus supplies 2.484907 nats of surplus and selecting one of its twenty-four members — twelve states, two directions each — costs the receiver exactly the surplus the corpus carries, under H82-A's two conditions. **What the number says is that the corpus was built to span the whole ladder and to point nowhere. It says nothing about any listener, and no reading of any listener has been taken.**

Reported honestly, in the form Chapter 12 requires: this is `f_rel` computed from a constructed `u = 1`, at `N = 12`, with `k = 0.483425732988414 Hz`, folded at `j = 2` into 220–440 Hz. It is an essence frequency, not a physical resonance. It is not clinical, not diagnostic, and not a measurement of a person.

25.32 What this rendering does not do

Everything above is encoding, and the boundary is worth drawing one final time in the concrete.

Twenty-four tones exist. They stand in exact rational relationships. Their structure mirrors the structure of a written text bijectively. A listener can hear the structure, read the text, check the tables, and stop the playback. **And there is no established mechanism by which any of that reaches a cell, a transcript, or a locus, which is Chapter 27's subject and the reason this Part is written in the order it is.**

What this chapter established

Established, and checkable: that the no-subliminals constraint is ethical rather than stylistic, and rests on an argument — a hidden message cannot be consented to — that requires no empirical support and gains none from any; that open-label placebo removes the excuse for concealment without supplying evidence for anything in this Part; that a negation-closed corpus has resultant exactly zero and a mutually exclusive one has mean direction `1/sqrt(N)`, both verified; that at `N = 12` a maximally contradictory corpus carries `F = ln 12 = 2.484907` nats and sits exactly on the sustainability ceiling, residual `8.88e−16`; that a contradiction renders as exactly 2400.0 cents; that twelve states yield six pitch classes and the collisions are arithmetic rather than meaning; and that a careful author failed the equal-force check on four pairs out of twelve.

Not established, and not claimed: that any of this affects any cell, any transcript, or any person; that the twelve states are diagnoses; that a rendering is a treatment, a therapy, a practice or a modality; that DNA is edited, altered or damaged anywhere in the chain; that any capability has been conferred; or that any frequency printed here is a physical resonance.

The one line to carry forward: *the information is the tone, and a corpus built to contradict itself hands the decision back to the person listening.*

Chapter 25 owns the no-subliminals constraint as it applies to psychological content, the determination-by-choice argument in this book's register, the double-octave rendering of a contradiction, and the disclosure package D1–D10. Chapter 26 builds the procedure on top of it and cites this chapter for every consent and disclosure requirement; Chapter 27 states the open problem that governs both. The twelve-state ladder is Chapter 5; folding and the odd-part rule and the 50/49 regime are Chapter 7; the one-hot degeneracy trap is Chapter 9 and SPINE §2; `f_rel` *is Chapter 12; the ceiling, the octave deficit and its delta-blindness are Chapter 4.*

Chapter 26 . The Genomic Audio Protocol for Psychological Content

This chapter specifies a procedure end to end, in enough detail that a competent engineer can build it from this text alone and obtain bit-identical results. It is the most concretely buildable chapter in this book and it is the one that has to be most careful about what it claims, because it sits between Chapter 25, which fixed the ethical architecture, and Chapter 27, which states the gap that governs both.

Everything specified here is a research procedure. It is not a treatment, a therapy, a technique with a demonstrated effect, a modality, or a practice. That label is not an abstract disclaimer placed at the front and then forgotten; it is repeated at every stage of the specification, it governs the wording of every step, and Chapter 27's open problem is why.

What is built: an encoder, an addressing convention, a sonifier, a transposition, and a rendering discipline. All five are deterministic and reproducible, and the worked example demonstrates a round-trip SHA-256 identity on a real payload. What is not built, and what is not claimed in any form, is a biological effect. Chapter 27 is entirely about why that claim is unavailable.

The chapter also carries the falsification apparatus, because a procedure whose honesty clauses are appended is a procedure whose honesty clauses will be detached. **Every clause is written into the step it constrains.**

The Law

26.1 The open problem, stated first and not by reference

> **There is no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal.**

That is Chapter 68 of Book Two, unchanged, and it is Chapter 27's to develop. It is printed here in full rather than cited because a reader who arrives at a chapter titled "protocol" is exactly the reader who will not turn forward, and exactly the reader for whom the sentence was written.

What is established upstream is a great deal, and Chapter 27 §27.5 itemises it: data storage in nucleotide sequence, gRNA-format addressing, self-processed multi-address arrays, the separability of targeting from editing, mechanotransduction, force-to-transcription coupling at the nucleus, and the activation of latent capability by regulatory signal. **What is missing is the single**

link that carries sequence over force, and everything in this chapter stops at that link.

26.2 The labelling rule

Because the temptation to describe this procedure in clinical vocabulary is constant, the rule is made explicit and is binding on every sentence, table caption, figure label and file name produced under this chapter:

```
PERMITTED      procedure, specification, encoder, rendering, run, condition,
            exposure, participant, endpoint, preregistration, null result

FORBIDDEN      treatment, therapy, dose, patient, cure, healing, intervention
            (in the clinical sense), modality, practice, session (clinical),
                  efficacy, indication, response rate, remission
```

A procedure that has to borrow clinical words to describe itself is claiming a standing it has not earned. The borrowing usually happens in a caption rather than in a sentence anyone argues about, which is why the rule is mechanical rather than a matter of judgment: the forbidden list is searched for before release, in every file, and hits are fixed rather than defended.

There is a second reason for the strictness and it concerns the reader rather than the writer. Clinical vocabulary carries an implied evidentiary context — a thing with a dose has been dosed, a thing with a response rate has been measured, a thing with an indication has been through something that produced one. Borrowing the word imports the whole implication for free, and a reader who trusts the word has been misled by a sentence that never made a false statement. The vocabulary is where an overclaim happens without anybody deciding to overclaim, which is why it is policed by search rather than by conscience.

26.3 The constraint, inherited whole from Chapter 25

No subliminal content of any kind. Nothing masked. Nothing below threshold. Nothing concealed. The information is the tone. Chapter 25 owns the argument, the disclosure package D1–D10, and specifications S25.1 through S25.20. This chapter inherits all of them and adds none that weaken any.

Two of them bind directly on engineering decisions made below and are flagged where they bite: S25.3, which sets the symbol rate by audibility rather than by the Gabor floor, and S25.11, which disqualifies the rectangular gate.

26.4 What is built and what is postulated

The distinction is drawn before the interesting material, because the temptation to blur it increases as the chapter goes on.

Element	Status	Warrant
Text maps losslessly to a nucleotide string	**BUILT, VERIFIED**	§26.9, SHA-256 round trip

Element	Status	Warrant
The balanced free-choice rule holds GC within one base of L/2	**PROVED (arithmetic)**, measured	§26.9
The balanced rule raises symbol entropy from 1.244171 to 1.935874 bits/base	**MEASURED** on the payload	§26.9
RLL(3) capacity is 1.982354 bits/base	**COMPUTED** (transfer-matrix eigenvalue)	§26.8
A 20-nt prefix is a sufficient address	**COMPUTED**, 0.005638867 expected hits	§26.10
The prefix is a routing header, not a molecule	**STATED, structural**	§26.10
Route B index assignment n = 1,3,5,7 gives four distinct classes	**PROVED (arithmetic)**	§26.12
The naive assignment n = 1,2,3,4 collapses three symbols	**PROVED, REJECTED IN PRINT**	§26.12
Symbol alphabet minimum gap is 990.055901 Hz	**COMPUTED**	§26.13
Symbol layer and state layer are separated by 995.9922 cents	**COMPUTED**	§26.14
Gabor limit dt.df >= 1/(4 pi) = 0.079577 constrains the rate	**PROVED (classical Fourier)**	§26.15
Single-transposition audible capacity is N <= 25, at j = 6 only	**COMPUTED**, verified	§26.16
The folded-lattice cliff is at N = 17; second at N = 41; rate factor 8	**CITED**, Chapter 7 §7.6	§26.16
The rectangular gate has no finite spectral second moment	**COMPUTED**, divergent	§26.17
k is a chosen reference constant	**POSTULATE**, badged	CANON_SONIC §6
A transposed tone bears a physical relation to a base	**REFUSED**	H26-4
That any of this affects any cell	**NOT CLAIMED**	Chapter 27

Eleven computed or measured rows, two proofs, one rejection in print, one badged postulate, one refusal, and one open problem. A reader is entitled to see the accounting in that form before reading a word of the argument, and to check it against the chapter as they go.

26.5 Honesty clauses, binding

H26-1 — No clinical, medical or therapeutic claim. This chapter describes signal processing and an experimental design. No trial, no controlled observation, and no demonstrated physiological effect of the procedure exists, and none is asserted.

H26-2 — No claim that DNA is edited, altered or damaged. Nothing in the pipeline is a molecule. The output of the last stage is a waveform; the nucleotide string is a string in the computer-science sense. **At no point does this procedure synthesise, deliver, or modify any nucleic acid, and it contains no nuclease, no template, and no Cas protein.**

H26-3 — No claim that any capability has been conferred on anything.

H26-4 — The transposition is a mapping, not a resonance. This is the strongest clause here and the one a careless reader most often violates. The tones of §26.13 are essence frequencies obtained by applying a postulated square law to a chosen index assignment and then shifting register. **A tone at 1113.8129 Hz has no physical relationship to cytosine. Cytosine does not absorb at that frequency, does not resonate at it, and is not affected by it.** The mapping is a labelling convention, chosen because it is principled and reproducible. It is not a physical coupling and this book never says or implies that it is. This is CANON_SONIC §6 applied to this chapter's derivation.

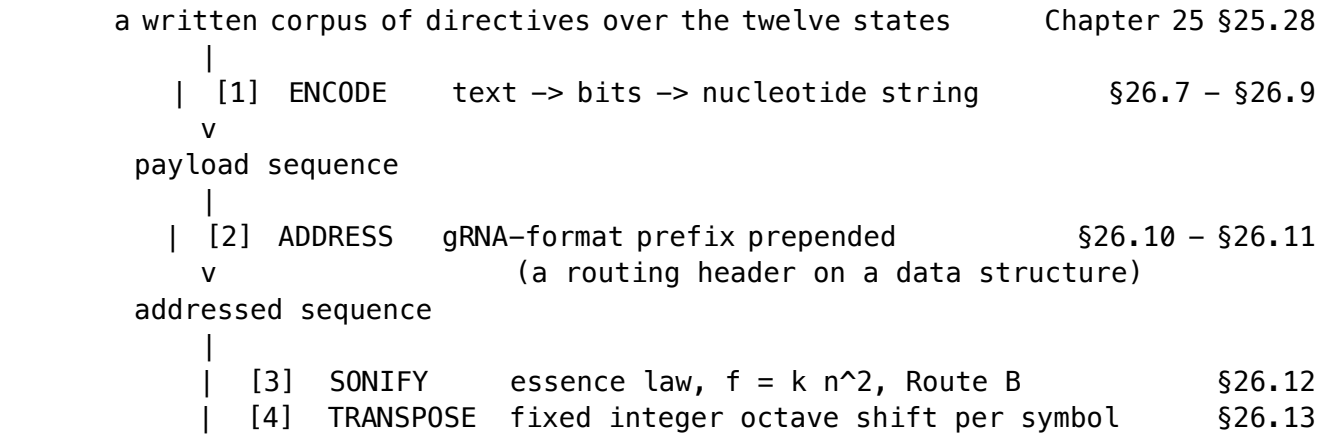
H26-5 — k is chosen, not measured. `k = (9/8)^2/phi^2 = 0.483425732988414 Hz` is a Pythagorean ditone divided by the golden section and is a design decision by the author. It is badged every time it appears. Result One of CANON_SONIC §2 explains why the badge costs so little: k cancels out of every ratio in the system.

H26-6 — Every number here was computed, and the computations are stated in enough detail to be redone. Anything that could not be computed is marked nominal or omitted.

H26-7 — The worked payload is CONSTRUCTED. It is the corpus of Chapter 25 §25.28, written for this book. It is not a transcript and not an observation of anyone.

The Engineering

26.6 The pipeline


a written corpus of directives over the twelve states
Chapter 25 §25.28
[1] ENCODE text -> bits -> nucleotide string
§26.7 – §26.9
payload sequence
[2] ADDRESS gRNA-format prefix prepended
§26.10 – §26.11
(a routing header on a data structure)
addressed sequence
[3] SONIFY essence law, f = k n^2, Route B
§26.12
[4] TRANSPOSE fixed integer octave shift per symbol
§26.13

```
        |  [5]  LAYER      the state layer of Chapter 25 §25.14       §26.14
        |  [6]  RENDER    windowed envelope, rate set by audibility  §26.15 –
§26.17
        v
    a disclosed audio file, published with D1–D10
        |
        |  [7]  TRANSMIT   playback in a room, with consent
        v
    Chapter 27's open problem begins here, and this chapter stops
```

Steps 1 through 6 are engineering and are demonstrated below with computed and verified numbers. Step 7 is playback of an audio file. **What happens after step 7 is not claimed by this book in any form.**

26.7 Stage 1 — text to bits

The corpus is written as plain text and the encoding to bits is ordinary. It must nonetheless be published in full: character set, byte width, byte order, whether directives are concatenated or kept separate, and what separator is used.

The house scheme is 8-bit ASCII, one byte per character, no terminator, directives separated by a single newline, both members of each pair present in the published rendering order.

For the corpus of Chapter 25 §25.28:

```
    characters, including newline separators      675
    bytes                                         675
    bits                                        5,400
    SHA-256 of the source text
        c7d98b1cfde5c515ce99ff7db77d484f71a09c5cd921994f15df66c50373b72d
```

Nothing interesting happens at this stage, and that is deliberate: **any cleverness here would be a place to hide something.** A published, standard, character-for-character mapping is auditable by anyone with a text editor, which is what D2 of the disclosure package is for.

26.8 The rate landscape, and an honest correction carried forward

Church, Gao & Kosuri (*Science* 337:1628, 2012) encoded a book into synthesised oligonucleotides and recovered it by sequencing. Book Two Chapter 69 makes a correction that must be carried: **they did not use two bits per base. They used one** — `A` or `C` encoded a zero, `G` or `T` encoded a one — and the free choice within each pair was spent on homopolymer avoidance and GC control. The base-4 alphabet is the theoretical ceiling; the Church architecture of block, address and common flanks is the part that was demonstrated. This procedure uses the alphabet and the architecture and says plainly that the doubling of rate relative to the 2012 paper is not something the 2012 paper did.

The homopolymer-constrained capacity is a transfer-matrix eigenvalue and is computed rather than quoted:

```
    unconstrained base-4                    2.000000 bits/base   100.000%
    RLL(2)  max run 2                       1.922688 bits/base    96.134%
```

```
RLL(3)  max run 3                       1.982354 bits/base    99.118%
RLL(4)  max run 4                       1.995717 bits/base    99.786%
RLL(5)  max run 5                       1.998939 bits/base    99.947%
this procedure, one bit per base        1.000000 bits/base    50.000%
```

Forbidding runs longer than three costs 0.882% of capacity. Discarding the second bit costs half of it, and buys something the compressed scheme cannot have.

26.9 Stage 1b — bits to bases, and what the discarded bit buys

With two acceptable bases per bit there is one free choice at every position. For a 5,400-bit payload the number of distinct nucleotide strings encoding exactly the same message is `2^5400`, a 1,626-digit count (`5400 . log10 2 = 1625.562`).

The house rule for spending the free choice is **balanced selection**: at each position choose, among the bases legal under the homopolymer constraint, the one used least often so far, ties broken alphabetically. The obvious alternative is greedy first-legal selection. The two were built and measured on the same payload, and the difference is not small:

```
rule      A     C     G     T     GC       max run   entropy (bits/base)
greedy   3233   266   1856   45   0.392963      3        1.244171
balanced 1750  1749   951   950   0.500000      1        1.935874
```

Both recover the source exactly:

```
SHA-256, source text              c7d98b1c...0373b72d
SHA-256, recovered from greedy    c7d98b1c...0373b72d      IDENTICAL
SHA-256, recovered from balanced  c7d98b1c...0373b72d      IDENTICAL
```

Two properties of the balanced rule are worth stating because they are structural rather than lucky. Because the rule always increments the currently smaller count, the two members of each bit class stay within one of each other; each class contains exactly one strong base and one weak one; therefore **GC content differs from L/2 by at most one base regardless of what the payload says.** Here it is exactly 2,700 of 5,400. And because consecutive symbols from the same bit class alternate while consecutive symbols from different classes are drawn from disjoint letter pairs, the measured maximum homopolymer run is 1 — comfortably inside RLL(3).

The entropy figure is the one that matters for rendering. A greedy string is 3,233 adenines out of 5,400 and sounds like a drone with occasional events; a balanced string uses all four symbols nearly equally. A payload whose symbol distribution is that skewed is a payload whose rendering a listener cannot follow, which is **an audibility failure under S25.1 arriving from an unexpected direction.**

The first sixty bases of the balanced encoding, published because D3 requires the string and not merely the rule:

```
AGCATGCACTACAGCTAGCTAGCACATCACACAGCTAGCACTACGACACTACAGCTACGA
```

A scheme whose rule is one-to-many must publish the string. The weaker encoding forces the stronger disclosure, and that is the right direction for the trade to run.

26.10 Stage 2 — the address, which is a routing header and not a molecule

Chapter 66 of Book Two established the addressing architecture and this chapter uses it without extending it. The prefix is formatted after the guide-RNA spacer convention, and its status must be stated in the same breath as its format:

The prefix is a field in a file. It is not delivered, it is not synthesised, it is not a molecule, no Cas protein is present anywhere in this procedure, and therefore nothing is edited.

Why twenty nucleotides, computed rather than asserted. The *Streptococcus pyogenes* Cas9 spacer is 20 nt and the arithmetic reason is address-space sizing against genome size:

```
L        4^L                 expected hits in 6.2e9 positions
 8       6.553600e+04              94604.5
12       1.677722e+07                369.549
16       4.294967e+09                  1.44355
18       6.871948e+10                  0.0902219
20       1.099512e+12                  0.005638867
22       1.759219e+13                  0.000352429
24       2.814750e+14                  0.0000220268

4^20 = 1,099,511,627,776
address space / genome positions = 177.3406
```

Below 16 nt an address is not an address — it names hundreds of places. At 20 nt the expected number of spurious matches is about one in 177. That is the whole justification; it is arithmetic, and it is the same arithmetic that sets hash lengths in ordinary computing.

It is worth being precise about what "addressing" does and does not mean in a procedure where nothing is delivered. In ordinary CRISPR practice a spacer is an address because a molecule carries it to a locus and binds there; the address does work because the delivery vehicle exists. Here there is no vehicle. The prefix is a field in a file, and its function within this procedure is bookkeeping: it says which of the twelve directives a given block of payload belongs to, it makes the transmission parseable by anyone reading the published string, and it fixes the format so that the disclosure of D4 is checkable against a standard. The 20-nt length is justified by the same arithmetic that would justify it in a molecular context, and the justification is retained because borrowing a format and then silently changing its parameters is how formats become meaningless. But the length is doing bookkeeping work here, not molecular work, and no reader should be left able to think otherwise.

Cas12a's native processing of its own crRNA array (Zetsche et al., *Cell* 163:759, 2015; Fonfara et al., *Nature* 532:517, 2016) is cited here for one thing only: that a **multicast destination list** is a real biological format. It is cited for the format and for nothing else, and no claim is made that any such parsing occurs in response to sound.

26.11 The addressed payload

One address per directive pair, twelve pairs:

```
address prefixes         12 x 20 nt  =    240 bases
payload                                 5,400 bases
```

```
TOTAL TRANSMISSION                          5,640 bases
```

The prefixes are published in full in D4 with their derivation, exactly as the payload is published in D3. **An address that is not disclosed is a routing decision the listener cannot inspect, which fails S25.4 whether or not it does anything.**

26.12 Stage 3 — sonify by the essence law

Four symbols must become four distinguishable tones, by the law this book already owns:

```
f(n) = k * n^2      k = (9/8)^2 / phi^2 = 0.483425732988414 Hz    POSTULATE, badged
```

Applying it requires a defensible ordinal on the four bases, which CANON_SONIC §6 demands of every application of the law. **An arbitrary ordering yields arbitrary frequencies, and the ordering must be argued in print or the route is unavailable.** The ordering used is alphabetical — A, C, G, T — which is not defended as meaningful, only as published, stable, and conventional in every sequence database in existence. That is a weaker warrant than the ladder of Chapter 5 carries, and it is stated as weaker.

The obvious index assignment is a trap, and it is rejected in print. Assigning `n = 1, 2, 3, 4` gives odd parts 1, 1, 3, 1: three of the four bases collapse onto a single pitch class, because `f(2) = 4 f(1)` is exactly two octaves and `f(4) = 16 f(1)` exactly four. By Chapter 7's odd-part rule a listener attending to pitch class hears three of the four symbols as the same note. The scheme is unusable and the reason is arithmetic.

The house assignment is the odd indices, which gives four distinct classes:

```
base    n     f(n) (Hz)          j     f_audible (Hz)
  A     1     0.4834257330       8      123.7570
  C     3     4.3508315969       8     1113.8129
  G     5    12.0856433247       8     3093.9247
  T     7    23.6878609164       8     6064.0924
```

This is Route B of Book Two Chapter 82 §82.9 at its house default, carried unchanged.

26.13 Stage 4 — transposition, and the separation it produces

Transposition is by a fixed integer number of octaves and preserves pitch class exactly (CANON_SONIC §3 and §5). Here a single `j = 8` serves all four symbols, so the ratio structure of the alphabet after transposition is identical to the ratio structure before it. **Nothing is added and nothing is distorted; it is a change of register, the only purely presentational operation in the chain.**

The resulting separation:

```
sorted tones (Hz)        123.7570   1113.8129   3093.9247   6064.0924
adjacent gaps (Hz)          990.0559    1980.1118    2970.1677
adjacent gaps (cents)      3803.9100    1768.7174    1165.0244
MINIMUM GAP                 990.055901 Hz  =  1165.0244 cents
all four inside 20 Hz – 20 kHz                       yes
```

The tightest pair in the symbol alphabet is 1165.0244 cents apart — very nearly an octave — a separation any device with frequency dependence at all can resolve, and one that no plausible implementation error can permute. **That property is the whole reason the odd-index assignment is the default: it is not chosen for elegance, it is chosen because the naive one fails.**

It is worth pausing on how different that is from the alternative route. Book Two Chapter 69 built the same alphabet from measured infrared band centres and found the four symbols occupying less than half a semitone, with two of them two cents apart — a code that a routine solvent shift could permute outright, and one whose Gabor floor forced a rate of five and a half symbols per second. Register spreading fixed it there, by giving each symbol its own octave index. The essence-law route used here arrives at a comparable separation by a different road: the indices are already spread, because squaring them spreads them, and a single transposition therefore suffices. Neither route is more physical than the other — H26-4 forbids reading either as physical — but the essence route is the one that needs no spectrometer and no calibration, and this book uses it for that reason and states the reason.

26.14 Stage 5 — the state layer, and channel separation

Chapter 25 §25.14 renders the twelve states themselves: each directive at its state's essence frequency folded into 220–440 Hz, each negation exactly two octaves above. That layer and the symbol layer are both audible, both disclosed, and must be separable by ear, which requires checking rather than assuming.

```
state layer band              220.0000 – 440.0000 Hz    (j = 6 for the ladder)
 nearest symbol tone below           123.7570 Hz  ->   995.9922 cents clear
 nearest symbol tone above          1113.8129 Hz  ->  1607.9178 cents clear
```

The state band is clear of every symbol tone by at least 995.9922 cents — better than an octave — so the two disclosed layers occupy disjoint registers and neither masks the other. That is a computed property of this particular pairing of `j` values, not a general fact, and any change to either transposition requires recomputing it. Note also that the negation register of Chapter 25 (880–1760 Hz) does overlap the symbol tone at 1113.8129 Hz; where both layers are rendered simultaneously they must be separated in time instead, and the run schedule of §26.18 does exactly that.

26.15 The design table

This is the section CANON3 §5's constraints exist to produce, and every row is computed or cited with its owner named.

The Gabor limit, which binds every rendering:

```
dt * df  >=  1 / (4 pi)  =  0.07957747154594767
```

Classical Fourier, no quantum content, measurable on an oscilloscope. Book Two Chapter 71 owns it.

Applied to the symbol alphabet, minimum gap 990.055901 Hz:

```
convention df = gap        dt >= 0.080377 ms
convention df = gap/2      dt >= 0.160753 ms
```

Both conventions are printed because both appear in the literature and quoting one without saying which is a reproducibility failure. The design table below uses `df = gap`.

dt (ms)	df at that window	safety over floor	symbols/s	total run, 5,640 bases
1	79.5775 Hz	12.44x	1000.00	5.640 s
5	15.9155 Hz	62.21x	200.00	28.200 s
10	7.9577 Hz	124.41x	100.00	56.400 s
20	3.9789 Hz	248.83x	50.00	112.800 s
25	3.1831 Hz	311.04x	40.00	141.000 s
50	1.5915 Hz	622.07x	20.00	282.000 s
100	0.7958 Hz	1244.14x	10.00	564.000 s

The house default is 20 ms per symbol, a safety factor of 248.83 over the floor, and the factor is large on purpose. The Gabor floor is where two tones become mathematically indistinguishable; a signal whose symbols no listener can resolve is a concealed signal by another route, and **S25.3 therefore sets the rate by audibility rather than by the mathematics.** At 20 ms the full 5,640-base transmission runs 112.800 seconds.

Applied to the state layer, whose binding separation is the tightest folded gap at `N = 12` — 7.734812 Hz between classes 7 and 5, Chapter 7 §7.6, cited and not re-derived:

```
dt >= 0.0795774715 / 7.734812 = 0.0102882 s = 10.2882 ms
```

The state layer is nearly 128 times more demanding than the symbol layer, and that ratio is the reason the two layers get different treatment in the run schedule.

26.16 The regime, the capacity, and the two cliffs

Three structural constraints, all owned elsewhere, all binding here.

(1) The 50/49 regime. Chapter 7 §7.6 derives it once and no other chapter re-derives it. At `N = 12` the minimum adjacent separation in the folded class lattice is 34.9756 cents, the interval `50/49`, between classes 7 and 5. At `N = 17` the minimum collapses to 6.0008 cents, the interval `289/288`, because `17^2 = 289` lands one sixth of a comma from `2 . 12^2 = 288`. The second cliff is at `N = 41`, where `41^2 = 1681` meets `2^4 . 105 = 1680` through class 29, giving `1682/1681` and 1.0296 cents. Both are exact rational coincidences.

The rate factor across the first cliff is exactly 8, because the binding pairs sit at different octave heights — `50/49` straddles `49/32` and `289/288` straddles `9/8` — so the absolute gap falls from `f/32` to `f/256`. Chapter 7 and Chapter 2 §2.11 own that figure; it is cited.

This book's `N = 12` sits below the first cliff. Every folded distinction asked of a listener or an instrument here is at least 34.9756 cents wide, roughly a third of a semitone, and lengthening the index past 16 would cost a factor of eight in symbol rate.

(2) Single-transposition audible capacity. A single integer octave transposition admits at most `N = 25` essences inside 20 Hz to 20 kHz, and only at `j = 6`. Verified:

```
N = 20    admissible j = [6]
N = 25    admissible j = [6]
N = 26    admissible j = []      the set is empty
```

`N = 12` is comfortably inside the limit. A longer index would require split transposition, which is permitted but must be published per symbol.

(3) No tempering. The essence ratios are exact ratios of squares. They are not adjusted to twelve-tone equal temperament anywhere in this procedure. 34.9756 cents is not 0 cents, and a rendering that rounds it to the nearest semitone has destroyed the distinction it was built to carry.

26.17 The envelope — the rectangular gate is disqualified

S25.11 forbids a rectangular gate and the reason is computable. The spectrum of a rectangular window of duration T is a sinc, whose squared magnitude decays as `f^−2`, so the second moment integral `int f^2 |S(f)|^2 df` diverges. Numerically, at `T = 5 ms`, integrating over successively wider bands:

```
band +−1e3 Hz     second moment  2.068251e+04
band +−1e4 Hz                    2.030538e+05
band +−1e5 Hz                    2.026834e+06
band +−1e6 Hz                    2.026465e+07
```

The value grows linearly with the integration band and does not converge, which means a rectangular gate has no defined `df` at all and the Gabor bound cannot be applied to it. Any implementation uses a windowed envelope — the specific window is published in D7, with its rise and fall times.

26.18 The run schedule

Because the two layers overlap in the negation register (§26.14), and because their Gabor requirements differ by a factor of nearly 128, the house schedule separates them in time:

```
PHASE 0   silence, 5 s                    the listener confirms the room
PHASE 1    the state layer, 24 tones          Chapter 25 §25.14
           directive and negation as a
           double octave, 2 s per pair        48.000 s
PHASE 2    silence, 5 s
PHASE 3    the symbol layer, 5,640 bases
           at 20 ms per symbol              112.800 s
PHASE 4    silence, 5 s
           TOTAL                            175.800 s
```

Phase 1 exists so that the listener hears the structure of the corpus — twelve opposed pairs, as double octaves — before hearing the encoded transmission of it. Putting the structure first is a disclosure decision, not an aesthetic one: a listener who has heard the pairs knows what the corpus is before the long phase begins, and can stop.

26.19 What this procedure would look like if it were a technique, and why it is not

A useful way to see the boundary is to write down what would have to change for any of this to be described as a technique rather than a research procedure.

It would need a demonstrated effect: a preregistered endpoint that moved, in a design with the four controls of S26.21 in place, replicated by a group that did not build the apparatus. It would

need that effect to survive the scrambled-sequence control of Chapter 27 §27.13, because an effect that does not distinguish an ordered signal from its permuted twin is an effect of sound, not of content, and the entire content architecture of Chapters 25 and 26 would be beside the point. It would need a mechanism, or at minimum a candidate mechanism with its own independent evidence, because a replicated effect with no mechanism is a phenomenon to be investigated rather than a technique to be applied. And it would need a magnitude large enough to matter against the detectable floor computed in Chapter 27 §27.9.

None of those four conditions is met. Not one of them is close to met. **The distance between "the procedure is fully specified" and "the procedure does something" is the distance this Part exists to keep visible**, and it is kept visible by writing the specification exactly as carefully as if the effect were real, and then saying, in the same breath, that it is not known to be.

The Specification

Part A — the encoding chain

S26.1 The character set, byte width, byte order and separator convention shall be published.

S26.2 The bit-to-base rule shall be published, and because it is one-to-many, the actual nucleotide string shall be published in full (D3).

S26.3 The free choice shall be spent by the balanced rule of §26.9, or by another published rule whose GC and entropy figures are computed and reported alongside the greedy baseline.

S26.4 Homopolymer runs shall be constrained to at most 3, and the achieved maximum run reported.

S26.5 A round-trip hash identity shall be demonstrated and both hashes published.

S26.6 The address length shall be justified arithmetically against the target genome size, and the prefixes published in full.

S26.7 Every document, caption and file name shall state that the prefix is a routing header and that nothing is delivered, synthesised, or edited.

S26.8 The index assignment for the four bases shall be published, with the ordinal argued, and the rejected naive assignment stated so that a reader knows it was considered.

S26.9 Every published frequency shall carry its `n`, its `j`, and its band.

S26.10 Minimum adjacent gaps shall be reported in Hz and in cents.

S26.11 Channel separation between the symbol layer and the state layer shall be computed in cents and published; where the registers overlap, the layers shall be separated in time.

S26.12 The symbol rate shall be reported with its Gabor safety factor and with the convention (`df = gap` or `df = gap/2`) stated explicitly.

S26.13 The envelope shall be windowed, published, and reported with rise and fall times. A rectangular gate is disqualified.

S26.14 No tempering of any interval, at any stage.

Part B — preregistration and falsification, per Paper E §5

This is where the procedure becomes a research procedure rather than a demonstration, and none of it may be retrofitted after data exist.

S26.15 — The five operational choices of Paper E §5.1 shall be recorded in writing before any measurement: the system boundary, the input variable, the order proxy, the time base, and the cost accounting. Book Two Chapter 24 Form A is the house form.

S26.16 — The observable and the SESOI. The endpoint shall be named, and the smallest effect size of interest declared, before any data are collected. Chapter 27 §27.9 computes what magnitudes are detectable at what sample sizes; a SESOI below the detectable floor is a preregistration that has already failed.

S26.17 — All four baselines shall be computed and reported:

```
   1.  PERSISTENCE           the endpoint predicts itself at the next period
  2.  DIRECT-INPUT        the input predicts the endpoint with no model between
  3.  MATCHED AUTOREGRESSIVE with the SAME number of free parameters as the model
   4.  SHUFFLED-INPUT CONTROL refit under the permutation, holdout permuted
                              independently, cost term NOT permuted
```

S26.18 The matched autoregressive baseline shall carry the same number of free parameters as the model. A suite is only a suite if all four are read together: evaluated alone, the matched autoregression false-passed 35.8% of the time in Book Two's simulation.

S26.19 — The structural ceiling test shall be added, and it is not optional. Any single-period increment claimed shall satisfy the theoretical maximum of Corollary 4.2. This is the only test that detects repackaging.

S26.20 — The three measured failure modes of the shuffled-input control shall be carried into every report that uses it. They were measured by simulation in Book Two Chapters 25, 55 and 81 rather than asserted, and two of the three cannot be repaired by any surrogate:

```
    (1) ANTI-CONSERVATIVE AS ORDINARILY RUN.
       500 null trials, nominal alpha = 0.05, measured false-rejection rates:
            iid permutation      0.108     2.2x nominal
            circular shift       0.120     2.4x nominal
            block permutation    0.060     near nominal
          A surrogate test taken off the shelf roughly DOUBLES the false-
positive rate,
           and the block variant -- the one usually skipped as fussy --
is the only one
      that behaved. REMEDY: calibrate by simulation on your own design (series
      length, autocorrelation, block structure) and preregister the calibrated
        threshold, not the nominal one.

    (2) BLIND TO REPACKAGING.
      A constructed filtered repackaging -- a rolling average of the input, and
      nothing else -- passed all four baselines including the shuffle at p = 0.0005,
       beat persistence by 89.4%, and beat the matched autoregression by 89.2%.
        It was caught by the structural ceiling test and by NOTHING ELSE: its
```

```
        single-step jump was 38.5 times the theoretical maximum.
     REMEDY: the structural ceiling test of S26.19. No surrogate detects this.

    (3) BLIND TO A SHARED SLOW DRIVER.
        Where input and endpoint are both driven by a third slow variable --
a funding
            cycle, a seasonal effect, a drifting incubator -- a 4,000-
trial simulation
       found the shuffled-input control PASSING 98.35% OF THE TIME. The same
     simulation reports the control working honestly in the clean case at 48.17%
     detection, which is what makes 98.35% a measurement rather than a complaint.
        REMEDY: design controls. No surrogate detects this either.
```

S26.21 — The consolidated rule, which supersedes any reading of the shuffle as a general guarantee: the shuffle is a weak, mis-calibrated empirical audit. More surrogates do not fix any of this. DESIGN CONTROLS DO. The four that answer failure modes (2) and (3):

```
BLINDING          the person measuring the endpoint does not know the condition
CHANNEL SEPARATION   the endpoint is measured through a different channel, by
                   different people, from different records, than the input
PROSPECTIVE SAMPLING  the sampling schedule is fixed before exposure and is not
                        adapted to what is observed
YOKED CONTROL      a control unit receives the same schedule, the same handling
                  and the same environment, and differs only in the ordering
                        of the signal
```

Every one of those four is a decision about the instrument, not about the analysis. **They must be written into Part 1 of the preregistration and cannot be added once the series exists.**

S26.22 — Kill conditions shall be declared in advance and honoured. The framework is rejected for a given run if any of the following holds:

```
K1  the shuffled-input control, at the CALIBRATED threshold, matches the fitted
      model on held-out data
K2  the structural ceiling test fails -- a claimed increment exceeds the maximum
K3  the scrambled-sequence control (Chapter 27 §27.13) is indistinguishable from
      the ordered condition
 K4   the yoked control moves with the exposed condition
 K5   the fitted parameters are not this system's, per FR3
```

S26.23 — A null result shall be reported, in full, with the same prominence as a positive one. Chapter 27 §27.11 computes how little of the design space one null bounds — 0.000801% of a modest 124,800-cell grid — so a null is a small, honest, publishable increment and must not be inflated into a refutation or buried as a disappointment.

S26.24 — The labelling rule of §26.2 applies to every output, including preprints, slides, captions, file names, and any summary written for a non-technical reader.

The Failure Modes

FM-26.1 — The clinical drift in the caption. The body text is careful and the figure caption says "response." Captions are read more than bodies.

Fix: S26.24 and §26.2's forbidden list, applied mechanically with a search before release.

FM-26.2 — The address becomes a molecule in the retelling. "gRNA-format prefix" is compressed to "guide RNA" in a summary, and the summary now describes a delivered molecule.

Fix: S26.7. The full phrase "routing header, not a molecule" appears in every document, not once per book.

FM-26.3 — The transposition read as a resonance. Somebody concludes that cytosine "has a frequency of 1113.8129 Hz."

Fix: H26-4. The number is a label produced by a postulated square law applied to an alphabetical ordering and then shifted eight octaves. **It is not a property of cytosine.**

FM-26.4 — The naive index assignment reintroduced. A later implementer assigns `n = 1,2,3,4` because it is obvious, and three symbols collapse to one pitch class.

Fix: §26.12, which rejects it in print with the arithmetic, so the rejection is findable.

FM-26.5 — The greedy encoder shipped. The first-legal rule is simpler and passes the round-trip test, so it looks fine. It produces a 60% adenine string that no listener can follow.

Fix: S26.3. Report GC and entropy against the greedy baseline every time.

FM-26.6 — The rate set by the Gabor floor. An engineer reads 0.080377 ms, sets the symbol rate near it, and produces a signal that is technically not masked and practically inaudible as symbols.

Fix: S25.3 and §26.15. **The floor is where two tones become mathematically indistinguishable; it is not a perceptual threshold, and a rendering at the floor is concealment achieved by speed.**

FM-26.7 — The shuffle treated as a guarantee. The control passes, the result is announced, and the two failure modes that a surrogate cannot see are never addressed.

Fix: S26.20 and S26.21, carried in the report rather than in the methods appendix.

FM-26.8 — Design controls added after the fact. Blinding is "not practical" and is replaced with a fifth surrogate.

Fix: S26.21. Surrogates and design controls answer different questions and one cannot substitute for the other at any exchange rate.

FM-26.9 — The preregistration adapted after the first look. The endpoint is redefined, the window moved, or the SESOI lowered, all of them defensibly.

Fix: S26.15 and S26.16, timestamped and deposited, with any deviation reported as a deviation.

FM-26.10 — The null buried. A run returns nothing and is not written up.

Fix: S26.23. A null bounds a real, computable, small fraction of the design space and is worth exactly that much — which is more than zero and much less than a refutation.

FM-26.11 — The scrambled-sequence control omitted because it "obviously" makes no difference. This is the control that decides the entire question and it is the one most likely to be dropped as redundant.

Fix: K3, and Chapter 27 §27.13, which explains why no other control substitutes for it.

FM-26.12 — Layer overlap unchecked. A later implementer changes one transposition and the state layer lands on top of a symbol tone.

Fix: S26.11. The 995.9922-cent clearance is a property of one specific pair of `j` values and must be recomputed after any change.

FM-26.14 — The corpus rewritten between runs. The payload changes because a directive read poorly, and two runs are pooled as though they were the same condition. Because the encoding is deterministic, a single changed character changes the payload hash, the base string, and every symbol after the change point.

Fix: S26.5. The payload hash is the run's identity. Two runs with different payload hashes are two conditions, and pooling them requires the same justification as pooling any two conditions.

FM-26.13 — The procedure described as having worked. Somebody who ran it reports that "it went well," meaning the audio rendered correctly, and a reader hears a claim about a biological effect.

Fix: §26.2, and a reporting convention that separates *the procedure executed as specified* from *the endpoint moved*, which are different sentences about different things.

Worked Example

CONSTRUCTED. The payload is the corpus of Chapter 25 §25.28, written for this book. It is not a transcript and not an observation of anyone. No exposure has been run and no endpoint has been measured.

26.25 End to end, with every number

```
STAGE 1     text
            characters (with newline separators)          675
            bytes                                         675
            bits                                        5,400
       SHA-256  c7d98b1cfde5c515ce99ff7db77d484f71a09c5cd921994f15df66c50373b72d

STAGE 1b    bases, 1 bit/base, balanced free choice
            payload bases                               5,400
            A / C / G / T                  1750 / 1749 / 951 / 950
          GC                                0.500000   (= L/2 exactly)
        maximum homopolymer run                  1   (RLL(3) satisfied)
            symbol entropy                     1.935874 bits/base
            greedy baseline entropy            1.244171 bits/base
            first 60 bases
```

```
              AGCATGCACTACAGCTAGCTAGCACATCACACAGCTAGCACTACGACACTACAGCTACGA
              SHA-256 of recovered text
           c7d98b1cfde5c515ce99ff7db77d484f71a09c5cd921994f15df66c50373b72d
              ROUND TRIP IDENTICAL

  STAGE 2     address
              12 prefixes x 20 nt                              240 bases
              expected spurious matches per 20-mer     0.005638867
              address space / genome positions           177.3406
              TOTAL TRANSMISSION                          5,640 bases

 STAGE 3    sonify, essence law, Route B odd indices, k = 0.483425732988414 Hz
              A  n=1   f =  0.4834257330 Hz
              C  n=3   f =  4.3508315969 Hz
              G  n=5   f = 12.0856433247 Hz
              T  n=7   f = 23.6878609164 Hz

  STAGE 4     transpose, j = 8 for all four symbols
              A     123.7570 Hz
              C    1113.8129 Hz
              G    3093.9247 Hz
              T    6064.0924 Hz
              minimum adjacent gap    990.055901 Hz  =  1165.0244 cents

  STAGE 5     state layer, Chapter 25 §25.14
              directives      220.0000 -  440.0000 Hz   (ladder at j = 6)
           negations     880.0000 - 1760.0000 Hz  (+2 octaves, 2400.0 cents)
              clearance to nearest symbol tone below     995.9922 cents

  STAGE 6     render
              symbol duration                              20 ms
              Gabor floor (df = gap)                  0.080377 ms
              safety factor                              248.83x
           envelope                        windowed; rectangular disqualified
              tempering                                  none
              phase 1 (state layer)                      48.000 s
              phase 3 (symbol layer)                    112.800 s
              total with silences                       175.800 s
```

26.26 The disclosure package, as delivered

D1 through D10 of Chapter 25 §25.19, complete, before exposure. D1 contains the twelve pairs in full including the four that fail the equal-force check of Chapter 25 §25.29 — pairs 3, 5, 7 and 12, 33.33% of the corpus — named as failing, because S25.8 permits rendering an unrepaired corpus only if the failures are disclosed. D3 contains all 5,400 payload bases. D9 contains H26-1 through H26-7 and Chapter 27's open problem in full.

26.27 The preregistration, filled in

No data exist. This is the form as it would stand before a first run, and it is included so that a reader can see what a complete one looks like.

```
1. SYSTEM BOUNDARY    cultured cells, in vitro, named line, named passage range.
                  Deliberately excluded: any human participant, any endpoint
                        that is not a transcript abundance.
2. INPUT           the rendered waveform of §26.25, as a file, with its hash.
3. ORDER PROXY       preregistered transcript abundances at the 12 addressed loci.
4. TIME BASE        fixed sampling schedule, declared before exposure (S26.21).
 5. COST ACCOUNTING      declared per Book Two Chapter 24 Form A.

 OBSERVABLE              log2 abundance at the 12 preregistered loci.
SESOI              declared in advance; must exceed the detectable floor of
                     Chapter 27 §27.9 at the planned n, or the design fails
                        before it starts.

 ARMS                    ORDERED       the waveform as specified
                  SCRAMBLED    symbol-permuted twin: identical total energy,
                                duration, symbol counts and long-term power
                               spectrum; ordering destroyed  (Ch 27 §27.13)
                         SILENT        handling-matched, no signal
                   YOKED       same schedule, same handling, ordering varied

DESIGN CONTROLS       blinding of the measurer to condition        REQUIRED
                    channel separation of endpoint from input      REQUIRED
                     prospective sampling schedule                 REQUIRED
                       yoked control                               REQUIRED

 BASELINES              persistence, direct-input, matched autoregressive,
                  shuffled-input -- all four, read together (S26.17, S26.18)

 STRUCTURAL TEST       ceiling test per S26.19, single-period increment vs
                          theoretical maximum

SURROGATE CALIBRATION  block permutation, calibrated by simulation on THIS design;
                  the calibrated threshold is preregistered, not the nominal
                          0.05 (S26.20 failure mode 1)

 KILL CONDITIONS         K1 - K5 of S26.22, declared and honoured

 REPORTING             null reported in full, with the same prominence as a
                     positive result, and with the design-space fraction it
                          bounds computed (S26.23)
```

26.28 What this procedure has and has not produced

Has produced: a deterministic encoder with a verified round-trip identity; a balanced free-choice rule that holds GC at exactly half and raises payload entropy from 1.244171 to 1.935874 bits per base; an arithmetically justified 20-nt routing header; a four-symbol alphabet whose tightest pair is 1165.0244 cents apart; two disclosed layers separated by 995.9922 cents; a symbol rate 248.83 times above the Gabor floor; and a 175.800-second audio file with a complete disclosure package.

Has not produced, and does not claim: any biological effect; any evidence that a cell responds to the ordering of the signal; any clinical, medical or therapeutic result; any alteration of DNA; any conferred capability; or any physical resonance. **The procedure ends at a loudspeaker, and Chapter 27 is about the gap on the other side of it.**

What this chapter established

Established, and checkable: that a written corpus maps losslessly to a 5,400-base nucleotide string with a verified SHA-256 identity; that the balanced free-choice rule holds GC within one base of `L/2` by construction and measured exactly 0.500000 here, with maximum homopolymer run 1 and entropy 1.935874 against the greedy rule's 1.244171; that a 20-nt prefix has 0.005638867 expected spurious matches in 6.2e9 positions; that the naive base index assignment collapses three of four symbols onto one pitch class and is rejected; that the odd-index assignment gives a minimum separation of 990.055901 Hz; that the symbol and state layers are 995.9922 cents apart; that at 20 ms per symbol the rendering sits 248.83 times above the Gabor floor and runs 175.800 seconds; that the rectangular gate has no finite spectral second moment; and that the shuffled-input control fails in three measured ways, of which two are answerable only by design controls.

Not established, and not claimed: that any of this affects any cell, transcript or person; that any tone bears a physical relation to any base; that the procedure is a treatment, therapy or practice of any kind; that DNA is edited, altered or damaged; or that any capability has been conferred.

The one line to carry forward: *the whole procedure is buildable, auditable and disclosed, and it stops at a loudspeaker.*

Chapter 26 owns the operational procedure, the design table, the run schedule, and the preregistration and falsification apparatus for this Part. Chapter 25 owns the no-subliminals constraint, the disclosure package and the consent specifications, all of which this chapter inherits whole; Chapter 27 owns the open problem that governs both. Folding, the odd-part rule, the 50/49 regime and the 34.9756-cent minimum are Chapter 7 §7.6; the rate factor of 8, the `N = 41` second cliff and the `N <= 25` capacity are CANON3 §5 and Chapter 2 §2.11; the essence law and k are Chapter 2 §2.1 and CANON_SONIC §1; the compound rule `f = k^2 G^4` is Chapter 20 and is not used here.

Chapter 27 . The Open Problem, Restated

Everything in this book up to this point has been either computed, proved, cited, or badged as a postulate in the sentence that stated it. The keystone of Part I is an exact substitution. Part II computes on a count. Part III computes on an interaction parameter whose scale status is stated every time it is used. Part IV audited a tradition and printed what failed. Part V has specified an encoder, an addressing convention, a sonifier and a rendering discipline, all of them deterministic and all of them verified.

There is one thing missing from that chain and it is not a detail. It is the link between the signal and the archive.

> **There is no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal.**

That sentence is this chapter. It is stated in CANON_BIO §5, restated in CANON3 §5, carried in every chapter of this Part, and it is not softened here, not worked around, and not argued down. This chapter spends its length making the gap as precise as it can be made, because a gap stated precisely is a research programme and a gap stated vaguely is an excuse.

The reason for the length is a matter of intellectual accounting. A book that carries a chosen reference constant, a postulated square law, a differentiation index whose defensibility it admits is contestable, and a sonification procedure whose biological effect is unknown, has to earn its reader's attention somewhere. It earns it here or it does not earn it at all. **A book that names its own load-bearing gap this precisely earns the right to everything else in it;** a book that declines to name it forfeits the right to all of it, including the parts that are correct.

The Law

27.1 The gap, stated four ways

The same proposition at four levels of precision, because each level is the one somebody will need.

Colloquial. Nobody knows how a cell could read a message out of a sound.

Engineering. No known biological structure performs demodulation of a symbol stream from an acoustic carrier into a sequence-specific output. There is no identified transducer with the

required frequency selectivity, no identified memory element that preserves symbol order across the symbol interval, and no identified mapping from a decoded symbol string to sequence-specific binding at a genomic locus.

Formal. Let `s(t)` be an acoustic pressure waveform encoding a symbol string `x = (x_1, ..., x_L)` over an alphabet of size 4, and let R be the cell's transcriptional response. There is no established mechanism producing an R that depends on `x` as an *ordered string*, as distinct from depending on functionals of `s(t)` that are invariant under permutation of the symbols — total energy, duration, long-term power spectrum, envelope statistics, symbol counts. **The claim that requires a mechanism is the claim that R is a function of the ORDER, and that is exactly the claim for which no mechanism exists.**

Operational. Take a stimulus and its symbol-permuted twin. The two have identical total energy, identical duration, identical long-term power spectrum, and identical symbol counts. If a cell cannot be shown to distinguish them, no sequence-specific channel has been demonstrated, and nothing else in the experiment matters.

27.2 What the statement does not say

Two symmetric errors sit on either side of the sentence and both have to be refused explicitly, because each is comfortable in a different direction.

It is not a claim of impossibility, and impossibility is not claimed. Absence of a known mechanism is a statement about the current state of the literature. Several real channels between physics and biology were identified late and were initially treated as implausible; the burden of demonstration sits entirely on the claimant, which is the ordinary position for any unestablished claim and carries no special hostility toward this one.

Nor is absence of a known mechanism a licence. The symmetric error is to treat "not proven impossible" as though it were partial support. It is not support of any kind. **A claim with no mechanism and no demonstration has zero evidential standing,** and the correct number of qualifiers to attach to it before demonstration is not "few" but "it is not asserted."

Both errors share a structure worth naming: they convert a statement about evidence into a statement about the world. The sentence at the head of this chapter is about what is known. It is not a prediction, and it will be settled by an experiment rather than by an argument.

27.3 Why this chapter is the book's warrant

The ordering of Part V is deliberate and it runs against the commercial instinct. Chapter 25 built the ethical architecture, Chapter 26 built the procedure, and Chapter 27 says that nobody knows whether the procedure does anything. A book optimising for persuasion would reverse the last two, or merge this chapter into a paragraph, or place it in an appendix labelled "limitations."

The reason for the ordering is that the alternative is not available to a book that wants to be checked. Every number in Chapters 1 through 26 is checkable by someone with a Python interpreter, and a reader who checks a hundred of them and finds them right acquires a reasonable prior that the hundred and first is right too. That prior is exactly what an unmarked overclaim would spend. **The restraint is not modesty and it is not a legal posture; it is the only way the arithmetic retains any value to a reader who cannot personally verify all of it.**

27.4 Honesty clauses, binding on this chapter and inherited by every chapter that cites it

H27-1 — No clinical, medical or therapeutic claim, and no intervention on any person is described here. Every experiment specified in this chapter is *in vitro*, on cultured cells, with transcript abundances as endpoints.

H27-2 — No claim that DNA is edited, altered, or damaged. Nothing in the pipeline of Chapter 26 carries a nuclease, a template, or a Cas protein, and nothing is delivered.

H27-3 — No claim that any capability has been conferred on anything.

H27-4 — Essence frequencies are essence frequencies, not physical resonances. Nothing in this chapter or any other claims that a person, tissue, cell or molecule resonates at any value printed in this book.

H27-5 — The concordance columns carry different things. Where a traditional source is cited anywhere in this Part, it carries *precedent*: that an idea was arrived at independently and preserved. It does not carry *evidence*. CANON_BIO §1 requires the role to be stated in the sentence that draws the concordance, and this chapter draws none.

H27-6 — Every number in this chapter was computed from stated assumptions, and the assumptions are printed with the number. The power calculations of §27.9 in particular are arithmetic on declared inputs. They are not estimates of anything measured, and the variance figures in them are inputs a reader supplies from their own system, not values this book is asserting about biology.

27.5 What is real upstream, itemised

The gap is a single missing link, not a general absence of evidence, and it matters to say so with the same precision used to state the gap. Otherwise an honest reader cannot tell how much of the architecture is standing.

Component	Status	Source
Arbitrary digital data can be stored in and recovered from nucleotide sequence	**ESTABLISHED**	Church, Gao & Kosuri, *Science* 337:1628 (2012)
Higher-rate DNA storage codes exist and are demonstrated	**ESTABLISHED**	Goldman et al., *Nature* 494:77 (2013); Erlich & Zielinski, *Science* 355:950 (2017)
A ~20-nt spacer is a sufficient genomic address	**COMPUTED**, and standard CRISPR practice	Chapter 26 §26.10
A CRISPR effector self-processes a multi-address array	**ESTABLISHED**	Zetsche et al., *Cell* 163:759 (2015); Fonfara et al., *Nature* 532:517 (2016)
Targeting and editing are separable functions	**ESTABLISHED**	Qi et al., *Cell* 152:1173 (2013); Gilbert et al., *Cell* 159:647 (2014)
Mechanical force gates ion channels	**ESTABLISHED**, Nobel 2021 (Patapoutian)	PIEZO1/PIEZO2

Component	Status	Source
Surface force reaches the nucleus, stretches chromatin, and alters transcription	**ESTABLISHED**	Tajik et al., *Nature Materials* 15:1287 (2016)
Ultrasound can activate specific engineered channels	**ESTABLISHED**	Ibsen et al., *Nature Communications* 6:8264 (2015); Duque et al. (2022)
Biology builds exquisite mechanical receivers	**ESTABLISHED**	cochlear hair cells
Non-coding sequence is a large, structured, addressable archive	**ESTABLISHED**	CANON_BIO §5; Book Two Chapter 67
Regulatory input reorganises expression without adding sequence	**ESTABLISHED**, Nobel 2012	Takahashi & Yamanaka, *Cell* 126:663 (2006)
A cell demodulates sequence-specific content from sound	**NO ESTABLISHED MECHANISM**	**this chapter**

Eleven established rows and one open row. Both halves belong in any honest report. Reporting only the eleven is advocacy; reporting only the one is a different dishonesty, because it implies the whole edifice is speculation when most of it is textbook.

27.5b The archive the signal would be addressing

One of the eleven rows deserves expansion, because it is the one that makes the architecture worth specifying at all and it is also the one most often described loosely.

The phrase “junk DNA” is retired. Non-coding sequence is roughly 98% of the human genome and is largely regulatory: promoters, enhancers, silencers and insulators, together with transposable elements at roughly 45% of the genome and endogenous retroviral sequence at roughly 8%, and the large families of long non-coding and micro RNAs. That is not filler around the genes; it is the control surface, and it is the part of the genome that regulatory input acts on.

The single most instructive item in it, for a book about narrative, is syncytin. The gene indispensable to placenta formation derives from a retroviral envelope gene: ancient foreign text, inherited, later read out as vital function. **A sequence that arrived from outside became a sequence the organism could not do without, which is documented molecular biology and not a metaphor** — though it is very hard to read about it without reaching for one, and the reaching is worth resisting in print.

What this establishes for the chapter is narrow. There is a large, structured, addressable archive, and regulatory input demonstrably reorganises which parts of it are expressed — Yamanaka reprogramming is the strongest available demonstration that latent capability is activated rather than added. **The archive is real and the regulatory layer is real. What is missing is any route by which an acoustic symbol stream reaches either.**

27.6 The shape of the missing link

Look at the eleven established rows and sort them by what they carry.

Rows 1 through 5 and rows 10 and 11 are **sequence** operations. They store, address, parse, target, archive and regulate, and every one of them operates on symbol strings or on molecular structures that read symbol strings.

Rows 6 through 9 are **force** pathways. Mechanical deformation opens a channel, ions move, signal propagates; force at the surface reaches the nucleus and changes what is transcribed; engineered channels respond to ultrasound; hair cells transduce sound exquisitely.

Nothing in the established literature carries sequence over force. A pressure wave that opens a PIEZO channel does so by deforming the membrane, and the channel does not care what melody deformed it. Tajik et al. applied force and measured transcription; the force carried a magnitude and a direction, not a string. Sonogenetics gets its specificity from the protein that was engineered into the cell, not from the structure of the waveform — which is a point worth dwelling on, because sonogenetics is the result most often produced as though it closed this gap. It does the opposite: it is the clearest existing demonstration that acoustic specificity in biology has so far been achieved by putting the specificity in the receiver, and never by putting it in the signal.

27.7 Three requirements, and which are unmet

Stating the gap as a single sentence is right for a canon file and too coarse for a research programme. Decomposed, a sequence-specific acoustic channel needs three distinct things, and they fail differently:

```
    D1  SYMBOL SEPARATION
        The receiver must distinguish the symbols of the alphabet.
      STATUS: satisfiable in principle. Chapter 26 §26.13 builds an alphabet whose
        tightest pair is 1165.0244 cents apart -- a separation any device with
      frequency dependence at all can resolve. D1 is an engineering requirement
        and the engineering exists.

    D2  ORDER MEMORY
      The receiver must preserve symbol ORDER across the symbol interval, so that
        the response depends on the sequence and not on the histogram.
      STATUS: NO IDENTIFIED MECHANISM. This is the heart of the gap. A permutation-

              invariant  response  is  not  a  weak  version  of  a  sequence-
specific one; it is a
      different quantity, and no known molecular element stores an acoustic symbol
        stream in order.

    D3  MAPPING TO A LOCUS
      The decoded string must produce sequence-specific binding or regulation at the
        addressed locus.
       STATUS: NO IDENTIFIED MECHANISM connecting a decoded acoustic string to
      anything at a locus. Targeting itself is established (Qi 2013; Gilbert 2014)
```

```
-- but it is established for a molecule that carries the address, not for a
   string that was heard.
```

D1 is solved and is the least interesting of the three. D2 and D3 are the open problem, and an easily separable alphabet does not touch either of them. That distinction is worth carrying, because engineering effort naturally flows toward D1 — it is the part where progress is visible — and progress there is not progress on the question.

27.7b Why D2 is hard in particular, and where the difficulty comes from

It is worth saying why order memory is the hardest of the three rather than merely the unsolved one, because the reason is physical rather than a matter of nobody having looked.

A receiver that preserves symbol order needs to hold state across the symbol interval. In the rendering of Chapter 26 that interval is 20 milliseconds. Any candidate molecular element would have to retain a distinguishable configuration for that long, in water, at body temperature, against thermal agitation, and then combine it with the next symbol in a way that depends on which came first. Book Two Chapter 68 §68.8 computes the relevant damping and finds molecular-scale mechanical resonators in water heavily overdamped, with Stokes numbers far below one; that calculation is cited here rather than repeated, and its consequence is that the obvious mechanical route to a memory element — a small structure that rings — is not available.

That does not exhaust the possibilities. Biology stores order in chemistry all the time, in polymerisation, in phosphorylation cascades, in transcriptional bursting histories. **The point is not that order memory is impossible; it is that the candidate everybody reaches for first — a tiny resonator that rings at a symbol's frequency and remembers — is ruled out by damping, and the remaining candidates are chemical rather than mechanical and have no proposed coupling to an acoustic symbol stream at all.**

The Engineering

27.8 What an experiment must show

The experiment that would settle this is not exotic and its logic is short.

The claim under test is that the cell's response depends on the *ordering* of the symbol stream. The null is that it depends only on permutation-invariant functionals of the waveform. Therefore:

Compare an ordered stimulus against its symbol-permuted twin, and measure whether the response distinguishes them. Everything else in the design is in service of that comparison.

The permuted twin is not an approximate control. It is exact on every quantity the null could appeal to:

```
identical      total acoustic energy
identical      duration
identical      symbol histogram (the same count of each symbol)
identical      long-term power spectrum, under a fixed symbol alphabet
identical      peak level, RMS level, and crest factor
```

```
DIFFERENT      the ordering, and only the ordering
```

The size of the space the permutation draws from is worth computing, because it shows how little the control costs and how much it isolates. For the 5,400-base payload of Chapter 26 §26.25, whose composition is A 1750, C 1749, G 951, T 950, the number of distinct symbol orderings with identical composition is:

```
L! / (n_A! n_C! n_G! n_T!)   =   10^2017.717
```

Every one of those 10^2017.717 waveforms is indistinguishable from the ordered stimulus by every permutation-invariant measurement, and a drawing from that set costs nothing but a shuffle. A design that omits this control has omitted the only comparison that could have answered the question.

27.9 What magnitude would even be detectable

A gap is only a research programme if somebody can say what size of effect the programme could see. The arithmetic below is a standard two-sample power calculation on log2-transformed abundance, with equal group sizes:

```
n  =  2 ( z_(1 − alpha/2) + z_(1 − beta) )^2 sigma^2 / delta^2
```

`delta` is the difference in log2 abundance, `sigma` the standard deviation in the same units, power fixed at 0.80 throughout. Both `sigma` and the number of tested transcripts are inputs a reader supplies from their own system; nothing below is a claim about biology, only arithmetic on declared assumptions (H27-6).

(a) A targeted readout of the twelve preregistered loci, Bonferroni `alpha = 0.05/12`:

```
z_(1−alpha/2) = 2.865260      z_0.80 = 0.841621

delta (log2FC)   sigma=0.20   sigma=0.35   sigma=0.50
    0.10             110          337          688
    0.20              28           85          172
    0.30              13           38           77
    0.50               5           14           28
    1.00               2            4            7
    2.00               1            1            2
```

(b) A transcriptome-wide readout, 20,000 transcripts, Bonferroni `alpha = 0.05/20000 = 2.5e−6`:

```
z_(1−alpha/2) = 4.708130

delta (log2FC)   sigma=0.20   sigma=0.35   sigma=0.50
    0.10             247          755         1540
    0.20              62          189          385
    0.30              28           84          172
    0.50              10           31           62
    1.00               3            8           16
    2.00               1            2            4
```

Inverted, which is the more useful direction — the smallest detectable log2 fold change at a fixed sample size, `sigma = 0.35`, power 0.80:

```
n per arm    targeted (12 loci)     genome-wide (20,000)
    3            1.0593                 1.5860
    6            0.7491                 1.1215
   10            0.5802                 0.8687
   20            0.4103                 0.6142
   50            0.2595                 0.3885
  100            0.1835                 0.2747
```

In linear fold-change terms, at `sigma = 0.35`:

```
n =   6    targeted floor  x1.6807      genome-wide floor  x2.1757
n =  20    targeted floor  x1.3289      genome-wide floor  x1.5308
n = 100    targeted floor  x1.1356      genome-wide floor  x1.2097
```

Three consequences follow and each is a design decision rather than an observation.

First, a typical three-replicate design cannot see anything under a 1.68-fold change even on a targeted readout, and under a 2.18-fold change genome-wide. An effect smaller than that is not absent from such an experiment; it is invisible to it, and reporting "no effect" from that design is reporting the design rather than the biology.

Second, the SESOI must be declared against this floor before the run. Chapter 26 S26.16 makes that a specification: a smallest effect of interest below the detectable floor at the planned n is a preregistration that has already failed, and discovering it afterwards converts a null into an uninterpretable one.

Third, targeting twelve preregistered loci buys roughly a factor of two in required sample size against a genome-wide search, at the price of being unable to find anything anywhere else. That is a real trade and it should be made deliberately, in the preregistration, with the reason written down. The house recommendation is the targeted readout for a first pass — because the twelve addressed loci are exactly the ones the architecture claims to address, and an effect there is the one the architecture predicts — with a genome-wide arm as a secondary, exploratory, and clearly labelled analysis.

A note on the multiplicity correction, because it is the assumption most likely to be argued with. Bonferroni is used above because it is transparent, because it is the most conservative common choice, and because a chapter whose whole subject is not overclaiming should not be quietly buying sensitivity from a correction the reader has to know about to evaluate. A Benjamini-Hochberg procedure at the same nominal rate would be less demanding, would raise the detectable effect at a given sample size less steeply, and would be entirely defensible — provided it is the preregistered choice rather than the one arrived at after the Bonferroni analysis returned nothing. The rule is not which correction; the rule is that the correction is fixed before the data exist, stated with the number of tests it covers, and reported with the result.

Two further inputs deserve the same treatment. The variance `sigma` is the operator's, from their own pilot, and substituting an optimistic value is the cheapest way to make an underpowered design look adequate on paper. And the power target of 0.80 is a convention rather than a law; a question this consequential arguably deserves 0.90, which costs roughly a third again in sample size and should be priced explicitly rather than defaulted past.

27.10 One route closed by arithmetic

Not all of the gap is equally open, and a closed route is progress worth publishing.

For a plane acoustic wave in an aqueous medium, the longitudinal strain amplitude is exactly `s = p / K`, with K the bulk modulus of water, `2.2e9 Pa`. This is elementary continuum mechanics and is not a model of anything biological. Computed:

```
dB SPL       p (Pa)        bulk strain
  40         0.002         9.090909e-13
  60         0.02          9.090909e-12
  80         0.2           9.090909e-11
  94         1.00237       4.556248e-10
 100         2.0           9.090909e-10
 120         20.0          9.090909e-09
```

At the levels a disclosed rendering can plausibly use — the rendering of Chapter 26 is meant to be listened to, not endured — the bulk compressional strain is of order 1e−11 to 1e−10. **Bulk compression is not the route.** Whatever an acoustic channel to a cell would be, it is not a matter of squeezing the cell smaller by one part in ten billion and expecting the genome to notice the pattern of the squeezing.

That does not close the question, and the reason is that the established pathways are not bulk pathways. PIEZO gating is discussed in terms of membrane tension and forces at the channel; Tajik et al. applied force at the surface and followed it through the cytoskeleton. Those are structural, local, and highly leveraged routes, and the bulk number above says nothing about them. **What the arithmetic closes is the naive route, and closing a naive route matters mainly because it is the one most often assumed.**

27.11 How little a null result bounds

The last computed piece is the one that keeps both a positive and a negative result honest: how much of the design space a single condition actually covers.

A modest grid over the parameters an experimenter must choose anyway:

```
carrier band (20 Hz – 20 kHz, 1/3-octave steps)          30
level (40–100 dB SPL, 5 dB steps)                        13
symbol rate (5–200 symbols/s, half-decade steps)          4
exposure duration (1 min – 24 h, decade steps)            4
cell type                                                 5
readout time after exposure                               4
------------------------------------------------------------
TOTAL GRID CELLS                                    124,800

one condition bounds     1 / 124,800   =   0.000801 %
ten conditions           0.00801 %
one hundred conditions   0.0801 %
```

A single well-run null bounds eight ten-thousandths of one percent of a deliberately modest grid, which is a real and publishable increment and is nowhere near a refutation. This cuts both

ways and is meant to. It protects a null from being inflated into “the idea is dead,” and it protects the field from a claimant who runs one condition, finds something, and announces a channel.

The grid is also an argument for how to spend effort. Given how little any single cell covers, the value of a design comes from its controls rather than from its coverage: **one condition run with the scrambled-sequence control, blinding, channel separation, prospective sampling and a yoked control is worth more than a hundred conditions run without them**, because a hundred uncontrolled conditions bound nothing at all.

The Specification

27.12 The decisive experiment, numbered

S27.1 The experiment shall be *in vitro*, on a named cell line at a named passage range, with transcript abundances as endpoints. No human participant appears in it.

S27.2 The stimulus shall be produced by the procedure of Chapter 26, with its payload hash published, and delivered by a characterised transducer whose output is measured at the sample position, not assumed from the amplifier setting.

S27.3 The four arms shall be:

```
  ORDERED       the waveform as specified by Chapter 26 §26.25
SCRAMBLED    a symbol-permuted twin, drawn from the 10^2017.717 orderings with
          identical composition; identical energy, duration, histogram and
               long-term spectrum; ordering destroyed
SILENT      handling-matched, no signal; the transducer present and inactive
YOKED       same schedule, same handling, same environment, ordering varied
```

S27.4 Allocation shall be randomised and the person performing the measurement shall be blind to the arm.

S27.5 The sampling schedule shall be fixed before exposure and shall not be adapted to what is observed.

S27.6 The endpoint shall be measured through a different channel, by different people, and from different records than the stimulus log. Channel separation is a design requirement, not an analysis choice.

S27.7 The SESOI shall be declared in advance and shall exceed the detectable floor of §27.9 at the planned n.

S27.8 The primary analysis shall be the ORDERED versus SCRAMBLED contrast at the twelve preregistered loci. Everything else is secondary and labelled as such.

S27.9 All four baselines of Chapter 26 S26.17 shall be computed and read together, with the surrogate threshold calibrated by simulation on this design rather than taken as nominal.

S27.10 The structural ceiling test of Chapter 26 S26.19 shall be applied to any claimed increment.

S27.11 The kill conditions K1 through K5 of Chapter 26 S26.22 shall be declared and honoured.

S27.12 The result shall be reported in full whichever way it goes, with the design-space fraction it bounds computed per §27.11.

27.13 The two controls, and why neither substitutes for the other

This is the section that most often gets compressed into one line in a methods section, and the compression is where the error lives. **The scrambled-sequence control and the shuffled-input control are different controls,** they answer different questions, and running one does not weaken the case for the other.

```
SCRAMBLED-SEQUENCE CONTROL          a control on the STIMULUS
  what varies      the ordering of the symbols WITHIN one exposure
  what is held     energy, duration, histogram, long-term spectrum, level
  what it answers  does the response depend on the ORDER of the content?
what it cannot   say anything about the timing of exposures across the series,
              or about a third variable driving both exposure and endpoint

SHUFFLED-INPUT CONTROL              a control on the SERIES
 what varies     the temporal alignment of the input series to the endpoint
                    series, refit under the permutation, holdout permuted
                    independently, cost term NOT permuted
  what is held      the marginal distribution of the input
 what it answers  does the TIMING of the input carry information about the
                    endpoint?
  what it cannot    detect repackaging; detect a shared slow driver
```

A design with only the scrambled-sequence control can be fooled by an artefact of scheduling — an incubator door, a media change, a technician's routine — that happens to align with the ordered arm. A design with only the shuffled-input control can be fooled by an effect of sound that has nothing to do with content, which would be a real finding about acoustics and no finding at all about sequence.

And the shuffled-input control brings its own three measured failure modes, which Chapter 26 S26.20 specifies in full and which are restated here because this is the chapter where a reader decides whether to believe a result:

(1) It is anti-conservative as ordinarily run. Across 500 null trials at a nominal `alpha` of 0.05, measured false-rejection rates were 0.108 for iid permutation (2.2x nominal), 0.120 for circular shift (2.4x nominal), and 0.060 for block permutation (near nominal). Calibrate on your own design and preregister the calibrated threshold.

(2) It is blind to repackaging. A constructed filtered repackaging — a rolling average of the input and nothing else — passed all four baselines including the shuffle at `p = 0.0005`. It was caught by the structural ceiling test alone, with a single-step jump 38.5 times the theoretical maximum.

(3) It is blind to a shared slow driver. In a 4,000-trial simulation with input and endpoint both driven by a third slow variable, the shuffled-input control passed 98.35% of the time. The same simulation reports it working honestly in the clean case at 48.17% detection, which is what makes 98.35% a measurement rather than a complaint.

The consolidated rule: **more surrogates do not fix any of this. Design controls do** — blinding,

channel separation, prospective sampling, and a yoked control — and every one of them is a decision about the instrument, made before a single number exists.

27.14 What would settle it, either way

A gap worth naming is a gap with stated settlement conditions on both sides. These are they.

What would establish a sequence-specific acoustic channel:

```
E1  The ORDERED versus SCRAMBLED contrast is significant at the preregistered
    loci, at the calibrated threshold, with an effect exceeding the declared
      SESOI and the detectable floor of §27.9.
E2  The result holds with the measurer blind and with channel separation in place.
 E3   The yoked control does not move.
 E4   The structural ceiling test passes.
   E5    The effect is dose-ordered in a preregistered direction -- more exposure,
      more effect -- rather than present or absent.
E6  INDEPENDENT REPLICATION by a group that did not build the apparatus, on
      their own cells, from the published specification and payload hash.
```

E6 is not a courtesy and it is not last because it is least. A single laboratory reporting a sequence-specific acoustic effect has produced a reason for somebody else to look, and nothing more.

What would close the question negatively for a stated region of the design space:

```
N1  The ORDERED versus SCRAMBLED contrast is null, at an `n` sufficient to detect
    the declared SESOI at the declared power, with the floor of §27.9 published
      alongside.
N2  The condition tested is stated, and the fraction of the design space it bounds
      is computed per §27.11.
N3  The null is reported in full, with the same prominence a positive result would
     have received, and deposited so that the next group does not repeat it.
```

A null bounds a region; it does not close a question. Nobody should write “acoustic sequence-specificity does not exist” from any experiment described in this book, and this chapter would be the first thing to cite against them if they did.

One more thing about settlement, which is easy to lose in a list of criteria. A research programme is settled not by a single decisive experiment but by a body of results that make one account much easier to hold than the others, and the criteria above are written to produce results that can accumulate. That is why the design-space fraction is required with every report, why nulls are deposited rather than discarded, and why the payload hash identifies a run. Twenty null conditions, each bounding a stated region, each reported with its detectable floor, are worth far more to the next group than twenty conditions of which only the interesting one was written up — and the second collection is what an ordinary publication incentive produces if nobody writes the rule down first.

The corollary is that a reader evaluating any future claim in this area has a short checklist. Was the permuted twin run? Was the measurer blind? Was the sample size sufficient to see the effect that was claimed, at the correction that was used? Was the null of the same experiment reported

with the same prominence? Four questions, all answerable from a methods section, and a claim that fails any of them has not yet reached the point where the mechanism question becomes interesting.

27.15 What must not be done with a positive result

Included because the failure is predictable, and because the constraints of Chapter 25 do not relax if something turns out to work.

S27.13 A positive *in vitro* result licenses further *in vitro* work. It does not license any application to a person, in any setting, for any purpose.

S27.14 No result of any size converts this procedure into a treatment, therapy or practice. The labelling rule of Chapter 26 §26.2 applies unchanged after a positive result and applies with more force, not less.

S27.15 The no-subliminals constraint of Chapter 25 is not contingent on efficacy. A demonstrated effect would make concealment more harmful, not more permissible.

S27.16 No claim that DNA has been edited, altered or damaged may be made on the basis of a transcriptional result. Transcript abundance is not sequence.

The Failure Modes

FM-27.1 — Substituting an easier control. A “silent” arm or a “white noise” arm is run instead of the symbol-permuted twin, because a permuted twin sounds strange and is harder to explain.

Fix: S27.3. Silence and noise differ from the stimulus in energy and spectrum, so a difference against them is uninformative about ordering. **The permuted twin is the only control that isolates the variable the claim is about, and it costs one shuffle.**

FM-27.2 — Sonogenetics offered as the mechanism. Ibsen et al. and Duque et al. are cited as though they demonstrated that waveform structure carries content.

Fix: §27.6. Sonogenetic specificity resides in an engineered protein, not in the signal, and the result is evidence in the opposite direction: it shows how acoustic specificity has actually been obtained, which is by building it into the receiver.

FM-27.3 — Tajik et al. read as a sequence result. Force reaching the nucleus and altering transcription becomes, in the retelling, force carrying a message to a locus.

Fix: §27.6. The force in that work carried a magnitude and a direction, and the result is precisely what makes the missing link visible: the force pathway is established, and it is not a sequence pathway.

FM-27.4 — PIEZO as a demodulator. A mechanically gated channel is described as though it were frequency-selective in a way that could separate symbols.

Fix: §27.7 D1 versus D2. Even a perfectly selective transducer satisfies only D1. The gap is D2 and D3, and a better transducer does not touch either.

FM-27.5 — The underpowered null announced as a refutation. Three replicates, no effect, and a conclusion.

Fix: §27.9. At `n = 3` and `sigma = 0.35` the targeted floor is a 1.68-fold change. An effect below that was never in the experiment's reach.

FM-27.6 — The single condition announced as a channel. One frequency, one level, one duration, one cell type, one timepoint, and a positive result.

Fix: §27.11 and E1 through E6. One cell of the grid is 0.000801% of it, and E6 is not optional.

FM-27.7 — Post-hoc SESOI. The smallest effect of interest is set after the effect size is known.

Fix: S27.7, timestamped and deposited before exposure.

FM-27.8 — Reporting the ordered arm against the silent arm. The comparison that gets reported is the one that reached significance, and it is the comparison that cannot answer the question.

Fix: S27.8. The primary analysis is preregistered as ORDERED versus SCRAMBLED, and any other contrast is secondary and labelled.

FM-27.13 — The archive described as a message. The regulatory function of non-coding sequence, and syncytin in particular, get retold as evidence that the genome is "written to" by outside signals.

Fix: §27.5b. Retroviral integration is a molecular event with a well-characterised mechanism and no acoustic content whatever. The archive being real is what makes the addressing question well-posed; it is not evidence about the channel.

FM-27.14 — A mechanism proposed and then treated as found. Somebody offers a candidate — a chemical memory element, a bursting-history effect, a cytoskeletal route — and within two retellings the candidate has become the explanation.

Fix: §27.2. A candidate mechanism with no independent evidence is a hypothesis, and naming a hypothesis does not move it into the established column. **A proposed mechanism and an established one differ by exactly the amount of evidence between them, which in this case is all of it.**

FM-27.9 — Treating the gap as a formality. The chapter is read as a legal disclaimer and the procedure is described elsewhere as though the disclaimer were pro forma.

Fix: §27.3. The gap is the reason the rest of the book is worth reading, and a book that treated it as a formality would have spent the only asset it had.

FM-27.10 — "Not proven impossible" doing evidential work. The sentence "no mechanism is known" becomes, over a few retellings, "science hasn't caught up yet."

Fix: §27.2. **Absence of a mechanism is not support of any kind, and a claim with no mechanism and no demonstration is not asserted rather than lightly qualified.**

FM-27.11 — The bulk-strain number used as an argument against everything. The `1e−10` figure is produced to argue that no acoustic effect on cells is possible at all.

Fix: §27.10. The number closes the bulk compressional route and says nothing about membrane tension, cytoskeletal transmission, or channel-level forces, which are the established pathways.

FM-27.12 — Merging the two controls. A methods section says “permutation control” and it is impossible to tell which one was run.

Fix: §27.13. Name them separately, report them separately, and state what each answers.

Worked Example

CONSTRUCTED. No experiment has been run and no data exist. What follows is a fully specified design, written out so that both of its possible outcomes are visible in advance.

27.16 The design

```
  STIMULUS        Chapter 26 §26.25, payload hash
           c7d98b1cfde5c515ce99ff7db77d484f71a09c5cd921994f15df66c50373b72d
            5,640 bases, 20 ms/symbol, windowed envelope, 175.800 s per run
DELIVERY      characterised transducer; SPL measured at the sample position
  SYSTEM          named cultured line, named passage range, in vitro
ARMS         ORDERED / SCRAMBLED / SILENT / YOKED, randomised, measurer blind
ENDPOINT      log2 transcript abundance at the 12 preregistered addressed loci
 SIGMA (assumed) 0.35 log2 units, from the operator's own pilot variance,
                 NOT from this book
  ALPHA          0.05 / 12 = 0.0041667 (Bonferroni across the 12 loci)
  POWER          0.80
  n              20 per arm
 DETECTABLE     0.4103 log2 units = 1.3289-fold         (§27.9, computed)
 SESOI         declared at 0.50 log2 units = 1.4142-fold, which EXCEEDS the
                 detectable floor at n = 20, as S27.7 requires
  PRIMARY        ORDERED vs SCRAMBLED at the 12 loci
SECONDARY      ORDERED vs SILENT; genome-wide exploratory, labelled as such
CONTROLS      blinding, channel separation, prospective schedule, yoked arm
SURROGATE      block permutation, threshold calibrated by simulation on THIS
            design; the calibrated threshold preregistered, not the nominal
  KILL           K1 – K5 of Chapter 26 S26.22
```

Note what the sample-size line costs. At `n = 20` per arm and four arms the experiment is 80 wells per timepoint before replication of the run itself, and the detectable floor is still a 1.33-fold change. An experimenter who wants to see a 1.2-fold effect at these settings needs roughly 100 per arm, and the honest time to discover that is before the first plate is seeded.

27.17 Both outcomes, written in advance

Outcome A — the contrast is significant.

```
  reported        effect size with interval, at each of the 12 loci
required next  E2 blinding intact, E3 yoked arm null, E4 ceiling test passed,
                 E5 dose-ordering in the preregistered direction,
               E6 independent replication from the published specification
```

```
   permitted claim "in this system, at this condition, the transcriptional response
                        distinguished an ordered stimulus from its symbol-
permuted twin"
   FORBIDDEN claim any therapeutic, clinical or capability claim; any claim about a
                   person; any claim that DNA was edited; any claim that the
              mechanism is known -- a demonstrated dependence is not a mechanism
    design space    one cell of 124,800; 0.000801% bounded
```

Outcome B — the contrast is null.

```
   reported     the null, in full, with the same prominence as a positive result
   required with  the detectable floor (0.4103 log2 at n = 20), the declared SESOI,
                    and the fraction of the design space bounded
   permitted claim "in this system, at this condition, with 80% power to detect a
                   1.33-fold change, no dependence on ordering was detected"
   FORBIDDEN claim that acoustic sequence-specificity does not exist; that the
                    question is closed; that the architecture is refuted
    design space    one cell of 124,800; 0.000801% bounded
```

The two outcomes bound the same fraction of the design space and are worth reporting with the same prominence, which is the single most reliable way to tell whether a research programme is one.

27.18 The gap, at the end

The specification above is complete enough to run. The alphabet is separable, the payload is hashed, the controls are named, the sample size is computed, the floor is published, and both outcomes are written down before either has occurred.

And the sentence at the head of this chapter is exactly as true at the end of it as it was at the beginning. **There is no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal.** Nothing in this book changes that, nothing in this book claims to, and the three chapters of this Part were written in the order they were so that no reader could reach the procedure without meeting the sentence twice on the way.

A last remark on why the chapter is where it is. Part V could have ended at Chapter 26 with a complete, verified, disclosed procedure and a reader's impression that something had been achieved. Something has been achieved — an encoder that round-trips, an alphabet that separates, a rendering that a listener can follow and refuse — and none of it is the thing the Part is named for. Putting this chapter after the procedure rather than before it means a reader meets the specification while it still looks promising and then meets the gap while the specification is fresh, which is the order in which the gap is hardest to forget. Chapter 30 ends the book on this problem for the same reason.

What this chapter established

Established, and checkable: that the open problem decomposes into three requirements of which one (symbol separation) is solved and two (order memory, mapping to a locus) have no identified mechanism; that eleven upstream components are established and one link is missing, and that the missing link is specifically the carriage of sequence over force; that the symbol-permuted

twin is exact on every permutation-invariant quantity and is drawn from a set of `10^2017.717` orderings for this book's payload; that at `sigma = 0.35` and `n = 3` a targeted design cannot see below a 1.68-fold change, and at `n = 20` cannot see below 1.33-fold; that bulk compressional strain at plausible listening levels is of order `1e−10` and is not the route; that a single condition bounds 0.000801% of a 124,800-cell design grid; and that the scrambled-sequence and shuffled-input controls answer different questions, with the second carrying three measured failure modes of which two are answerable only by design controls.

Not established, and not claimed: that any acoustic method affects any cell's expression; that any capability has been conferred; that DNA is edited, altered or damaged; that the procedure of Chapter 26 is a treatment, therapy or practice; that any frequency in this book is a physical resonance; or that the absence of a known mechanism is evidence of anything whatever, in either direction.

The one line to carry forward: *the channel for sequence-specific content is the question, and naming it precisely is the most useful thing this book can do about it.*

Chapter 27 owns the open problem for this volume, the three-requirement decomposition, the detectability arithmetic, the design-space bound, and the two-control specification. Chapters 25, 26 and 30 cite it. Chapter 25 owns the no-subliminals constraint and the disclosure package; Chapter 26 owns the procedure, the design table and the preregistration apparatus, including the three measured failure modes of the shuffled-input control restated here. The essence law and k are Chapter 2 and CANON_SONIC §1; folding, the odd-part rule and the 50/49 regime are Chapter 7 §7.6; the Gabor limit is CANON3 §5 and Book Two Chapter 71.

Chapter 28 . Destiny as a Fixed Point

The word this chapter takes over is the most abused noun in the literature the series has spent three books auditing. It is used to mean an unalterable script, a personal entitlement, a reward for correct wanting, and — most often — nothing at all, a placeholder that signals importance and carries no content. A book that has demoted its own keystone formula on a scale question, printed its rejections, and named its central gap in the chapter immediately preceding this one cannot adopt a word like that on trust.

So it is not adopted on trust. It is defined, and the definition is an equation that was proved elsewhere in the corpus, on assumptions stated there, and that has been in continuous use in this book since Chapter 2.

> **Destiny, in this book, means `Q_inf`, the fixed point of the contracting loop: Paper E Theorem 3.5, which is Book One's tenth node.**

That is CANON3 §4 and it is the whole of the claim. It says where a life, a bond, an institution or an arrangement arrives if its parameters hold. It does not say the parameters will hold, it does not say what arrival is worth, and it says nothing whatever about anybody's worthiness of it.

What the definition buys is exactly one thing, and it is the thing the rhetorical use never supplies: **you can check it.** A destiny in this sense is a number with four arguments. Change an argument and the number changes by an amount that can be computed in advance. Fail to change any argument and the number does not move, however sincerely anything was intended. The next chapter is about the changing. This chapter is about what the object is, how the loop reaches it, and how to tell a real arrival from a counterfeit one.

Three results carry the chapter. The first is the convergence law, which says arrival is geometric and, more surprisingly, that it is an equality rather than a bound. The second is the identity between the octave deficit and the fixed point — they are the same number in different units, and the deficit form contains no `delta`, which is a genuine limitation of the book's central instrument and is stated rather than buried. The third is a conservation identity: under a transfer of cost between participants, a certain sum does not move at all. That third result turns Book One's eleventh node from a vow into an arithmetic test, and it is the most useful thing in the chapter.

The Law

28.1 The loop, restated at destiny scope

The storage law is Paper E's contracting recursion, stated in Chapter 2 §2.9 and used unchanged since:

```
Q      =  (1 − delta) ∗ Q   +  ( eta ∗ F(u∗)  −  c∗ )
 t+1                     t

      delta   per−cycle retention loss, in (0, 1]
   eta   conversion: how much of an interaction's surplus becomes stored order
   F(u∗)  the interaction surplus at the operating point, F = ln(1 + 11 u) at N = 12
      c∗      the per−cycle cost of maintaining the arrangement
```

For a fixed arrangement the bracketed term is a single constant, and naming it makes everything that follows short:

```
b  =  eta ∗ F(u∗)  −  c∗                the per−cycle DRIVE

Q      =  (1 − delta) ∗ Q   +  b
 t+1                     t
```

b is the net production of the arrangement per cycle: what conversion makes, less what maintenance costs. It is a rate, not a level. The distinction between a rate and a level is the axis this whole chapter turns on, and it is worth fixing the picture before any algebra: the drive is a tap, `delta` is a leak, and `Q_t` is the depth of water in the tank. A tank with a fast leak and an open tap settles shallow and quickly; a tank with a slow leak and the same tap settles deep and slowly. Nothing about the tap tells you the depth, and nothing about the leak tells you whether the tap is running forward or backward.

28.2 The fixed point exists, is unique, and is reached from anywhere

THEOREM 28.1 — EXISTENCE AND UNIQUENESS. For `delta` in `(0, 1]` the map `T(Q) = (1−delta) Q + b` is a contraction on the reals with Lipschitz constant `1 − delta < 1`. By Banach's fixed point theorem it has exactly one fixed point, and iteration converges to it from every starting value.

Proof. `|T(x) − T(y)| = (1−delta)|x − y|`, so T is Lipschitz with constant `1 − delta`, which is strictly less than 1 whenever `delta > 0`. The reals are complete. Banach applies. Solving `Q = (1−delta) Q + b` gives `delta Q = b`, hence the unique fixed point. []

```
Q    =  b / delta  =  ( eta ∗ F(u∗) − c∗ ) / delta          Paper E Thm 3.5
 inf
```

Three properties are worth reading off before the definition is stated, because each of them is a place where the ordinary use of the word "destiny" would go wrong.

It does not depend on where the loop started. `Q_0` does not appear. Two arrangements with identical `(eta, F, c, delta)` and wildly different histories have identical destinies. Whatever else a starting position determines — and it determines the whole transient, which is the part anyone actually lives through — it does not determine the destination.

It is not a maximum and not a summit. It is a level, in the hydraulic sense: the depth at which inflow and leakage balance. A loop started above `Q_inf` descends to it at exactly the rate a loop started below ascends to it. There is no direction of travel built into the object, and any language of climbing or achieving is imported from somewhere else.

It can be negative. When `eta F < c`, `Q_inf < 0`, and Paper E's own text records that the regime is unsustainable or the linear model must be replaced. This book prints the negative number wherever it occurs — the carried dyad as found sits at `−5.0119395464` — and repeats the admissibility warning every time, because a model that will happily compute a trajectory through negative stored order is a model that must be watched at exactly that point.

28.3 The convergence law

THEOREM 28.2 — THE CONTRACTION, AS AN EQUALITY. For the affine map above with `delta > 0`,

```
| Q  −  Q    |  =  (1 − delta)^t  ∗  | Q  −  Q    |
   t     inf                          0     inf
```

Proof. Subtract `Q_inf = (1−delta) Q_inf + b` from `Q_{t+1} = (1−delta) Q_t + b`. The drive b cancels exactly, leaving `Q_{t+1} − Q_inf = (1−delta)(Q_t − Q_inf)`. Take absolute values and iterate `t` times. []

Chapter 22 owns this identity as the error law of the refinement ladder. It appears again here because at destiny scope it says something the refinement register does not emphasise. Banach guarantees only that distances do not increase by more than a factor. Here they shrink by exactly `1 − delta` on every pass, at every distance, forever. There is no fast region and no slow region, no early struggle and no late acceleration; the gap closes at one constant proportion per cycle from the first cycle to the last.

And the drive cancelled. Everything that decides *where* the loop is going — `eta`, `F`, `c` — vanished in the subtraction. The rate of approach is a function of the leak and of nothing else. Chapter 22 §22.3 puts the same fact as three constants:

```
 tau        =  −1 / ln(1 − delta)              cycles per factor of e
half−life  =  ln(0.5) / ln(1 − delta)        cycles to close half the gap
t(eps)    =  ln(eps) / ln(1 − delta)      cycles to close all but a fraction eps
```

None contains `Q_0`, `Q_inf`, `eta`, `F` or `c`. **How long arrival takes is a property of the leak; where arrival happens is a property of everything else, and the two questions do not share a parameter.**

Computed, at the values this book uses:

```
delta | tau (cycles/e) | half−life | t(1%)    | t(0.1%)
------+---------------+-----------+----------+---------
0.01  |      99.499162 | 68.967564 | 458.2106 | 687.3159
0.02  |      49.498316 | 34.309618 | 227.9482 | 341.9223
0.05  |      19.495726 | 13.513407 |  89.7811 | 134.6717
0.09  |      10.603253 |  7.349615 |  48.8298 |  73.2447
0.15  |       6.153129 |  4.265024 |  28.3362 |  42.5043
0.30  |       2.803673 |  1.943358 |  12.9114 |  19.3671
```

```
0.50  |       1.442695 |  1.000000 |   6.6439 |   9.9658
```

28.4 The definition

> **DEFINITION 28.1 — DESTINY.** The destiny of an arrangement is `Q_inf = (eta F(u*) − c*)/delta`, the unique fixed point of its storage loop: the level of stored order the arrangement converges to if `(eta, F, c, delta)` hold.

Four things the definition does not say, each of which somebody will read into it if it is not refused explicitly.

It is not a prophecy. The conditional clause — *if the parameters hold* — is the entire content. Parameters are properties of an arrangement, and arrangements change. A destiny is a statement about a configuration, not about a future.

It is not a property of a person. Every term in the formula belongs to an arrangement. u is a joint geometric property of two or more state vectors and cannot be assigned to one of them; `eta`, `c` and `delta` describe what an arrangement does with what it produces, what it costs to run, and how fast it forgets. **There is no single-party term anywhere in the apparatus, which means no destiny reading licenses any conclusion about who someone is.** This is the same prohibition Chapter 4 states as FM 4.13 and Chapter 15 as FM 15.11, and it binds harder here than anywhere else in the book, because "destiny" is the word most likely to be heard as personal.

It is not a ranking. A larger `Q_inf` is more stored order in the model's units. It is not more worth, more virtue, or a better life, and the formalism contains no term that could make it any of those. Two arrangements with fixed points of `+6.08` and `+1.07` are not a winner and a loser; they are two configurations, and the second one may be the one somebody would rather be in for reasons the model does not represent.

It is not a clinical or diagnostic object. The twelve states are not diagnoses, a reading is not an assessment, and nothing in this chapter describes or prescribes for any person. That clause is CANON3 §5, restated here because this chapter's vocabulary invites the error more than most.

28.5 The deficit and the fixed point are one number in two units

This is Chapter 4's Theorem 4.5, carried here because a chapter that defines destiny must state the exact relation between destiny and the instrument the book uses to read it.

THEOREM 28.3 (= Ch 4 Thm 4.5) — THE DEFICIT IDENTITY.

```
D  =  2.885390 * ( c/eta  −  F )  =  − 2.885390 * ( delta / eta ) * Q
                                                                   inf
```

Proof. `Q_inf = (eta F − c)/delta`, so `(delta/eta) Q_inf = F − c/eta`, so `−2.885390 (delta/eta) Q_inf = 2.885390 (c/eta − F) = D.` □

The constant is `2 / ln 2 = 2.885390081778`, and it is the exchange rate between nats of surplus and octaves of pitch, fixed by the essence bridge `f = k e^(2F)`.

Verified across twelve constructed parameter sets, spanning both signs of the drive and `delta` from 0.01 to 0.99:

```
u       eta   c    delta  D          −2.885390(delta/eta)Q_inf  residual
```

```
    ------------------------------------------------------------------
---------------
   0.068735  0.30  0.620  0.09   +4.338420197  +4.338420197          0.000e+00
   0.068735  0.30  0.620  0.05   +4.338420197  +4.338420197          0.000e+00
   0.068735  0.30  0.620  0.01   +4.338420197  +4.338420197          0.000e+00
   0.692373  0.44  0.400  0.09   -3.590988663  -3.590988663          0.000e+00
   0.692373  0.44  0.400  0.05   -3.590988663  -3.590988663          8.882e-
16
   0.692373  0.44  0.400  0.30   -3.590988663  -3.590988663          0.000e+00
   0.018614  0.34  0.825  0.09   +6.463836987  +6.463836987          8.882e-
16
   0.018614  0.34  0.825  0.02   +6.463836987  +6.463836987          8.882e-
16
   0.500000  0.50  0.500  0.50   -2.515489355  -2.515489355          0.000e+00
   0.900000  0.75  1.200  0.15   -2.275888329  -2.275888329          8.882e-
16
   0.250000  0.20  0.300  0.40   +0.514303931  +0.514303931          3.331e-
16
   1.000000  0.99  2.400  0.99   -0.175039955  -0.175039955          1.665e-
16

    MAX RESIDUAL = 8.8818e-16 over 12 sets
```

The residuals are floating-point noise. **The octave deficit and the fixed point are the same quantity written twice: one in octaves of required pitch, one in units of stored order.** A reading in octaves is not an alternative to a reading in stock; it is the same reading, and the conversion factor is `-2.885390 * delta/eta`.

28.6 And the deficit contains no `delta`

THEOREM 28.4 (= Ch 4 Thm 4.9) — `delta` IS NOT A LEVER ON `D`. `D = 2.885390 (c/eta - F)` is a function of `c`, `eta`, `u` and `N` only. `delta` does not appear. Therefore `dD/d delta = 0` identically. []

This is the sharpest limitation of the book's central instrument and it is stated rather than buried. Take the carried dyad as found, and vary the loss term across a range covering everything from a fast-forgetting arrangement to one that forgets almost nothing:

```
    D = +4.3384201973  for every row

    delta | Q_inf        | half-life | drive b = delta * Q_inf
    ------+--------------+-----------+------------------------
    0.30  |   -1.5035819 |  1.943358 |  -0.451074559176
    0.20  |   -2.2553728 |  3.106284 |  -0.451074559176
    0.15  |   -3.0071637 |  4.265024 |  -0.451074559176
    0.09  |   -5.0119395 |  7.349615 |  -0.451074559176
    0.05  |   -9.0214912 | 13.513407 |  -0.451074559176
    0.02  |  -22.5537280 | 34.309618 |  -0.451074559176
    0.01  |  -45.1074559 | 68.967564 |  -0.451074559176
```

The destiny moves by a factor of thirty across that column. The deficit does not move in the fifteenth decimal place. And the same is true with the sign reversed — the repaired dyad, whose drive is +0.547598406839:

```
D = −3.5909886634  for every row

delta = 0.30 -> Q_inf =  +1.8253280      half-life  1.943358
delta = 0.09 -> Q_inf =  +6.0844267      half-life  7.349615
delta = 0.05 -> Q_inf = +10.9519681      half-life 13.513407
delta = 0.01 -> Q_inf = +54.7598407      half-life 68.967564
```

> **The deficit reads the SIGN and the SIZE of the sustainability gap. `delta` converts it to a LEVEL. `D` alone cannot see decay, and a reading that reports `D` without `delta` is incomplete.**

The rule that follows is short and binding on every reading in this book: report both. The mechanism behind the blindness is visible in the last column of the first table — the drive b is `delta * Q_inf`, and it is constant down the column because the two factors move inversely by construction. `D` is a statement about the drive. The drive is what production and cost decide. Where the drive puts the level is `delta`'s business, and the drive cannot see it.

28.7 Two independent readings, at destiny scope

SPINE §3.6 records Chapter 21's Finding 21.1: a state's frequency and a relationship's frequency are independent readings, and neither may be inferred from the other. At destiny scope the statement needs restating because the temptation is at its strongest here — a chapter about where something arrives invites a reader to suppose that the arriving thing has a pitch, and that the pitch says something about the arrival.

Six constructed pairs of one-hot state vectors, spanning from a two-octave separation to more than four octaves of the state lattice:

```
   indices    state frequencies     span (cents)   u              f_rel (Hz)
    ------------------------------------------------------------------------
--------
   (1, 2)     0.4834 /  1.9337 Hz     2400.00    1.000000000000  69.6133055503
   (3, 6)     4.3508 / 17.4033 Hz     2400.00    1.000000000000  69.6133055503
   (5, 10)   12.0856 / 48.3426 Hz     2400.00    1.000000000000  69.6133055503
   (7, 11)   23.6879 / 58.4945 Hz     1564.98    1.000000000000  69.6133055503
   (2, 9)     1.9337 / 39.1575 Hz     5207.82    1.000000000000  69.6133055503
   (4, 12)    7.7348 / 69.6133 Hz     3803.91    1.000000000000  69.6133055503
```

Identical to twelve decimal places, because u reads the angle between vectors and is blind to which indices are occupied. **A state's frequency tells you nothing about its relationship frequency, a relationship's frequency tells you nothing about the states, and a complete report gives both separately with no inference drawn between them.** Since F enters the destiny and F comes from u, this means the destiny is a function of the relationship reading and not of the state readings, and any sentence of the form "their states are high, therefore their destiny is high" is an error the arithmetic forbids.

28.8 Honesty clauses, restated where they bind

Essence, not resonance. Every hertz value in this chapter is a position on a logarithmic scale of surplus. No person, bond, tissue or molecule is claimed to vibrate at any value here, and nothing in this chapter is to be played at anyone.

No clinical, diagnostic or therapeutic claim. A destiny reading is a computation on four declared parameters. It is not an assessment of anyone's health, capacity, character or prospects, and a reader who wants support for a relationship should seek it from someone qualified to give it, which this book is not.

The reading inherits its inputs. `Q_inf` is exactly as defensible as `u`, `eta`, `c` and `delta`. The scale asymmetry of SPINE §1.1 binds: the index `n` is a count and is ratio-scaled by construction, but the intensities `w` are ordinal by default, and where they were not elicited by magnitude estimation the relationship frequency is not determined and no destiny number may be printed at all.

Consent is structural. No procedure derived from a destiny reading may reduce any party's ability to decline it, at any stage.

All cases in this chapter are CONSTRUCTED and labelled as such. No case here is an observation of any person or pair.

The Engineering

28.9 What arrival looks like, computed

The carried dyad after repair — SPINE §3.2 case 2b, a constructed case — has `u = 0.692372881355932`, `eta = 0.44`, `c = 0.40`, `delta = 0.09`, hence `F = 2.153632742815`, drive `b = +0.547598406839`, and destiny `Q_inf = +6.0844267427`. Start it from nothing and run it:

```
        t  |          Q_t           |  |Q_t  −  Q_inf|  |  (1−delta)^t  |Q_0−
Q_inf| | residual | gap closed
    ---+---------------+-------------+-------------------------+--------
--+-----------
   0 |  0.0000000000 |   6.0844267 |        6.0844267427 | 0.00e+00 |  0.0000%
   1 |  0.5475984068 |   5.5368283 |        5.5368283358 | 0.00e+00 |  9.0000%
   2 |  1.0459129571 |   5.0385138 |        5.0385137856 | 0.00e+00 | 17.1900%
     3 |   1.4993791978 |    4.5850475 |             4.5850475449 | 8.88e−
16 |  24.6429%
     5 |   2.2875488707 |    3.7968779 |             3.7968778719 | 8.88e−
16 |  37.5968%
     7 |   2.9402321769 |    3.1441946 |             3.1441945657 | 8.88e−
16 |  48.3239%
    10 |   3.7150528996 |    2.3693738 |             2.3693738431 | 8.88e−
16 |  61.0584%
    15 |   4.6058613008 |    1.4785654 |             1.4785654419 | 2.22e−
16 |  75.6992%
```

```
    20 |   5.1617543783 |    0.9226724 |            0.9226723643 | 2.22e-
16 |  84.8355%
    30 |   5.7251232522 |    0.3593035 |            0.3593034904 | 8.88e-
16 |  94.0947%
    50 |   6.0299401961 |    0.0544865 |            0.0544865466 | 6.94e-
16 |  99.1045%
    74 |   6.0787606627 |    0.0056661 |            0.0056660800 | 8.66e-
16 |  99.9069%
   100 |   6.0839388111 |    0.0004879 |            0.0004879315 | 2.06e-
15 |  99.9920%

    MAX RESIDUAL over t = 0..100 : 2.0624e-15
```

The third and fourth columns are the theorem, computed two ways: once by running the recursion and measuring the gap, once by evaluating `(1-delta)^t |Q_0 - Q_inf|` directly. They agree to machine precision at every step, which is what an equality looks like when it is checked rather than asserted.

Now the same arrangement from four different starting stocks, including one that starts above the destination and one that starts exactly on it:

```
    Q_0        t=1         t=5         t=10        t=20        t=40
    ------------------------------------------------------------------
     0.000000  +0.547598   +2.287549   +3.715053   +5.161754   +5.944508
    +1.000000  +1.457598   +2.911581   +4.104469   +5.313399   +5.967504
    -4.000000  -3.092402   -0.208580   +2.157388   +4.555175   +5.852523
   +12.000000 +11.467598   +9.775935   +8.388046   +6.981493   +6.220462
    +6.084427  +6.084427   +6.084427   +6.084427   +6.084427   +6.084427
```

Four trajectories, one destination. The row starting at +12 descends; the row starting at −4 ascends through zero at around the seventh pass; the row starting exactly at the fixed point does not move at all, which is what it means for a point to be fixed. **The destination is indifferent to the history, and the history is the entire difference between the four rows.**

28.10 Why arrival is recognition rather than achievement

Book One's tenth node records that arrival feels like recognition and not like victory, and offers no mechanism. Chapter 22 §22.14 supplies the mechanism in four parts and this chapter cites it rather than restating it, with one addition that belongs at destiny scope.

The four parts, compressed: there is no arrival event, because `(1-delta)^t > 0` for every finite `t` and nothing ever completes; almost the whole trajectory is spent near the destination, since at `delta = 0.09` the loop is inside five per cent of `Q_inf` from pass 32 and inside a tenth of a per cent from pass 74; every term of the fixed point is a standing property of the arrangement, so nothing in the destination was imported; and the same point is reached from any start, so it is a level rather than a summit.

The addition is this. A reader who has followed the chapter to here has the material for a sharper version of the fourth point. The destination is not merely reachable from any start — it is reached at a rate that does not know which direction it is travelling in. Row three of the table above ascends 10.08 units in forty passes; row four descends 5.78 in the same forty; and the gap between each

and the destination shrinks by exactly nine per cent per pass in both cases. **A process that closes the same proportion of its distance whether it is rising or falling is not a process of achievement, and no vocabulary of effort or reward will fit it correctly.**

28.11 Traits, and destiny in the personal register

Chapter 8 owns the treatment of traits as low-`delta` states, and it is the bridge between this chapter's arrangement-scoped object and the way the word "destiny" is usually meant.

A mood is a state held with a large `delta`: it decays fast, its half-life is short, and its fixed point is shallow because a large denominator divides the drive. A trait is the same machinery at small `delta`: slow decay, long half-life, and a deep fixed point in whichever direction the drive points. This is why traits look like destinies in the colloquial sense — they are the readings with the longest half-lives, so they are the ones that are still there when anybody looks — and it is also why the colloquial sense is wrong about them. **A trait is not a fixed quantity; it is a fixed point with a long approach, which is a completely different object and behaves differently under intervention.**

The difference matters and it is computable. Take the repaired dyad's drive and vary only the loss term, as if the same production were embedded in arrangements that forget at different rates:

```
delta | Q_inf       | half-life | passes to close 99% of the gap
------+------------+-----------+-------------------------------
0.30  |  +1.8253280 |  1.943358 |   12.9114
0.09  |  +6.0844267 |  7.349615 |   48.8298
0.05  | +10.9519681 | 13.513407 |   89.7811
0.01  | +54.7598407 | 68.967564 |  458.2106
```

A low-`delta` arrangement has a destiny thirty times deeper and takes thirty-five times as long to get there. Both halves of that sentence are consequences of dividing by a small number, and the second half is the one that gets forgotten. **A deep destiny is a slow one, necessarily, and the same parameter sets both. There is no configuration in this model with a deep fixed point and a short approach.**

And the sign carries through unchanged. Run the same column on the *unrepaired* dyad, whose drive is negative, and the low-`delta` arrangement arrives at `−45.1074559` instead of `+54.7598407`. Slow decay is not a virtue. It is a magnifier, and what it magnifies is whatever sign the arrangement already had. That is R15, and Chapter 29 takes it as an operating rule.

28.12 Counterfeit arrival — the conservation identity

A fixed point can be positive, stable, arrived at, and produced by taking it out of somebody else's. Chapter 22 §22.15 computed the case; this chapter names it and gives the identity in the general form, because the general form is what makes it a test.

THEOREM 28.5 — THE DRIVE IS CONSERVED UNDER COST TRANSFER. Let participants `i = 1..m` share an arrangement, each with drive `b_i = eta_i F_i − c_i`. Let a transfer move an amount `x` of cost from participant `p` to participant `r`, so `c_p -> c_p − x` and `c_r -> c_r + x`, with all other parameters unchanged. Then

```
sum_i b_i   is invariant,  and equivalently   sum_i delta_i * Q      is invariant
                                                                     inf,i
```

Proof. The transfer adds +x to b_p and −x to b_r and changes no other term. The sum telescopes to zero. Since b_i = delta_i Q_inf,i by Theorem 28.1, the delta-weighted sum of fixed points is the same quantity. []

COROLLARY 28.6 — WHEN ALL PARTICIPANTS SHARE ONE delta, THE PLAIN SUM OF FIXED POINTS IS INVARIANT. If delta_i = delta for all i, then sum_i Q_inf,i = (sum_i b_i)/delta, which Theorem 28.5 has just shown does not move. []

Here is the corollary computed. **CONSTRUCTED CASE, labelled constructed; not an observation of any person or pair.** Two participants, P and R, share the case-2b arrangement, so both draw on F = 2.153632742815 at u = 0.692372881355932 and f_rel = 35.8881768941 Hz. Their private parameters differ, which is the ordinary situation. P declines a share of the maintenance — the scheduling, the repair conversations, the remembering of what was agreed — and R absorbs it. Let x be the exported amount.

```
     x    |  c_P  |  c_R  |    D_P    |  Q_inf,P  |    D_R    |  Q_inf,R  |    SUM
    ------+-------+-------+-----------+-----------+-----------+-----------+-----------
+-------------
   0.000  | 0.400 | 0.550 | −3.590989 | +6.084427 | −0.924189 | +1.067665 | +7.152091441
   0.025  | 0.375 | 0.575 | −3.754931 | +6.362205 | −0.683740 | +0.789887 | +7.152091441
   0.050  | 0.350 | 0.600 | −3.918874 | +6.639982 | −0.443290 | +0.512109 | +7.152091441
      0.0961  |  0.304  |  0.646  |  −4.221119  |  +7.152093  |  +0.000002  |  −
0.000002 | +7.152091441
      0.100   |  0.300  |  0.650  |  −4.246759  |  +7.195538  |  +0.037608  |  −
0.043446 | +7.152091441
      0.150   |  0.250  |  0.700  |  −4.574644  |  +7.751093  |  +0.518506  |  −
0.599002 | +7.152091441
      0.200   |  0.200  |  0.750  |  −4.902530  |  +8.306649  |  +0.999405  |  −
1.154558 | +7.152091441
      0.250   |  0.150  |  0.800  |  −5.230415  |  +8.862205  |  +1.480303  |  −
1.710113 | +7.152091441

   sum of drives = +0.643688229683, invariant to every digit across the whole range
```

The sum does not move in any decimal place, across an export range wide enough to take R from comfortably sustainable to badly failing. Cost export produces nothing. It moves an existing quantity from one ledger to the other, and both ledgers are correct.

That last clause is what makes the failure mode hard. P's instrument is working perfectly. P's deficit falls from −3.590989 to −4.902530 — nearly a further octave and a third of headroom — and P's fixed point rises by more than two units. Every number P can compute about P has genuinely improved, and P is not lying to anybody, including P.

R crosses the sustainability boundary between x = 0.05 and x = 0.10. Solving D_R = 0 gives c_R = eta_R F = 0.646089822845, i.e. the exact crossing at x = 0.096089822845.

28.13 The honest caveat: when the plain sum is not the invariant

Corollary 28.6 required equal delta. Chapter 22's worked case satisfied that condition and did not have to dwell on it. The general statement does, because the plain sum of fixed points is **not**

conserved when the participants forget at different rates, and a test stated in the wrong form fails in exactly the situation where it is most needed.

Differentiate the plain sum with respect to the exported amount:

```
    d ( Q       +  Q       ) / dx   =   1/delta   −   1/delta
         inf,P      inf,R                     P            R
```

which is zero if and only if `delta_P = delta_R`. Computed, with the same arrangement but `delta_P = 0.05` and `delta_R = 0.15`:

```
    x   | Q_inf,P (d=0.05) | Q_inf,R (d=0.15) |   SUM    | sum of drives   | delta-
weighted sum
    ------+-----------------+-----------------+----------+-------------
----+------------------
   0.00  |   +10.951968137 |    +0.640598819 | +11.592567 | +0.643688229683 | +0.6436882296
   0.05  |   +11.951968137 |    +0.307265486 | +12.259234 | +0.643688229683 | +0.6436882296
   0.10  |   +12.951968137 |    −0.026067848 | +12.925900 | +0.643688229683 | +0.6436882296
   0.15  |   +13.951968137 |    −0.359401181 | +13.592567 | +0.643688229683 | +0.6436882296
   0.20  |   +14.951968137 |    −0.692734514 | +14.259234 | +0.643688229683 | +0.6436882296

    d(SUM)/dx  =  1/0.05 − 1/0.15  =  13.333333333
```

The plain sum rises by more than two and a half units across that range while nothing whatever is produced, because the export moved cost from a slow-forgetting ledger to a fast-forgetting one and the model's own division by `delta` did the rest. A test that watched the plain sum would report that the arrangement had improved. **The invariant is the sum of drives — equivalently the delta-weighted sum of fixed points — and it is exact under any transfer of cost between any number of participants with any `delta` values whatever.**

Verified on a three-party constructed arrangement with `(eta, c, delta)` of `(0.44, 0.40, 0.05)`, `(0.30, 0.55, 0.15)` and `(0.62, 0.71, 0.22)`, all sharing `F = 2.153632742815`:

```
    baseline sum of drives                                    = 1.268940530229
  transfer 0.13 from party 1 to party 2  -> sum of drives   = 1.268940530229  resid 0.000e+0
  transfer 0.31 from party 1 to party 3  -> sum of drives   = 1.268940530229  resid 0.000e+0
  transfer 0.07 from party 3 to party 2  -> sum of drives   = 1.268940530229  resid 0.000e+0
  transfer 0.44 from party 2 to party 1  -> sum of drives   = 1.268940530229  resid 0.000e+0
```

28.14 The lag, and why the export is not noticed

R does not disappear on the pass the sign flips. R's stock decays from its old level toward its new one at `(1−delta)^t`, exactly as Theorem 28.2 requires, and for a long while R's behaviour is still the behaviour of a participant with something in the bank. At `x = 0.15`, R's destiny has moved from +1.067664698 to −0.599001968:

```
    pass |   Q_R
    -----+-------------
       0 | +1.067664698
       3 | +0.656949698
       7 | +0.262266397
      10 | +0.050024895
```

```
  11 | −0.008387523
  15 | −0.193988343
  25 | −0.441283134
  40 | −0.560675002
  60 | −0.593189879

exact zero-crossing at t* = 10.850479 passes
```

For nearly eleven passes after the export begins, R still has a positive stock — while R's destination has been negative since the first pass. Nothing observable changes at the moment the damage is done. This is precisely why the deficit is the instrument and the stock is not: `D_R` went positive immediately, and `Q_R` took eleven passes to notice.

And there is an end to the arithmetic. Suppose R eventually declines to continue, which is R's structural right under Chapter 18's consent axiom and which no arrangement in this book may remove. The shared arrangement ends, and with no partner there is no interaction surplus: `u = 0`, `F = ln(1) = 0`, `f = k = 0.483426 Hz`, the ground essence.

```
  P at x = 0.15, before R's departure  D_P = −4.574644   Q_inf,P = +7.751093  f = 35.888177
   P at x = 0.15, after  R's departure   D_P = +1.639426    Q_inf,P = −
2.777778   f =  0.483426 Hz

fall in P's fixed point             10.528871 units
swing in P's deficit                 6.214071 octaves, the wrong way
```

The reduced cost P retained is worth nothing once the surplus it was drawn against is gone. The swing is exactly `2.885390 * F = 6.214071` octaves, which is to say: **what P loses at the end is precisely the whole of the surplus P was exporting the cost of.**

28.15 The arithmetic test for Book One's eleventh node

Book One's eleventh node is a vow, and vows are the part of a practice that this series has been most suspicious of, because a vow is unfalsifiable in exactly the way a testimonial is: it reports an intention and the intention is always available. Theorem 28.5 converts it into something else.

> **THE EXPORT TEST. Compute the deficit and the fixed point for every participant separately. Then compute the sum of drives. If a participant's reading improved and the sum of drives did not move, the gain was exported and nothing was produced.**

Three properties make this a test rather than an exhortation.

It is arithmetic, not attribution. It does not ask what anybody meant. It asks whether a conserved quantity moved. A well-intentioned export and a deliberate one produce the same reading, which is correct, because the harm is the same.

It runs on pass one. The stock-based version of the same question — "is anyone worse off?" — returns a false negative for nearly eleven passes in the worked case. The drive-based version returns the right answer immediately, because the drive is where the transfer happened.

It costs one column. The reporting format of §28.19 adds the sum of drives to what a reading already contains. Nothing new has to be measured; the parameters were already collected for

each participant's own reading.

What the test does not do is also worth stating. It detects export of *cost*, which is the operation the formalism represents. It does not detect every way one party can benefit at another's expense, because the formalism does not represent every such way. A participant who degrades another's `u` by suppressing what they hold, or who lowers another's `eta` by refusing to let their work become permanent, is doing something the model can express only as a change in somebody's parameters, and the conservation identity does not cover it — those changes are not transfers and the sum of drives does move. The test is exact for what it covers and silent outside it, and a reader who takes it for a general detector of exploitation will be misled by their own instrument.

28.16 The sensitivity of a destiny

A destiny is a quotient, and quotients with small denominators are sensitive. The partials, at the carried dyad as found:

```
dQ_inf / d eta    =  F / delta              =   +6.256498  per unit of eta
dQ_inf / d F     =  eta / delta           =   +3.333333  per nat of surplus
dQ_inf / d c      =  −1 / delta             =  −11.111111  per unit of cost
dQ_inf / d delta  =  −(eta F − c) / delta^2  =  +55.688217  per unit of delta

and the chain into the geometry:
dF / du           =  (N−1) / (1 + (N−1)u)     =   +6.263947
dF / dN           =  u / (1 + (N−1)u)         =   +0.039141
```

The fourth line is nearly nine times the third and nearly nine times the first. **The parameter with the largest partial derivative on the destiny is the one with no partial derivative at all on the deficit,** which is Theorems 28.3 and 28.4 stated as a pair of numbers rather than as an algebraic fact, and it is the reason both readings have to be published.

The partials are also in four different units and Chapter 4 §4.17b forbids ranking them against one another on size. What they are good for is error propagation: a destiny reported without an interval, when `delta` is known to `± 0.02`, is reporting a number whose uncertainty at this operating point is of order `± 1.1` units. Chapter 15's rule applies here unchanged — **a verdict is resolved only when it exceeds its own uncertainty** — and §28.20's gates enforce it.

The Specification

28.17 PROCEDURE DEST — reading a destiny

Step 1. Declare the arrangement. Name the participants, the cycle length, and what one pass is. A destiny is per relation, per site, per period; ratios do not average and readings do not aggregate (Chapter 4, SPEC 4.14).

Step 2. Check the scale precondition. Were the intensities elicited by magnitude estimation (Chapter 6's rule, SPINE §1.1)? If not, stop. The relationship frequency is not determined, and a destiny computed from an undetermined `F` is an artefact of the labelling. Publish the raw L2 norms so the free multiplier stays recoverable.

Step 3. Check for one-hot degeneracy. If the participants' state vectors have disjoint support, `u = 1` automatically and the surplus is meaningless (Chapter 9's trap). Re-elicit with overlapping support or stop.

Step 4. Compute u, then F, then f_rel. Use the single construction of SPINE §2: `u = 1 − (x.y)^2`, `F = ln(1 + 11u)`, `f_rel = k(1+11u)^2`. Carry twelve or more decimal places throughout and recompute derived values from the full-precision u, never from a rounded print (SPINE §3.4).

Step 5. Run the ceiling test before anything else. If `c/eta > ln N`, no value of u makes the arrangement sustainable and the case is structural, not tunable. Report it as structural and go to Chapter 29's restructuring branch.

Step 6. Compute the deficit. `D = 2.885390 (c/eta − F)`. Report its sign, its size, and its sensitivity to each input.

Step 7. Compute the destiny. `Q_inf = (eta F − c)/delta`. Report `delta` in the same sentence.

Step 8. Compute the approach. `tau`, half-life, and `t(1%)` from `delta` alone. Say how many passes of the declared cycle length that is in calendar time.

Step 9. Run the export test. For every participant separately, compute `D_i` and `Q_inf,i`. Compute `sum_i b_i`. Compare to the previous reading's sum of drives if one exists.

Step 10. Publish both readings and both refusals. The state frequencies and the relationship frequency, separately, with no inference between them; the essence clause; the no-clinical-claim clause; and the constructed label on every case.

28.18 Definitions and theorems owned here

```
 destiny = Q_inf                              Def 28.1     "Definition 28.1"
 existence, uniqueness, start-independence  Thm 28.1     "Theorem 28.1"
the convergence equality             Thm 28.2   "Theorem 28.2 (= Ch 22 Thm 22.1)"
the deficit identity                Thm 28.3   "Theorem 28.3 (= Ch 4 Thm 4.5)"
delta is absent from D              Thm 28.4   "Theorem 28.4 (= Ch 4 Thm 4.9)"
 conservation of drive under cost transfer  Thm 28.5     "Theorem 28.5"
 equal-delta sum invariance                 Cor 28.6     "Corollary 28.6"
 counterfeit arrival, defined              Def 28.2     "Definition 28.2"
the export test                       Sec 28.15   "the export test (Ch 28)"
```

DEFINITION 28.2 — COUNTERFEIT ARRIVAL. A fixed point is counterfeit when it is positive in one participant's accounts and the sum of drives across the arrangement did not move to produce it.

Not owned here and to be taken from their owners: the differentiation index and its count argument (Chapter 5); the elicitation procedure (Chapter 6); `f_rel = k(1+11u)^2` (Chapter 12); the R15 order-of-operations rule (Chapter 15); the four levers and how to pull them (Chapters 4 and 18, and Chapter 29 for what each one costs).

28.19 Reporting format

Every published destiny reading carries all of the following, in full:

```
    ARRANGEMENT      participants, cycle length, what one pass is
   SCALE STATUS    magnitude-estimated? raw L2 norms? -- if no, NO NUMBER IS PRINTED
    GEOMETRY         u to 12+ dp, cos theta, support overlap
   STATE READING   the occupied indices, their raw frequencies, their pitch classes
      RELATION  READING  F  in  nats,  N_eff,  f_rel  in  Hz                --
separately, no inference drawn
    CEILING         c/eta against ln N, and the verdict structural / tunable
    DEFICIT          D in octaves, with its sensitivity to each input
    LOSS             delta, stated in the same block as D and never omitted
    DESTINY          Q_inf, with its uncertainty
    APPROACH         tau, half-life, t(1%), converted to calendar time
    EXPORT           per-participant D_i and Q_inf,i, and the sum of drives
    LABEL            CONSTRUCTED or MEASURED, on every case
   REFUSALS      essence-not-resonance; no clinical claim; consent structural
```

28.20 Gates

```
   GATE 28-1  intensities magnitude-estimated and L2 norms published   FAIL -
> no number
   GATE 28-2  support overlap checked for one-hot degeneracy          FAIL -
> re-elicit
   GATE 28-3  ceiling test run before any lever is discussed          FAIL -
> incomplete
   GATE 28-4  D and delta reported together                           FAIL -
> incomplete
   GATE 28-5  Q_inf reported with an uncertainty                      FAIL -
> incomplete
   GATE 28-6  state and relationship readings reported separately     FAIL -
> incomplete
   GATE 28-7  sum of drives computed across all participants          FAIL -
> export unchecked
   GATE 28-8  every case labelled CONSTRUCTED or MEASURED             FAIL -
> withdraw
   GATE 28-9  no single-party term appears anywhere in the report     FAIL -
> rewrite
```

The Failure Modes

FM 28.1 — DESTINY READ AS PROPHECY. *Signature:* a fixed point reported as what will happen. *Mechanism:* the conditional clause dropped. `Q_inf` is where the loop goes if the parameters hold, and parameters are properties of arrangements that people change. *Fix:* the conditional is part of the number. Print it.

FM 28.2 — DESTINY READ AS A PROPERTY OF A PERSON. *Signature:* "my destiny is −5.01." *Mechanism:* u is a joint geometric property and eta, c, delta describe an arrangement. There is no single-party term in the apparatus. *Fix:* Gate 28-9. State the arrangement, or print nothing.

FM 28.3 — THE DEFICIT REPORTED ALONE. *Signature:* a reading consisting of D and a verdict. *Mechanism:* Theorem 28.4. D cannot see decay at all; the same D is consistent with fixed points differing by a factor of thirty. *Fix:* Gate 28-4.

FM 28.4 — delta REPORTED ALONE. *Signature:* a fast half-life presented as good news. *Mechanism:* the rate cannot see the destination's sign. A fast approach to a negative fixed point is a fast decline. *Fix:* Gate 28-4, the other way round.

FM 28.5 — TRUST AS A DESTINY LEVER IN A FAILING ARRANGEMENT. *Signature:* "we will lower delta and our destiny will improve." *Mechanism:* R15. delta divides the drive; when the drive is negative a smaller delta makes the fixed point more negative. Computed: −5.0119395464 at delta = 0.09 becomes −45.1074559 at delta = 0.01. *Fix:* Chapter 15's order rule, and Chapter 29 §29.5.

FM 28.6 — THE STOCK MISTAKEN FOR THE DESTINY. *Signature:* "we are at +1.07, so we are fine." *Mechanism:* the stock lags the destination by up to t∗ passes; in the worked export case t∗ = 10.850479. *Fix:* compute D_i, not Q_t.

FM 28.7 — COUNTERFEIT ARRIVAL. *Signature:* one participant's readings improve steadily and genuinely, pass after pass. *Mechanism:* cost export; the sum of drives did not move. *Fix:* Gate 28-7, the export test.

FM 28.8 — THE EXPORT TEST RUN ON THE PLAIN SUM. *Signature:* the sum of fixed points is watched, the participants have different delta, and the test passes an export. *Mechanism:* §28.13. d(sum)/dx = 1/delta_P − 1/delta_R, which is zero only when the rates match; in the constructed case the plain sum rises by 2.67 units while nothing is produced. *Fix:* watch the sum of drives, which is invariant unconditionally.

FM 28.9 — THE EXPORT TEST TREATED AS A GENERAL DETECTOR. *Signature:* an arrangement declared free of exploitation because the sum of drives held. *Mechanism:* §28.15. The identity covers transfers of cost. Degrading another party's u, eta or delta is not a transfer, and the sum moves. *Fix:* state the test's scope in the same sentence as its result.

FM 28.10 — THE DEEP FIXED POINT WANTED WITHOUT THE SLOW APPROACH. *Signature:* a plan for a durable arrangement that also arrives quickly. *Mechanism:* §28.11. Q_inf scales as 1/delta and the half-life scales as −ln 2 / ln(1−delta); one parameter sets both, in opposite directions. *Fix:* choose. There is no configuration with both.

FM 28.11 — delta = 0. *Signature:* a model of something incorruptible, unchanging, permanent. *Mechanism:* at delta = 0 the map is an isometry, Banach does not apply, there is no fixed point, and Q_t = Q_0 + bt grows without bound. The image of permanence is the one setting under which the apparatus stops working. *Fix:* name the leak. Chapter 8's low-delta trait is the honest version.

FM 28.12 — INFERRING DESTINY FROM STATE FREQUENCY. *Signature:* "their states are high, so their destiny is high." *Mechanism:* SPINE §3.6; six constructed pairs spanning 1565 to 5208 cents return identical u and identical f_rel to twelve decimal places. *Fix:* Gate 28-6. Two readings, reported separately, no inference between them.

FM 28.13 — THE PRECISION LEAK. *Signature:* a destiny that disagrees with a colleague's in the fourth significant figure. *Mechanism:* SPINE §3.4; recomputing from a six-decimal u shifts `f` and `Q_inf` in the fifth significant figure. *Fix:* recompute from the full-precision u. Every time.

FM 28.14 — THE READING USED AS COUNSEL. *Signature:* a destiny presented as advice about whether to stay in a relationship, or as an evaluation of anyone. *Mechanism:* register slip. `Q_inf < 0` says a configuration loses stored order at a computable rate. It does not weigh what the relation is for, what else it produces, or what ending it would cost. *Fix:* §28.8. This is not counselling, therapy or clinical assessment, and the book says so here.

Worked Example

28.21 The carried dyad, both phases, as full destiny reports

CONSTRUCTED. Not an observation of any person or pair. SPINE §3.2.

```
   ARRANGEMENT    two parties A and B; one pass = one week; the pass is their standing reviev
      SCALE  STATUS        constructed  intensities,  treated  as  magnitude-
estimated for the worked case
    RAW L2 NORMS     |A| = 0.948683298051    |B| = 0.890224690738

    ---- PHASE 2a, AS FOUND ----------------------------------------------
-----
    GEOMETRY         cos theta = 0.965020898600      u = 0.068734665264634
                  support overlap = 3 of 4 occupied indices -- not degenerate
    STATE READING    A holds {4, 6, 9}; B holds {4, 6, 9, 10}
                     raw 7.7348 / 17.4033 / 39.1575 / 48.3426 Hz
                     pitch classes {1, 3, 9, 5}
   RELATION READING F = 0.563084802746 nats  N_eff = 1.756081  f_rel = 1.4907987150 Hz
    CEILING        c/eta = 2.066667 vs ln N = 2.484907  ->  TUNABLE in principle
     DEFICIT         D = +4.3384201973 octaves       f_min = 30.1587698405 Hz
     LOSS            delta = 0.09
     DESTINY         Q_inf = -5.0119395464            DECAYS
    APPROACH       tau = 10.603253 passes; half-life = 7.349615; t(1%) = 48.8298
                   at one pass per week: half-life about seven and a half weeks
     LABEL           CONSTRUCTED
```

The pair agree about almost everything — `cos theta = 0.965` — and that is the whole of their problem. The relationship frequency sits at 1.49 Hz against a required 30.16 Hz, which is a gap of 4.34 octaves, and the destiny is negative.

The intuitive repair is to add trust, which lowers `delta`. Computed:

```
   delta 0.09 -> 0.05    Q_inf  -5.0119395464 -> -9.0214911835     WORSE by 4.0095516
   delta 0.09 -> 0.01    Q_inf  -5.0119395464 -> -45.1074559      WORSE by 40.0955164
    D at every one of those values               +4.3384201973       UNCHANGED
```

The deficit is flat across all three. A pair reading only their deficit would conclude that a successful, expensive, well-intentioned intervention had done nothing; a pair reading only their stock would

watch it fall and not know why. **A pair reading both would see immediately that they had multiplied a negative number by a larger factor and touched no mechanism.** That is the case for Gate 28-4 in one paragraph.

```
    ---- PHASE 2b, AFTER ONE STRUCTURAL CHANGE ----------------------------
----
   THE CHANGE     B takes on two states A does not hold; the pair adopt a decision log
    GEOMETRY          u = 0.692372881355932
   RELATION READING F = 2.153632742815 nats  N_eff = 8.616102  f_rel = 35.8881768941 Hz
    CEILING        c/eta = 0.909091 vs ln N = 2.484907  ->  TUNABLE, with margin
     DEFICIT          D = -3.5909886634 octaves        f_min = 2.9782153324 Hz
     LOSS             delta = 0.09
     DESTINY          Q_inf = +6.0844267427             ACCUMULATES
    APPROACH        half-life 7.349615 passes; inside 5% of Q_inf from pass 32;
                     inside 0.1% from pass 74
     LABEL            CONSTRUCTED
```

And now the same trust intervention, applied second instead of first:

```
   delta 0.09 -> 0.05    Q_inf  +6.0844267427 -> +10.9519681368    BETTER by 4.8675414
    D unchanged at          -3.5909886634
```

The symmetry is exact and worth pausing on. Lowering `delta` from 0.09 to 0.05 moved the unrepaired dyad's destiny by −4.0095516 and the repaired dyad's by +4.8675414. **The same intervention, the same magnitude of change in the same parameter, and the sign of the result was decided entirely by the sign of the drive it was multiplying.** Chapter 29 owns the operating rule that follows; this chapter's contribution is the reason it is true, which is that `delta` is a denominator and denominators do not have opinions.

28.22 The export case, in report format

CONSTRUCTED. Not an observation of any person or pair. The two-participant arrangement of §28.12, read at x = 0.15.

```
    ARRANGEMENT     P and R share the case-2b arrangement; one pass = one week
   SHARED GEOMETRY  u = 0.692372881355932  F = 2.153632742815  f_rel = 35.8881768941 Hz

    PARTICIPANT P    eta = 0.44   c = 0.250   delta = 0.09
                     D_P     = -4.574644 octaves
                     Q_inf,P = +7.751093
                     drive   = +0.697598

    PARTICIPANT R    eta = 0.30   c = 0.700   delta = 0.09
                     D_R     = +0.518506 octaves
                     Q_inf,R = -0.599002
                     drive   = -0.053910

    SUM OF DRIVES    +0.643688229683
    PREVIOUS SUM     +0.643688229683          ->  UNCHANGED
   EXPORT TEST    P improved by 1.666666 units of fixed point; the sum did not move.
```

```
                    VERDICT: the gain was exported. Nothing was produced.

  R's crossing    exact at c_R = eta_R F = 0.646089822845, i.e. x = 0.096089822845
   R's stock lag    t* = 10.850479 passes before Q_R goes negative
   LABEL            CONSTRUCTED
```

Read the block as a whole and notice what it does that a per-party reading cannot. P's three lines are the lines of a well-run arrangement improving steadily. R's three lines are the lines of an arrangement that has just crossed into decline. The sum line is the only place in the report where the relation between those two facts appears, and it appears as a quantity that did not change.

The terminal arithmetic completes the case. If R declines to continue — R's structural right, and no arrangement here may remove it — the shared surplus goes to zero, `f` falls to the ground essence `k = 0.483426 Hz`, and P's reading becomes `D_P = +1.639426`, `Q_inf,P = −2.777778`. P's fixed point falls by `10.528871` units and P's deficit swings `6.214071` octaves the wrong way, which is `2.885390 * F` exactly. **The exporter's terminal loss is the full value of the surplus whose maintenance was exported, and the model computes it without needing any account of anybody's feelings about it.**

28.23 What this chapter establishes, and what Chapter 29 takes

Established, and checkable. That the storage loop has exactly one fixed point for `delta > 0` and reaches it from every start (Theorem 28.1); that the approach is a geometric equality rather than a bound, verified to 2.06e-15 across a hundred passes (Theorem 28.2); that the octave deficit and the fixed point are one quantity in two units, verified to 8.88e-16 across twelve parameter sets (Theorem 28.3); that the deficit contains no `delta`, so the same `D` is consistent with destinies differing by a factor of thirty (Theorem 28.4); that the sum of drives is exactly conserved under any transfer of cost among any number of participants (Theorem 28.5), and that the plain sum of fixed points inherits that invariance only when the participants share a `delta` (Corollary 28.6 and §28.13); that a counterfeit fixed point is therefore detectable on pass one by an arithmetic test costing one column of a report; and that a state's frequency and a relationship's frequency remain independent readings at destiny scope.

Not established, and not claimed. That any arrangement's parameters will hold; that a larger fixed point is a better life; that any reading here describes any actual person or pair; that the formalism detects every way one party can gain at another's expense; that any frequency in this chapter is a physical resonance; or that any of this constitutes counselling, therapy, diagnosis or clinical assessment.

The one line to carry forward: *a destiny is a number with four arguments, and everything that can be done about one is done to those four.*

Chapter 29 takes the four arguments as levers, prices each of them, gives each its frequency reading and its failure mode, and states the two rules that govern the order in which they are pulled. It also applies the belief postulate of Book Two Chapters 76 and 77 in full, because the question "what actually moves a destiny" has a large and confident literature attached to it, and the postulate is what lets this book say precisely which parts of that literature have a channel and which have none. Chapter 4 owns `f_min`, the deficit and the four levers; Chapter 15 owns the

order-of-operations rule and the lever reserves; Chapter 22 owns the refinement ladder and the approach; Chapter 18 owns the consent axiom and the intervention procedure.

Chapter 29 . Changing the Fixed Point

Chapter 28 defined destiny as `Q_inf`, the fixed point of the contracting loop, and spent its length on what the object is: unique, reached from any start, indifferent to history, negative whenever conversion fails to cover cost, and detectable as counterfeit by an arithmetic test on a conserved sum. Everything in that chapter was a statement about a number that was holding still.

This chapter moves it. And the first thing to say is how few ways there are to do that.

```
Q     =  ( eta * F(u*)  -  c* ) / delta
 inf
```

Four symbols on the right. **Changing a destiny means changing `(eta, F, c, delta)` and nothing else** — not because the book disapproves of other approaches, but because the expression has four arguments and a function of four arguments changes when one of them changes and at no other time. That is CANON3 §4, it is a definition rather than a promise, and its whole value is that it makes the question answerable: *which one are you moving, and by how much?*

A proposal that cannot be assigned to one of the four is not an intervention on the destiny. It may be many other worthwhile things. It is not that.

The chapter takes the four in turn, and for each one gives three things the practical literature almost never gives together: the frequency reading that says whether the lever is the binding one, the cost of moving it, and the specific way plans built on it fail. Two results govern the whole exercise and both are inherited. The first is Chapter 15's reserve theorem, which says a lever can be **absent** rather than merely expensive — and it is demonstrated on a lever with nearly five times the elasticity of its rival. The second is R15's order rule, which says fix the numerator before lowering the loss, and which comes with a price for disobedience computed to six decimal places.

Then the chapter does what a chapter with this title has to do. There is a large, confident and commercially successful literature claiming that a destiny can be changed by believing in a new one. Book Two Chapters 76 and 77 built the apparatus for adjudicating exactly that claim, and it is applied here in full, with its conclusion in both directions: where a channel with real gain exists, belief moves the world by an amount the channel's own mechanism determines, and where no channel exists, nothing happens at all. Book Two's R23 supplies the companion number, and it is the harshest number in the corpus.

The Law

29.1 There are four arguments, and there is no fifth

THEOREM 29.1 — THE COMPLETENESS OF THE LEVER SET. `Q_inf` is a function of exactly `(eta, F, c, delta)`. Therefore any change to `Q_inf` is a change to at least one of those four, and any proposal that changes none of them changes `Q_inf` by zero.

Proof. Inspection of `Q_inf = (eta F − c)/delta`. There are no other free symbols. []

The proof is trivial and the consequence is not, because it is a closure statement: it says the list is complete, and a complete list is what turns a diagnostic into a procedure. Chapter 18 §18.1 states the same closure for interventions on the deficit and reaches a list of five handles rather than four, and the two statements have to be reconciled explicitly or a reader will think one of them is wrong.

They are both right, and the difference is one level of nesting:

```
 THE FOUR ARGUMENTS OF Q_inf          THE FIVE HANDLES ON AN ARRANGEMENT
----------------------------        ------------------------------------
 eta   conversion                      L2  raise eta
 F     surplus                        L1  raise u       }  the two routes
                                      L4  raise N       }  into F
 c     cost                           L3  cut c
 delta loss                           L0  lower delta
```

F is not directly manipulable. It is `ln(1 + (N−1)u)`, and it moves only when the geometry u moves or the category count N moves. So the four arguments give five handles, and Chapter 4's list and CANON3's list are the same list read at two depths. The reconciliation matters because the two sub-levers into F behave completely differently — one has the largest reserve in the apparatus and the other has almost none — and a plan that says "raise F" without saying which route has not yet said anything.

29.2 The partials, and why they may not be ranked

At the carried dyad as found — SPINE §3.2 case 2a, a constructed case, with `u = 0.068734665264634`, `eta = 0.30`, `c = 0.62`, `delta = 0.09`, `Q_inf = −5.0119395464`:

```
 dQ_inf / d eta    =  F / delta             =   +6.256498   per unit of eta
dQ_inf / d F     =  eta / delta          =   +3.333333   per nat of surplus
 dQ_inf / d c     =  −1 / delta            =  −11.111111   per unit of cost
dQ_inf / d delta  =  −(eta F − c)/delta^2   =  +55.688217   per unit of delta

 the two routes into F:
 dF / du          =  (N−1)/(1 + (N−1)u)      =   +6.263947   per unit of u
dF / dN         =  u /(1 + (N−1)u)       =   +0.039141   per added category
```

Chapter 4 §4.17b forbids ranking these against one another on size, and the prohibition is not pedantic. They are in four different units. `eta` is a dimensionless efficiency in `[0, 1]`; `c` is a cost in the same units as `eta F`; `delta` is a rate in `(0, 1]`; F is in nats. The statement "`delta` has

the largest derivative" is true and means nothing, because a unit of `delta` and a unit of `c` are not comparable quantities and no plan can spend one to buy the other.

What is legitimate is Chapter 15's elasticity comparison, which divides the units out. That is §29.9. What is also legitimate is the reserve comparison, which asks how far each lever can go before it runs out, and that is §29.10. The two answer different questions and the most expensive planning error in this chapter is confusing them.

29.3 The ceiling test comes before every lever

Before any of this is worth computing there is a test that can rule the entire exercise out, and it belongs at the top of the procedure because a plan built after skipping it is a plan for a case that has no solution.

THE CEILING (Chapter 4 Theorem 4.6; Paper E Corollary 3.4 in the frequency register). Since `F <= ln N`, sustainability requires `c/eta <= ln N`. Equivalently `f_min <= k N^2`. At `N = 12`, `c/eta` may not exceed `2.484907`, i.e. `f_min` may not exceed `69.6133055503 Hz`.

```
    c/eta > ln N   ->   NO value of u makes the arrangement sustainable.
                      Not a better pairing, not a better schedule, nothing.
                The only remedies: raise eta, cut c, or add genuinely distinct categories.
```

Run on the four arrangements this book carries, plus one constructed case built to fail it:

```
    arrangement                  c/eta      ln N       verdict
    ------------------------------------------------------------------------
    carried dyad 2a           2.066667   2.484907    tunable in principle
    carried dyad 2b           0.909091   2.484907    tunable, with margin
   the commons             2.426471   2.484907    tunable in principle, barely
    constructed family        1.400000   2.484907    tunable
  CONSTRUCTED over-ceiling  3.000000   2.484907   STRUCTURAL -- no u closes it
```

The last row is worth working, because a book that only showed tunable cases would be lying by selection. **CONSTRUCTED, labelled constructed.** An arrangement at `eta = 0.20` and `c = 0.60` sits at `c/eta = 3.000000`. Push its geometry to perfect orthogonality — `u = 1`, the theoretical maximum, two parties with no overlapping state whatever:

```
    at u = 1, N = 12:   F = ln 12 = 2.484907   D = 2.885390 * (3.000000 -
2.484907) = +1.486245
```

Still failing, by nearly a octave and a half, with the geometry lever fully spent. The only remaining route through F is N, and the requirement is exact: `N >= e^3 = 20.0855`, so twenty-one categories.

```
    at u = 1, N = 21:   F = ln 21 = 3.044522   D = 2.885390 * (3.000000 -
3.044522) = -0.128465
```

Sustainable, by an eighth of an octave, and only at perfect orthogonality in a twenty-one-category description. That is what "restructure rather than tune" looks like when it is priced. The commons of SPINE §3.3 is the same lesson at a milder setting: it passes the ceiling test by `0.058436` and is still short by +6.463837 octaves, and Chapter 18 records that its tuning procedure halts.

29.4 `delta` is a multiplier on a sign, not a lever on a gap

This is R15, and it is the correction to naive practice that this series has repeated more often than any other result, because it is the one every reader's intuition gets backwards.

THEOREM 29.2 (= Ch 4 Thm 4.9, restated for `Q_inf`). `delta` appears in `Q_inf` only as a denominator. Therefore `sign(Q_inf) = sign(eta F − c)` for every `delta` in `(0, 1]`, and reducing `delta` scales `|Q_inf|` upward without ever changing its sign. []

Computed on three drives, one negative, one positive, one exactly zero:

```
    drive b              delta 0.30    delta 0.09    delta 0.05    delta 0.01
     ------------------------------------------------------------------------
-----
   2a as found  −0.451075   −1.503582    −5.011940    −9.021491    −45.107456
   2b repaired  +0.547598   +1.825328    +6.084427   +10.951968    +54.759841
   exactly at hold   0.000   +0.000000    +0.000000    +0.000000    +0.000000
```

Three readings of that table. In the top row, the intervention that everyone reaches for first — build trust, add continuity, institute the ritual, keep the record — takes the destiny from −5.011940 to −45.107456, **a ninefold deterioration, while every party involved is doing something admirable and nothing is going visibly wrong.** In the bottom row the same intervention does the same thing with the sign reversed, and it is the single largest available improvement. In the middle row it does nothing at all, at any magnitude, because trust multiplies an existing sign and zero has none.

And the deficit is flat across the entire table. `D = +4.338420` in the top row at every `delta`; `D = −3.590989` in the second at every `delta`. A pair reading only their deficit through that whole sequence would see a completely unchanging instrument. That is Chapter 28 Theorem 28.4, and it is why this chapter's reporting format prints `delta` in the same block as `D`, every time.

29.5 The reserve theorem: a lever can be absent rather than expensive

Chapter 15 owns this result and it is the most counter-intuitive thing in Part III.

THEOREM 29.3 (= Ch 15 Thm 15.4) — THE LEVER RESERVES. The maximum number of octaves each lever can supply, alone, from an operating point `(u_0, eta_0, c_0, N)`:

```
     R_u     = 2.885390 ∗ ( ln N − F(u_0) )                        ( u −
> 1 )       the pitch headroom
    R_eta  = 2.885390 ∗ c_0 ∗ ( 1/eta_0 − 1/eta_max )      ( eta −> eta_max )
    R_c    = 2.885390 ∗ ( c_0 / eta_0 )                     ( c −> 0 )
   R_N   unbounded in principle; 2.885390 ∗ u/(1+(N−1)u) per category in practice
```

Computed on the carried dyad as found, against its requirement of +4.338420 octaves:

```
    R_u    ( u −> 1 )        5.545206 octaves       CLOSES the deficit
    R_eta  ( eta −> 0.40 )   1.490785 octaves       does not
    R_eta  ( eta −> 0.50 )   2.385256 octaves       does not
    R_eta  ( eta −> 0.75 )   3.577884 octaves       does not
      R_eta   ( eta −> 1.00 )    4.174198 octaves          does not −−
and eta_max = 1 is generous
```

```
R_c    ( c -> 0 )        5.963140 octaves       CLOSES the deficit
R_N    per added category at u = 0.068735 :  0.112937 octaves
```

Now set that beside the elasticities, which measure how fast each lever moves the deficit per *proportional* change and are therefore dimensionless and comparable:

```
point         c/eta      1 − 1/N_eff   |dD/d ln eta|  |dD/d ln u|   ratio
 ------------------------------------------------------------------------
----
dyad 2a      2.066667     0.430550      5.963140     1.242306    4.8001
dyad 2b      0.909091     0.883938      2.623082     2.550507    1.0285
the commons  2.426471     0.169955      7.001314     0.490387   14.2771
```

At the carried dyad as found, `eta` has 4.80 times the elasticity of `u` — and 4.174198 octaves of reserve against a requirement of 4.338420, falling short by 0.164222. The lever that moves the deficit nearly five times faster per proportional unit cannot close it at all, even taken to `eta = 1`, which would mean an arrangement converting every unit of interaction into permanent structure with nothing lost anywhere.

Chapter 4 §4.24 records the same fact in its other form. The `eta` required to close the deficit is `c/F = 0.62/0.563085 = 1.101077`. A conversion efficiency above one is not an expensive lever; it is an absent one, and no amount of effort, budget or commitment reaches it, because it would mean creating more structure than the interaction supplied.

> **THE HONEST DYADIC LAW (Ch 15). The cost-side levers have the higher elasticity and the lower ceiling; the geometry lever has the lower elasticity and the only large reserve. Small deficits are closed on the cost side. Large ones require difference.**

The reserve table also tells you where the cost-side lever is worth having. `R_eta` is large only where `eta` is very low — where the arrangement is converting almost nothing and there is a great deal of headroom in the mechanism:

```
eta_0    c        R_eta (octaves, at eta_max = 1)
-----------------------------------------------
0.20     0.400          4.6166
0.20     0.825          9.5218
0.30     0.620          4.1742
0.40     0.620          2.6834
0.50     0.400          1.1542
0.50     0.825          2.3804
```

An arrangement already converting half of what it does has under two and a half octaves available on the `eta` side no matter how expensive it is. The cost-side lever is the cheap one, the first one, and the one that is spent quickly.

29.6 The order rule, and its price

R15's design rule is one line: **raise `eta` or cut `c` until `eta F > c`, and only then invest in `delta`.** Chapter 15 owns it. What that chapter also owns, and what belongs here because this is the chapter about changing things, is the price of doing it backwards.

CONSTRUCTED CASE, labelled constructed. The carried dyad, starting stock `Q_0 = 1.0`, eight periods of the wrong order first. Path A lowers `delta` from 0.09 to 0.05 and runs eight periods before fixing the numerator. Path B fixes the numerator first, runs eight periods at the original `delta`, and then lowers it. Both end at the identical parameters and therefore, by Chapter 28 Theorem 28.1, at the identical fixed point.

```
 t |   Path A   |   Path B   |    gap
---+------------+------------+-----------
 0 |  +1.000000 |  +1.000000 |  +0.000000
 1 |  +0.498925 |  +1.457598 |  +0.958673
 3 |  -0.429315 |  +2.252950 |  +2.682265
 5 |  -1.267052 |  +2.911581 |  +4.178633
 8 |  -2.373029 |  +3.693462 |  +6.066491
10 |  -1.073842 |  +4.401167 |  +5.475008
13 |  +0.641339 |  +5.335475 |  +4.694135
16 |  +2.111893 |  +6.136527 |  +4.024634
20 |  +3.751671 |  +7.029761 |  +3.278090
24 |  +5.087281 |  +7.757306 |  +2.670025
32 |  +7.061215 |  +8.832564 |  +1.771349

 terminal fixed point, both paths          +10.951968
minimum stock reached, wrong order      -2.373029  (starting value +1.000000)
 periods spent below the starting value     13   (t = 1 through t = 13)
 permanent lag                               12 periods
 cumulative gap over 32 periods            111.133601 stock-periods
```

The lag is permanent and it is exactly twelve periods at every level: Path B reaches +3.693462 at `t = 8` and Path A first reaches it at `t = 20`; Path B reaches +5.039870 at `t = 12` and Path A at `t = 24`; +6.136527 at 16 and 28; +7.029761 at 20 and 32. **Eight periods of misordering cost twelve periods of permanent delay and 111.133601 stock-periods of cumulative loss, from doing exactly the same two things in the opposite order.**

The excess over eight is not a rounding artefact. While the numerator is negative, the lowered `delta` is *accelerating* the loss, so those eight periods do more damage than eight periods at the original `delta` would have done.

Two things about that table deserve to be read carefully. The first is that both paths converge to the same number, which means a planner who evaluates only the destination sees no difference between them whatever. The destination is order-independent; the trajectory is not; and the formalism will happily compute a path through negative stored order that a real arrangement may not survive. Chapter 4's FM 4.9 records that admissibility fails there, and thirteen periods below the starting stock is thirteen periods in which people make decisions.

The second is what the instrument said the whole time. `D` reads +4.338420 at the start of Path A and +4.338420 at the end of Path A's first eight periods. Whatever the pair were using, if it was the deficit alone, it reported that nothing had happened during the most damaging phase of the plan.

29.7 Honesty clauses, restated where they bind

Essence, not resonance. Every hertz value in this chapter is a position on a logarithmic scale of surplus. Nothing here is claimed to be a physical vibration and nothing here is to be played at anyone.

No clinical, diagnostic or therapeutic claim. A lever prescription is a statement about four declared parameters of an arrangement. It is not treatment, not counselling, and not an assessment of anybody.

Consent is structural (Chapter 18, Axiom 18.1). No intervention derived from any reading in this chapter may reduce any party's ability to decline it, at any stage, for any reason or none. The geometry lever in particular describes changes to what people hold and do, and it is worth saying plainly: an intervention that raises u by arranging for someone else to change is not an intervention this book licenses.

No subliminals. Nothing masked, nothing below threshold, nothing concealed. Open-label placebo works (Book Two Chapter 78), so disclosure costs nothing that matters.

All cases here are CONSTRUCTED and labelled as such.

The Engineering

29.8 Lever L1 — raise u, the geometry

The reading that says it binds. `R_u = 2.885390 (ln N − F(u_0))` exceeds the deficit while the cost-side reserves do not; equivalently, `1 − 1/N_eff` is small, meaning the arrangement is near parallel and has a great deal of unused pitch headroom. At the carried dyad as found, `1 − 1/N_eff = 0.430550` and `R_u = 5.545206` against a requirement of `4.338420`.

What it costs. The required u is exact: `u >= (e^(c/eta) − 1)/(N−1)`. For the carried dyad, `u >= 0.627132` from `0.068735`, a rise by a factor of `9.1240`. That is a large amount of real difference between two people currently at `cos theta = 0.965`.

What raising u actually is, and R8's trap. Chapter 14 owns the unison result and it is the single most important thing to carry into any attempt at this lever:

```
u = 1 − (x . y)^2   depends only on |rho|

perfect agreement     rho = +1   ->   u = 0
perfect opposition    rho = −1   ->   u = 0
```

Manufactured disagreement raises cost and returns exactly nothing. A party who adopts contrary positions in order to bring difference moves the pair toward `rho = −1`, which reads `u = 0` just as agreement does, while adding coordination overhead: the reading gets worse in both terms at once. The contrarian and the sycophant sound the identical note, and it is the ground essence `k = 0.483426 Hz`.

What does raise u is a change in what a party actually holds — states occupied that the other does not occupy, commitments taken on, work done in a domain the other is not in. **The test of a u intervention is whether it would still be true if nobody were reading the number.**

Failure mode. The representational one, which is Chapter 9's trap: two people described by disjoint sets of states are automatically orthogonal and read u = 1 regardless of content. A u raised by re-describing rather than by changing what is held is an artefact, and Gate 29-2 exists to catch it.

29.9 Lever L4 — raise N, the categories

The reading that says it binds. Only one: c/eta > ln N. Then the ceiling is the binding constraint, u is unavailable at every value, and adding categories is the only route into F that has anywhere left to go.

What it costs. Per added category, at fixed u, the gain is 2.885390 ∗ u/(1+(N−1)u) octaves. At the carried dyad that is 0.112937 octaves each; at the commons, where u = 0.018614, it is 0.044581 octaves each. **Adding categories to a near-parallel set buys almost nothing.** Computed, raising N alone on the carried dyad with u held at 0.068735:

N	F	D	
12	0.563085	+4.338420	
16	0.708538	+3.918731	
20	0.835496	+3.552406	
25	0.974421	+3.151556	
40	1.303090	+2.203217	
60	1.620446	+1.287520	
100	2.054730	+0.034441	
200	2.686363	−1.788066	OK

Two hundred categories to close a deficit that one geometry change closes. **The lever is unbounded in principle and nearly useless in practice at low u, which is exactly the regime in which a planner reaches for it.**

Failure mode. Index inflation, which is Chapter 21's FM-21.5: the count rises after a redescription that added no distinguishable component. An added category must satisfy Chapter 5's validity conditions and must carry variance the existing categories do not. Components that always co-occur are one component. And the rendering bounds bind if the reading is displayed: N = 17 is the folded-lattice cliff and single-transposition audible capacity is N <= 25.

29.10 Lever L2 — raise eta, the conversion

The reading that says it binds. A large c/eta with c already lean, and a reserve check that passes: D <= 2.885390 ∗ c ∗ (1/eta_0 − 1/eta_max). When that inequality holds, the cost side can close the gap alone. When it fails, difference is required, and no amount of work on conversion will substitute.

What it costs. The required eta is exact: eta >= c/F. At the carried dyad that is 0.62/0.563085 = 1.101077, which is impossible. At the repaired dyad it is 0.40/2.153633 = 0.185733, which is already exceeded. **The same lever is absent in one phase of the same constructed case and free in the other, and the only thing that changed between them was the geometry.** That is the order rule in a single sentence.

Failure mode, and it is specific to this lever. `eta` carries the entire cost side of the reading and its elasticity is the largest in the dyadic table, which means a verdict resting on it deserves an error bar rather than a decimal point. Chapter 17's constructed family carries a deficit of `+0.584737` octaves with `eta` known to `± 0.05`, giving an uncertainty of `± 0.9734` octaves — and the sign of the verdict is not resolved. A deficit near zero is the most dangerous reading in the book, because it is the one most likely to be an artefact of an imprecise input.

29.11 Lever L3 — cut `c`, the cost

The reading that says it binds. The largest reserve of the two cost-side levers, always: `R_c = 2.885390 (c/eta)` is by construction the entire required-octave figure, so cutting cost to zero always closes any deficit. That makes it the lever with the best arithmetic and the worst practical guarantee.

What it costs. The required cut is exact: `c <= eta F`. At the carried dyad, `c <= 0.30 * 0.563085 = 0.168925`, from `0.62` — a reduction of `72.754%` in coordination overhead. Reachable in principle; a large project in fact.

Failure mode, and it is the one that makes this lever's arithmetic misleading. Chapter 4's SPEC 4.11: before cutting `c`, ask whether the cost being cut is performing conversion work. If it is, `eta` falls with `c` and the ratio `c/eta` may not improve at all — or may worsen. The decision log in the carried dyad's repair is a cost that raises `eta`; removing it to "cut overhead" would cut `c` from 0.40 and take `eta` back toward 0.30, and the reading would move the wrong way while the arrangement looked leaner. **`R_c` is computed at fixed `eta`, and cost cuts do not generally hold `eta` fixed.**

29.12 The thing that is not a lever — `delta`

`delta` is the fourth argument of `Q_inf` and it is not a lever on `D` at all (Theorem 29.2). It belongs in a plan, and it belongs there second.

The reading that says it is ready. One condition only: `eta F > c`. Once the drive is positive, `delta` is the largest available improvement in the apparatus, and it is worth quantifying how large:

```
after the repair, drive b = +0.547598407
  delta 0.09 -> Q_inf  +6.084427
 delta 0.07 -> Q_inf  +7.822834     gain  +1.738408   half-life  9.5513
 delta 0.05 -> Q_inf +10.951968     gain  +4.867541   half-life 13.5134
 delta 0.03 -> Q_inf +18.253280     gain +12.168853   half-life 22.7566
 delta 0.01 -> Q_inf +54.759841     gain +48.675414   half-life 68.9676

the same moves before the repair, drive b = -0.451074559
  delta 0.09 -> Q_inf  -5.011940
  delta 0.07 -> Q_inf  -6.443922      change  -1.431983
  delta 0.05 -> Q_inf  -9.021491      change  -4.009552
  delta 0.03 -> Q_inf -15.035819      change -10.023879
  delta 0.01 -> Q_inf -45.107456      change -40.095516
```

The two columns are the same intervention producing gains of `+48.68` and `-40.10`, and the only thing that differs between them is the sign of a number that `delta` never touches.

Note also what the right-hand column of the first block costs. A destiny of +54.76 comes with a half-life of 68.97 passes and a 99% approach time of 458 passes. At one pass per week that is nearly nine years to arrive. A deep destiny is a slow one, necessarily; the same parameter sets both, and there is no configuration in this model with a deep fixed point and a short approach.

Failure mode. FM 22.12 in Chapter 22's vocabulary: adding trust to a failing arrangement in order to make it improve faster does the reverse of both. It makes the approach slower and the destination worse.

29.13 Which lever binds — the decision arithmetic

Put the reserves and the deficit side by side and the binding lever reads off directly. Computed across the four arrangements this book carries:

```
   arrangement          D        R_u      R_eta     R_c      R_N/cat   closes alone
    ------------------------------------------------------------------------
------------------
   carried dyad 2a    +4.338420  5.545206  4.174198  5.963140  0.112937  u, c
    carried dyad 2b     −3.590989       --          --          --          --
already sustainable
   the commons       +6.463837  6.632448  4.620867  7.001314  0.044581  u, c
    constructed family −0.516423       --          --          --          --
already sustainable
```

R_eta is computed at the generous eta_max = 1. In both failing arrangements the conversion lever cannot close the gap alone even at an impossible efficiency, and in both the two levers that can are the geometry and the cost.

The commons row is the instructive one and Chapter 18 carries it. Its deficit is +6.463837 octaves and **no single lever closes it in practice**, even though two of them close it in principle:

```
    raise u to 0.95         -> deficit −0.0332   (only just, and 0.95 is near-
orthogonality)
    raise eta to 0.90        -> deficit +2.1075    still failing
    cut kappa to 0.020       -> deficit +2.0085    still failing
```

R_u = 6.632448 exceeds the requirement by 0.168611 octaves, which is to say the geometry lever closes it only in the limit of six people who share almost no state at all — a configuration that is not a commons. R_c = 7.001314 closes it only at zero coordination cost, which is not a commons either. **Some systems cannot be tuned; they must be restructured, and the diagnostic is what tells you which case you are in.**

29.14 The belief postulate, applied in full

There is a large literature claiming that a destiny is changed by holding a different belief about it. Book Two Chapters 76 and 77 built the apparatus for adjudicating that claim and it is applied here without softening in either direction.

> **[POSTULATE — Book Two 76.1] BELIEF CREATES REALITY IF THE BELIEF IS GROUNDED IN TRUTH.**

The conditional clause is what makes this a scientific postulate rather than magical thinking, and the whole content sits in its formalisation:

```
BELIEF  ->  [ CHANNEL ]  ->  WORLD

"grounded in truth"  =  THE CHANNEL EXISTS AND IS REAL
```

> **[POSTULATE — Book Two 77.1]** A belief `B` affects a world-state `E` only through one or more channels. Each channel `i` has a gain `G_i` determined by the channel's own mechanism and not by the belief. Then `E = (sum_i G_i) B` for small `B` before saturation, and `|E| <= (sum_i G_i) B_max`. If no channel exists the sum is empty, `sum_i G_i = 0`, and `E = 0` at every `B`.

The badge stays on: this is a postulate, not a theorem. But the *bound* is arithmetic once the gains are estimated, and arithmetic is what makes a claim auditable in advance of any data.

Book Two 77.2 draws the correspondence with the storage law explicitly, and it is exact in form: `B` is the input where `S` was, `G` is the conversion where `eta` was, `E` is the output where the stock was. Where the correspondence stops, stated honestly: the storage law is a proven theorem about a recursion and the coupling relation is a postulate about the world. The form transfers; the warrant does not.

The four channel classes and their gains (Book Two §77.11): physiological, roughly 0.15 to 0.45 on subjective outcomes and approximately zero on objective disease endpoints; behavioural, the widest range and typically 0.1 to 0.6 in that chapter's worked cases; social, bimodal — small below a committed-fraction threshold and large above it; informational, zero to about 0.35 and entirely conditional on the target existing. Two readings of that list carry into this chapter. The behavioural channel dominates in every worked case, so the practical consequence of the postulate is not "believe harder" but: **find the action the belief produces, and check whether the action works.** And three of the four classes zero out on a fact about the world — no efferent pathway, ineffective action, absent target — which is where the conditional clause lives.

Composition. Series multiplies and parallel adds. A four-stage channel:

stage	gain	cumulative
belief registered as expectation	0.70	0.700000
expectation reaches the effector	0.50	0.350000
effector produces the action	0.62	0.217000
action lands on the outcome	0.35	0.075950

A chain of four individually respectable transmissions delivers 7.6% of its input, and the whole chain is bounded above by its weakest stage. Three such chains in parallel deliver `0.227850`.

29.15 The minimum viable gain, computed on this book's arrangement

Book Two §77.12 computes a minimum viable gain on its own worked group. Here is the same computation on the arrangement this book carries, which makes the result usable inside the frequency register.

Take case 2b as the arrangement: `S = F = 2.153633`, `C = c = 0.40`, `delta = 0.09`, `N = 12`, `ln N = 2.484907`. Then a belief-driven channel of gain `G` is worth holding this period only if it

clears the decay on the existing stock plus the maintenance cost:

```
G_min (growth this step)   =  (delta * Q_t + C) / S
G_min (headroom >= 0)      =  (delta * Q_t + C) / ln N

Q_t        G_min (growth)   G_min (headroom)
-----------------------------------------------
 0.0000       0.185733          0.160972
 1.0000       0.227523          0.197191
 3.0000       0.311102          0.269628
 6.0844       0.440000          0.381342
10.0000       0.603631          0.523158
```

Read the fourth row. At the fixed point, `G_min` equals `eta` exactly — `0.440000` against `eta = 0.44` — which is not a coincidence but the definition of a fixed point restated: the required gain at the destination is the gain that produced the destination. **The threshold rises with the stock, which means a belief channel sufficient to build an arrangement from nothing is not sufficient to maintain it once built.**

And the `G = 0` row deserves its own paragraph, because it is the arithmetic of an ungrounded belief:

```
no channel, F = 0, so the drive is  -C  and  Q_inf = -C/delta

C = 0.05, delta = 0.09  ->  Q_inf = -0.555556
C = 0.10                ->  Q_inf = -1.111111
C = 0.20                ->  Q_inf = -2.222222
C = 0.40                ->  Q_inf = -4.444444
```

A belief with no channel does not merely fail to build anything. Maintenance still bills, so **the fixed point is negative in proportion to what holding the belief costs, and the arithmetic says so without any need for moralising about it.**

Now the audit, run backwards, against the specific claim this chapter exists to adjudicate: *can believing differently close the carried dyad's 4.338420-octave deficit?* The question is well-posed, because the deficit's four inputs are the only routes:

```
through u    : u must reach 0.627132 from 0.068735, a factor of 9.1240
               -- a change in what the parties actually hold and do
through eta  : eta must reach c/F = 1.101077
               -- ABSENT. No belief reaches an efficiency above one.
through c    : c must fall to eta F = 0.168925, a cut of 72.754%
               -- a change in what the arrangement costs to run
through N    : ~200 categories at fixed u, or a real geometry change first
```

Two of the four routes are changes in what people do, one is arithmetically unreachable, and one is a change in a description that must satisfy Chapter 5's validity conditions. So the postulate gives a clean and deflationary answer: a belief that produces the behavioural change — the states actually occupied, the commitments actually taken on, the decision log actually kept — moves the destiny by exactly the amount that change is worth, which is computable in advance. A belief that produces no such change has an empty channel sum and moves the destiny by zero, while continuing to bill its maintenance cost.

29.16 R23, and what testimonials are worth

The companion result is Book Two's R23, and it is the sharpest thing in this chapter.

A corpus of testimonials is a record of successes, published because they were successes. The probability that a published account reports a success is therefore 1, under every hypothesis about how often the method works. The likelihood of the observed corpus is the product of those probabilities, and the likelihood ratio between any two hypotheses is that product divided by itself.

```
P(account reports a success | published, p = 0.900000)  =  1.000000
P(account reports a success | published, p = 0.010000)  =  1.000000
P(account reports a success | published, p = 1.0e-09 )  =  1.000000

likelihood ratio, at any corpus size n:                    1.000000
log10 LR                                                   0.000000
bits of evidence supplied                                  0.000000
```

A likelihood ratio of exactly 1.000000 means the evidence does not move the posterior at all. Not weakly, not a little — not at all. **A thousand accounts from people for whom a method worked carry precisely the weight of zero accounts, because the selection guarantees you observe successes whether or not the method does anything.**

The precedent is Abraham Wald's wartime work on aircraft armour: the bullet-hole data were conditioned on the aircraft returning, so the places with no holes were the places to armour. A manifestation corpus is the returning aircraft. It is a census of survivors presented as a census of attempts.

Three operating consequences, and they are the reason this book's protocols look the way they do. **The base rate is unobtainable from testimonials in principle, at any corpus size** — a thousand more accounts add a thousand numerators and no denominator, so the problem is not sample size and cannot be fixed by more of the same. Getting a denominator requires enrolling people before the outcome is known, which is a cohort design and is why preregistration is mandatory rather than advisable. And the contrast is enormous once the denominator exists: a constructed cohort of 100 enrolled with 90 successes yields a likelihood ratio of `9.66e+15` between `p = 0.9` and `p = 0.5`, against `1.000000` for any number of testimonials whatever.

R23's companion result completes the picture and it is the practical rule. Likelihood ratios measured at `2.0000` for a prediction specifying only a DIRECTION versus `11.3577` for one specifying a MAGNITUDE: **a prediction with a number attached is about 5.68 times more evidentially powerful, and a belief that predicts only a direction is not yet testable.** That is the precise justification for this chapter's requirement that a lever plan state its target value before it is executed.

The Specification

29.17 PROCEDURE CHANGE — planning a change of destiny

Step 1. Produce a complete destiny reading first (Chapter 28, PROCEDURE DEST, all ten steps). No lever may be discussed before `D`, `delta`, `Q_inf` and the export test are on the page.

Step 2. Run the ceiling test. If `c/eta > ln N`, stop. The case is structural. Report it as structural and go to Step 9.

Step 3. Compute all four reserves — `R_u`, `R_eta`, `R_c`, `R_N` per category — against the deficit. Report which levers close it alone and which do not.

Step 4. Compute the elasticities and report them separately from the reserves, with the statement that they answer different questions.

Step 5. Check the sign of the drive. If `eta F <= c`, no `delta` intervention may appear anywhere in the plan until a later step has made the drive positive. This is a step constraint, not advice.

Step 6. Assign every proposed action to exactly one handle — `u`, `N`, `eta`, `c`, `delta` — and state the target value with a number attached. A proposal that cannot be assigned moves nothing; a proposal without a target value is not yet testable (R23's companion result).

Step 7. For every action whose mechanism runs through what somebody believes, run the coupling audit. Name the channel, classify it, estimate its gain from the channel's own mechanism and not from the outcome, compose series and parallel, and compare the summed gain against the gain the claim requires. Report the audit whether it passes or fails.

Step 8. Test cost cuts for conversion damage. Before cutting `c`, ask whether the cost being cut performs conversion work. If it does, `eta` falls with it and the ratio may not improve.

Step 9. Order the actions: numerator first, loss second. Publish the projected trajectory as well as the projected fixed point, and check that the trajectory does not pass through negative stored order.

Step 10. Re-read after the intervention and publish the before-and-after in the Chapter 28 reporting format, including the export test, including `delta` alongside `D`, and including the cases where the plan did not work.

29.18 Gates

```
   GATE 29-1   destiny reading complete before any lever discussed     FAIL -
> stop
   GATE 29-2   u interventions checked against the one-hot artefact    FAIL -
> re-elicit
    GATE 29-3   ceiling test run first                                 FAIL -
> incomplete
   GATE 29-4   reserves AND elasticities both reported, separately     FAIL -
> incomplete
   GATE 29-5   no delta action scheduled while the drive is negative   FAIL -
> reorder
   GATE 29-6   every action assigned to one handle, with a target value FAIL -
> not a plan
   GATE 29-7   coupling audit run on every belief-mediated action      FAIL -
> unaudited
    GATE 29-8   cost cuts tested for conversion damage                 FAIL -
> incomplete
   GATE 29-9   trajectory published, not only the fixed point         FAIL -
> incomplete
```

```
   GATE 29-10  consent re-asked at every stage that changes the action  FAIL -
> halt
```

The Failure Modes

FM 29.1 — THE UNASSIGNED PROPOSAL. *Signature:* an intervention that cannot be attached to `u`, `N`, `eta`, `c` or `delta`. *Mechanism:* Theorem 29.1. `Q_inf` has four arguments and a proposal touching none of them moves it by zero. *Fix:* Gate 29-6.

FM 29.2 — TRUST FIRST. *Signature:* "we will build trust and things will improve." *Mechanism:* R15. In the constructed case this takes `Q_inf` from `−5.011940` to `−45.107456` while `D` does not move at all. *Fix:* Gate 29-5.

FM 29.3 — THE STRONGEST LEVER ASSUMED SUFFICIENT. *Signature:* a plan built on the largest elasticity. *Mechanism:* Theorem 29.3. At the carried dyad `eta` has 4.80x the elasticity and 4.174198 octaves of reserve against a 4.338420 requirement. *Fix:* Gate 29-4.

FM 29.4 — THE ABSENT LEVER PRICED AS AN EXPENSIVE ONE. *Signature:* a budget for raising `eta` to a required `1.101077`. *Mechanism:* a conversion efficiency above one would mean more structure created than interaction supplied. *Fix:* compute `c/F` before planning the lever.

FM 29.5 — MANUFACTURED DISAGREEMENT. *Signature:* a party adopting contrary positions to raise u. *Mechanism:* R8. u depends on `|rho|`; opposition reads `u = 0` exactly as agreement does, and the cost rises. *Fix:* §29.8's test — would it still be true if nobody were reading the number?

FM 29.6 — `u` RAISED BY RE-DESCRIPTION. *Signature:* a reading of `u = 1.000000` after a re-elicitation. *Mechanism:* disjoint support is automatically orthogonal (Chapter 9). *Fix:* Gate 29-2.

FM 29.7 — CATEGORIES ADDED TO A NEAR-PARALLEL SET. *Signature:* a plan to raise `N` in a system at low u. *Mechanism:* `0.112937` octaves per category at the carried dyad and `0.044581` at the commons; two hundred categories to close what one geometry change closes. *Fix:* raise u first, or accept that the case is structural.

FM 29.8 — INDEX INFLATION. *Signature:* `N` rises after a re-description that distinguishes nothing new. *Mechanism:* Chapter 21 FM-21.5; components that always co-occur are one component. *Fix:* Chapter 5's validity conditions, applied to each added category.

FM 29.9 — THE COST CUT THAT CUTS CONVERSION. *Signature:* overhead removed and the ratio `c/eta` unimproved or worse. *Mechanism:* SPEC 4.11; `R_c` is computed at fixed `eta` and cost cuts do not generally hold `eta` fixed. *Fix:* Gate 29-8.

FM 29.10 — THE MARGINAL VERDICT ACTED ON. *Signature:* a plan launched on a deficit of `+0.584737` with `eta` known to `± 0.05`, an uncertainty of `± 0.9734` octaves. *Mechanism:* the sign is unresolved. *Fix:* Chapter 15's rule — a verdict is resolved only when the deficit exceeds its own uncertainty.

FM 29.11 — THE DESTINATION EVALUATED AND THE PATH IGNORED. *Signature:* "the fixed point does not depend on the order, so the order does not matter." *Mechanism:* true and irrelevant;

the constructed case shows 12 periods of permanent lag, 13 periods below the starting stock, and 111.133601 stock-periods of cumulative loss. *Fix:* Gate 29-9.

FM 29.12 — BELIEF WITHOUT A CHANNEL. *Signature:* an intervention whose mechanism, when asked for, is that someone will believe it. *Mechanism:* the coupling requirement; an empty channel sum gives `E = 0` at every `B`, while `C` continues to bill, so `Q_inf = −C/delta`. *Fix:* Gate 29-7.

FM 29.13 — THE GAIN FITTED TO THE OUTCOME. *Signature:* a channel gain estimated from the result it is supposed to predict. *Mechanism:* Book Two 76.12's fifth gate — the apparatus is unidentifiable unless `G` comes from the channel's own mechanism. *Fix:* estimate `G` without touching the outcome, then predict.

FM 29.14 — THE MAGNITUDE ERROR. *Signature:* an effect promised that is larger than every named channel summed at maximum belief. *Mechanism:* the bound of 77.1. A four-stage series channel of individually respectable stages delivers `0.075950`. *Fix:* run the audit before collecting data.

FM 29.15 — TESTIMONIALS AS EVIDENCE. *Signature:* a lever recommended on the strength of accounts from people it worked for. *Mechanism:* R23. The likelihood ratio is exactly `1.000000` at any corpus size. *Fix:* a cohort with the denominator observed, preregistered.

FM 29.16 — THE DIRECTION-ONLY PREDICTION. *Signature:* "this will improve things." *Mechanism:* R23's companion result; direction-only predictions carry `LR = 2.0000` against `11.3577` for a prediction with a magnitude. *Fix:* Gate 29-6's target value.

FM 29.17 — A LEVER PULLED ON SOMEBODY ELSE. *Signature:* a plan that raises u by requiring another party to change. *Mechanism:* Chapter 18's consent axiom; the lever is real and the procedure is not available. *Fix:* Gate 29-10, and §29.7.

Worked Example

29.19 The four handles, one at a time, on the carried dyad

CONSTRUCTED. Not an observation of any person or pair. SPINE §3.2. The pair start as found and each row adds one handle to the row above it, in the order R15 requires.

```
   step                    u       eta   c    F       D       Q_inf    f_rel
    ------------------------------------------------------------------------
------------------------
   0  as found              0.068735  0.30  0.620  0.563085  +4.338420  -
5.011940    1.4908
    1  geometry only (u)          0.692373   0.30   0.620   2.153633   -
0.250931   +0.289887   35.8882
    2  + conversion (eta)         0.692373   0.44   0.620   2.153633   -
2.148294   +3.639982   35.8882
    3  + cost (c)                 0.692373   0.44   0.400   2.153633   -
3.590989   +6.084427   35.8882
```

```
    4  + trust (delta .09 -> .05) 0.692373   0.44   0.400   2.153633   -
3.590989  +10.951968   35.8882

   (Q_inf at delta = 0.09 for rows 0–3; row 4 is the same arrangement at delta = 0.05)
```

Read the columns. The geometry step alone flips the sign of both readings — a deficit of +4.338420 becomes −0.250931 and a destiny of −5.011940 becomes +0.289887 — and **it is the only step in the table that changes `f_rel` at all.** The two cost-side steps then buy 3.34 octaves of deficit and 5.79 units of destiny between them, and they are only available because the drive is already positive. The trust step buys 4.868 units and moves the deficit by nothing, which is Theorem 29.2 appearing on the page.

Now the same four handles in the order the intuition prefers:

```
   step                        u         eta    c     delta   D          Q_inf
    ------------------------------------------------------------------------
-------------
   0  as found                0.068735   0.30   0.620   0.09    +4.338420   -
5.011940
   1  trust first             0.068735   0.30   0.620   0.05    +4.338420   -
9.021491
   2  trust harder             0.068735   0.30   0.620   0.02    +4.338420   -
22.553728
   3  trust absolutely         0.068735   0.30   0.620   0.005   +4.338420   -
90.214912
```

Four rows of an unchanging instrument and an eighteenfold deterioration in the destiny. Every step in that table is an intervention a reasonable person would call good practice, and every step makes the arrangement worse, and the deficit — the book's own central instrument — reports nothing at any point. **That is the single strongest argument in this book for publishing `delta` alongside `D`, and it is why Gate 28-4 exists.**

29.20 The commons: the procedure halts

CONSTRUCTED. Not an observation of any group. SPINE §3.3: six members drawn from one doctrine, 15 pairs, mean pairwise `u = 0.018614`, cost `kappa = 0.055` per link, `eta = 0.34`, `delta = 0.09`.

```
    reading      F = 0.186272 nats   N_eff = 1.204754   f_rel = 0.7017 Hz
            f_min = 61.9349 Hz  D = +6.463837 octaves  Q_inf = -8.4630  DECAYS
    ceiling      c/eta = 2.426471 vs ln N = 2.484907  -> passes by 0.058436
   reserves    R_u = 6.632448  R_eta = 4.620867  R_c = 7.001314  R_N/cat = 0.044581
    single-lever raise u to 0.95    -> deficit -0.0332   at near-orthogonality
                 raise eta to 0.90  -> deficit +2.1075   still failing
                 cut kappa to 0.020 -> deficit +2.0085   still failing
    VERDICT      PROCEDURE HALTS. No single lever closes this in practice.
```

The reserves say two levers close it in principle. The single-lever table says what those in-principle closures actually require: six people who share almost no state, or coordination at near-zero cost. Neither describes a commons. And the roster ceiling is the reason: at `m(m−1) = 30` against a

bound of 2 eta ln N / kappa = 30.7225, a seventh member takes the group to 42 and makes the case structural outright.

This is the case the book keeps because it does not work. The lesson is not that the group should disband — the formalism prices stored order and nothing else, and a family, a team or a commons is for things the formalism does not represent. The lesson is that a lever plan for this arrangement would have been a plan for a case with no solution, and the ceiling test and the reserve table are what say so before anybody spends a year on it.

29.21 The coupling audit, run on a lever claim

CONSTRUCTED. Not an observation. Take the claim: *the pair will close their 4.338420-octave deficit by committing to a shared vision of the relationship they want.*

```
   Step 6  ASSIGN THE HANDLE.  The proposal names no handle. Ask which of u, N, eta, c, delta
         it moves. Two answers are possible and they audit completely differently.

   ANSWER A -- "it moves what we each take on."  -> handle L1 (u). Testable. Target value
         required: u >= 0.627132. Channel: behavioural, the dominant class. Gain estimated
          from behaviour records, not from the outcome. AUDITABLE, and Gate 29-
6 satisfied
             once the target is stated.

   ANSWER B -- "it moves how we feel about it."  -> no handle. The four arguments of Q_inf do
         not contain a term for how an arrangement is regarded. Channel sum is empty:
         E = 0 at every B. And the arrangement still bills: with F unchanged at 0.563085,
           eta 0.30, c 0.62, delta 0.09, the destiny remains -5.011940 exactly.

   Step 7  THE BOUND.  Required change in the drive to reach Q_inf = 0:  +0.451075 per pass.
          Available from Answer B's named channels, summed:            0.000000
         VERDICT: over budget by the entire amount, before any data are collected.
```

That is the postulate doing its work in both directions in one example. Answer A is a real channel with a real gain and the postulate says it moves the world by exactly what the channel can carry. Answer B has no channel and the postulate says nothing happens — and says it as arithmetic rather than as disapproval.

29.22 What this chapter establishes, and what Chapter 30 takes

Established, and checkable. That Q_inf has exactly four arguments and five handles, with F reached only through u and N (Theorem 29.1); that delta cannot change the sign of a destiny at any magnitude, so it is a multiplier on an existing sign rather than a lever on the gap (Theorem 29.2); that a lever can be absent rather than expensive, demonstrated on a lever with 4.80 times the elasticity of its rival and 4.174198 octaves of reserve against a 4.338420 requirement (Theorem 29.3); that reversing R15's order costs 12 periods of permanent lag, 13 periods below the starting stock, and 111.133601 stock-periods of cumulative loss in the constructed case; that an arrangement over the c/eta <= ln N ceiling is structural and no geometry closes it, worked at c/eta = 3.000000 where even perfect orthogonality leaves +1.486245 octaves and twenty-one categories are required; that a belief moves a destiny only through a channel whose gain is

estimable from its own mechanism, with an empty channel giving `Q_inf = −C/delta`; and that testimonial evidence carries a likelihood ratio of exactly `1.000000` at every corpus size.

Not established, and not claimed. That any lever prescription here applies to any actual person, pair or group; that a larger fixed point is a better outcome; that belief has any effect through any channel not named and estimated; that any reading here is diagnostic, therapeutic or clinical; or that the frequencies printed here are physical resonances.

The one line to carry forward: *there are four arguments, the reading says which one binds, and the ones that are absent are absent no matter how much anybody wants them.*

Chapter 30 closes the book and the series. It states the keystone once more in full — `f_min = k e^(2c/eta)`, the 2.885 octaves per unit of cost ratio, and the `c/eta <= ln N` ceiling — draws the arc from Book One's operator through Book Two's engineer to this book's tuning of the relation, and sets beside what the series produced the four or five results that constrain it. It ends on the open problem of Chapter 27 rather than on a summary of achievements, for the reason Chapter 27 gives. Chapter 4 owns `f_min` and the four levers; Chapter 15 owns the order rule, the elasticities and the reserves; Chapter 18 owns the intervention procedure and the consent axiom; Book Two Chapters 76 through 81 own the belief postulate, the coupling requirement and the grounding protocol.

Chapter 30 . The Sustaining Frequency

This chapter closes a book and a series. It has three jobs and they are in a deliberate order.

The first is to state the keystone once more, in full, with nothing left implicit — because the whole of Book Three exists to join two halves of one equation that Book Two laid down separately, and a closing chapter that gestured at the join instead of writing it out would have been a closing chapter about a book rather than the end of one.

The second is to draw the arc across all three books honestly: an operator learning to read a gradient, an engineer learning to build a circumstance, and then this volume tuning the relation between them. That arc is not a story of accumulating power. At each stage the instrument got sharper and the set of things it was entitled to say got smaller, and both halves of that sentence belong in the summary.

The third is to set beside the result the findings that constrain it. There are six of them and every one is a limitation. A surplus law that turns out to be a nineteenth-century psychophysical law with a threshold; a consonance that turns out to be redundancy; an interaction parameter that cannot tell perfect agreement from perfect opposition; a group whose whole surplus is available to a well-chosen pair; two frequency readings that cannot be inferred from each other; and a central diagnostic that is blind to decay. **Those six are the real yield of the series, and each of them narrows what the instrument can be used for.** A closing chapter that listed the affordances and omitted the constraints would be selling something.

Then the chapter stops on the open problem of Chapter 27, which is not solved, not softened, and not worked around. **The restraint is the argument.** A book that names its own load-bearing gap this precisely earns the right to everything else in it; a book that declines to name it forfeits the right to all of it, including the parts that are correct.

The Law

30.1 The keystone, in full

Book Two established both halves and never multiplied them.

```
    Volume Two, Ch 30     THE ESSENCE BRIDGE       f  =  k * e^(2F)
                                                  k  =  0.483425732988414 Hz
```

```
    Paper E, Thm 3.5      THE STORAGE FIXED POINT  Q     =  ( eta * F(u*) -
c* ) / delta
                                                      inf
```

The bridge says an interaction's surplus has an exact pitch. The fixed point says where a relation, a life or an institution arrives if its parameters hold. Sustainability means the fixed point is positive:

```
    Q     >  0     <=>     eta * F  >  c     <=>     F  >  c / eta
     inf
```

And the bridge inverts. If `f = k e^(2F)` then `F = (1/2) ln(f/k)`. Substitute:

```
    (1/2) ln( f/k )  >  c/eta        =>        f  >  k * e^( 2c / eta )
```

which is the keystone, and it is a substitution rather than a postulate — given the two halves, this line follows and nothing was added to obtain it.

THE MINIMUM SUSTAINABLE FREQUENCY

```
    f     =  k * e^( 2c / eta )
     min

    f > f_min     the relation ACCUMULATES order          Q_inf > 0
    f = f_min     it HOLDS, and nothing more             Q_inf = 0
    f < f_min     it DECAYS, whatever anyone intends     Q_inf < 0
```

The condition for a relation to pay for itself is not a ratio, not a policy, and not a virtue. It is a pitch.

In octaves above the ground essence k, the statement becomes an exchange rate:

```
    octaves required  =  2 (c/eta) / ln 2  =  2.885390 * (c/eta)
```

Every unit of cost-to-conversion ratio costs 2.885390 octaves of required pitch. Computed:

```
     c/eta      f_min (Hz)      octaves above k
     0.000        0.483426          0.000000
     0.250        0.797034          0.721348
     0.500        1.314087          1.442695
     0.750        2.166564          2.164043
     0.909091     2.978216          2.623082
     1.000        3.572060          2.885390
     1.500        9.709865          4.328085
     2.000       26.394151          5.770780
     2.066667    30.158790          5.963140
     2.484907    69.613354          7.169926   <- the ceiling at N = 12
     3.000      195.027860          8.656170
     4.000     1441.071800         11.541560
```

30.2 The ceiling

The third component of the keystone is a bound, and it is Paper E Corollary 3.4 in the frequency register. Since `F <= ln N`, the maximum sustainable cost ratio is capped:

```
c / eta  <=  ln N            equivalently        f     <=  k * N^2
                                                  min
```

At twelve categories, `c/eta` may not exceed `2.484907` and `f_min` may not exceed `69.6133055503 Hz`. Above that, **no arrangement of those twelve categories is sustainable** — not a better pairing, not a better schedule, not more effort. The only remedies are: raise `eta`, cut `c`, or add genuinely distinct categories.

N	ln N	max c/eta	f_max = k N^2 (Hz)	octaves
2	0.693147	0.693147	1.933703	2.000000
3	1.098612	1.098612	4.350832	3.169925
4	1.386294	1.386294	7.734812	4.000000
6	1.791759	1.791759	17.403326	5.169925
12	2.484907	2.484907	69.613306	7.169926
17	2.833213	2.833213	139.710037	8.174926
25	3.218876	3.218876	302.141083	9.287712
41	3.713572	3.713572	812.638657	10.715104
100	4.605170	4.605170	4834.257330	13.287712

The ceiling is the most useful single line in the apparatus for the purposes of refusing work. It converts a class of problems from difficult into unavailable, and it does so before any effort is spent. A test that can say "no arrangement of these categories works" in one division is worth more to a planner than any number of tests that say "try harder."

30.3 What the keystone unifies

Three laws that entered the corpus from three different directions land on one threshold.

The essence law gives a state a frequency: `f = k n^2`, with `k = (9/8)^2/phi^2 = 0.483425732988414 Hz` reverse-engineered from the published essence table and verified to fourteen significant figures. The constant is chosen and the square law is a postulate; both carry their badges here as everywhere.

The surplus law gives an interaction a magnitude: `F = ln(1 + (N−1)u)`, a theorem under Paper A's axioms, bounded above by `ln N`.

The storage law gives an arrangement a destination: `Q_{t+1} = (1−delta)Q_t + eta F − c`, with the fixed point `Q_inf = (eta F − c)/delta`, a proved property of a linear recursion.

The bridge `f = k e^(2F)` is what makes them commensurable, because it puts the surplus and the frequency in the same register — the logarithm — and once they are in the same register the sustainability condition of the third law can be written as a threshold in the units of the first.

The join is arithmetic and the join is the whole of Book Three's contribution. What it buys is that a question about whether an arrangement pays for itself becomes a comparison of two numbers in hertz, and both of them are computable from measurements a practitioner can actually take.

30.4 The diagnostic, and the two numbers that must travel together

```
octave deficit   D  =  2.885390 * (c/eta)  −  log2( f/k )
                   =  2.885390 * ( c/eta  −  F )
```

```
    D <= 0    sustainable as it stands
  D >  0   short by that many octaves; four levers: raise u, raise eta, cut c, raise N
```

And the identity that Chapter 28 owns:

```
    D  =  − 2.885390 ∗ ( delta / eta ) ∗ Q
                                          inf
```

verified to 8.88e−16 across twelve constructed parameter sets. **The octave deficit and the fixed point are the same number in different units.**

But D contains no delta. **The deficit reads the SIGN and the SIZE of the gap; delta converts it to a LEVEL; and a reading that reports one without the other is incomplete.** The carried dyad as found reads D = +4.3384201973 at delta = 0.30, at delta = 0.09, at delta = 0.05 and at delta = 0.01, while its destiny moves from −1.5035819 to −45.1074559 across the same column. This is a genuine limitation of the book's central instrument and it is stated here, in the closing chapter, rather than left in the chapter that derived it.

30.5 What the keystone does not say

Five refusals, and they are not decorative. Each one marks a place where the formalism has been mistaken for something larger.

It asserts no physical resonance. Every hertz value in three books is a position on a logarithmic scale of surplus. No person, bond, tissue or molecule is claimed to vibrate at any value anywhere in this corpus, and nothing here is to be played at anyone.

It makes no clinical, diagnostic or therapeutic claim. The twelve states are not diagnoses, a reading is not an assessment, and no procedure here treats anything. A reader who needs support for a relationship or for their health should seek it from someone qualified to give it, which this book is not.

It confers no capability and edits no DNA. Part V specified an encoder, an addressing convention, a sonifier and a rendering discipline. It specified a research procedure and labelled it as one.

It hides nothing. No subliminals, nothing masked, nothing below threshold. Open-label placebo works, so disclosure costs nothing that matters.

It removes nobody's ability to decline. Consent is structural, re-asked at every stage that changes what is done, and no reading in this book licenses an intervention on a person who has not agreed to it.

And one more, which is about the scale rather than the ethics. **The keystone inherits exactly the defensibility of its inputs.** The index n is a count and is ratio-scaled by construction, so it sidesteps R1's demotion; the intensities w are ordinal by default and must be magnitude-estimated before any relationship number is printed, because an ordinal-preserving relabelling moves u from 0.362115 to 0.525275 — 1065.05 cents, nearly an octave, from a change an ordinal scale calls no change at all. Where the intensities were not magnitude-estimated, the relationship frequency is not determined and no number may be printed.

30.6 Three through-lines, and what each one costs

Before the arc, three statements that hold in every volume of the series and that a reader should carry away even if nothing else survives. Each is a genuine finding and each has a price attached that the popular version of the same idea leaves off.

The loss term is where everything happens. `delta` appears in no diagnostic and decides every level. It sets the half-life, it sets the depth of the fixed point, it sets how long a repair takes to show, and it is the parameter the octave deficit cannot see. Three books of interventions reduce, in the end, to a question about a leak: how fast does an arrangement forget what it made? The price of this through-line is the one §30.12(f) names — because `delta` is absent from `D`, the instrument that reads sustainability is silent about the parameter that decides the level, and there is no version of the reading in which one number suffices.

Difference generates surplus, and only real difference does. `F = ln(1 + (N−1)u)` is increasing in `u`, so a relation's stored order comes from the non-parallelism of what its parties hold. That is the affirmative half and it is genuinely useful: it says that a bond, a team or an institution whose members converge on one description is losing its capacity to produce anything, and it says so with a number. The price is Result (c). `u` depends on `|rho|` alone, so opposition reads exactly as agreement does, and the difference that raises `u` has to be difference in what is actually held and done — states occupied, commitments taken on, work done in a domain the other is not in. The test of a `u` intervention is whether it would still be true if nobody were reading the number, and no rearrangement of the arithmetic will substitute for that.

Geometry does work that power does not. Every result in the corpus about arrangement — the table with no head, the silo, order without a centre, the roster ceiling, the dyad's interval — is a claim that the shape of a set of relations changes what it can store, independently of how much is put into it. The keystone is the compressed form: `f_min` depends on `c/eta` and on nothing else, so an arrangement that converts better or costs less needs less pitch, and an arrangement that converts badly cannot be rescued by volume. The price here is Result (d). Geometry's yield is bounded by `ln N`, an orthogonal pair already attains it, and beyond a pair the shape buys coverage and continuity rather than surplus. Geometry is powerful and it is capped, and the cap arrives at two participants.

The Engineering

30.7 Book One — the operator reads the gradient

Book One was twelve nodes in story form and asked no arithmetic of its reader. Its output was a perceptual skill: the ability to notice that a circumstance has a direction — that a room, a conversation or an arrangement is either accumulating something or losing it, and that the difference is visible before it is measurable.

Three of its nodes turn out to be theorems in the later corpus, and the correspondence was drawn in Chapter 2 §2.9 of this book rather than asserted. Node 09, RECURSE, is the transient `(1−delta)^t Q_0` decaying while the driven term builds. Node 10, FIXEDPOINT, is Paper E Theorem 3.5 — the object Chapter 28 defines as destiny. Node 11, SEAL, is the vow about when not to intervene, and Chapter 28 §28.15 converted it into an arithmetic test: compute the

deficit and the fixed point for every participant separately, then compute the sum of drives, and if one participant improved while the sum did not move, the gain was exported and nothing was produced.

That conversion is the clearest single illustration of what the series was for. A vow is unfalsifiable in the way a testimonial is — it reports an intention, and the intention is always available. The identity `sum_i b_i` invariant under any transfer of cost among any number of participants is not unfalsifiable at all. It costs one column of a report and it returns an answer on the first pass.

What should also be said about Book One is what it deliberately withheld. It asked no arithmetic of its reader and offered none, and a reader who finished it had a perceptual skill and no apparatus. That was not a simplification for a general audience; it was the correct ordering. A person who can compute an octave deficit for a circumstance they have not learned to perceive as having a direction has a number with nothing to point it at, and the failure is silent — the computation runs, the reading prints, and it is a reading of the wrong thing. Each stage of the series is unusable without its predecessor, and the failure at each boundary is different: **without the reading, a number with no referent; without the building, a diagnosis with no intervention; without the join, an intervention with no test of whether it paid.**

30.8 Book Two, Volume One — the engineer builds the room

Volume One took the storage law and built outward: the room as a circuit, acoustics, thresholds, light and material, the table with no head, the silo, order without a centre, institutions as storage. Its output was an apparatus for constructing circumstances rather than reading them.

Two of its results carry into the close. The first is the headroom gauge `h = eta ln N − delta Q − C`, which says when no arrangement of the available categories improves the system regardless of what anyone does. The second is R2's concavity rule — feasibility must be tested on the mean of `F(U_t)` and never on `F` of the mean, worth 6.2% in that volume's worked office — which is the reason this book forbids aggregating ratios across sites.

30.9 Book Two, Volume Two — the instrument turned inward

Volume Two turned the same apparatus on the person: the self as storage, the internal surplus, the personal formulary, the moment equation, psychophysics, the launch budget, the gradient, the echo, the contraction.

And it demoted its own keystone formula, at the gate its author wrote, on the formula the volume was built around. R1: the Moment equation is demoted on scale; `D/P` may be used ordinally only — direction and ordering, never a numeric magnitude — pending Rasch instrumentation at 123 items per term. Every psychological frequency derived through `D/P` inherits the demotion, and this book says so in place, every time.

That demotion is the reason Book Three could be written. Book Two spent a volume building the apparatus that could audit a join, and then used it on itself first. A book that had produced the keystone in its first hundred pages would have produced it without the means to check it. The omission was correct; it was closable only afterwards.

30.10 Book Two, Volume Three — carried to the cell

Volume Three carried the instrument down: the cell as receiver, addressing, genomic audio, the Gabor limit, the belief postulate, the coupling requirement, the grounding protocol. Its output was a specification and a catalogue of failures — the four channel classes with their gains, the places where belief demonstrably does nothing, and R23's likelihood ratio of exactly `1.000000` for testimonial evidence.

And it stopped. It named the gap — no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal — and did not go past it. Chapter 27 of this book restates that gap without softening, and this chapter ends on it.

30.11 Book Three — tuning the relation

This book multiplied the two halves and then spent twenty-six chapters finding out what the product would and would not support.

Part II carried the essence law into psychology on a differentiation index `n` — a count of the distinguishable components required to specify a state — and defended the count against the obvious objection that deciding what counts as one component is itself a judgment. The honest limit was stated wherever the index was used: **the frequency inherits exactly the defensibility of the count, and a contested decomposition yields a contested frequency.**

Part III built the dyad as an interval, `f_rel = k(1 + 11u)^2`, and produced the octave deficit — a computable reading of whether a bond pays for itself — together with the reserve theorem, the elasticities, the order rule and the constructed cases in which the procedure halts.

Part IV audited a tradition on its own material and printed what failed. The strongest single item is Chapter 19's Finding 19.2: Ripley's Twelve Gates of 1471, run as a schedule under this framework, places **the only numerator-acting gate ninth of twelve**. The tradition's canonical sequence lowers loss before it fixes the numerator, which is exactly the order R15 shows accelerates failure. And Chapter 20's Corollary 20.2: the element rule `f = k n^2` and the compound rule `f = k^2 G^4` cross at `n = 1/sqrt(k) = 1.438252`, not an integer, so they coincide at no index at all.

Part V specified the cellular procedure with every honesty clause in place and then, in Chapter 27, said that nobody knows how a cell could read a message out of a sound.

30.12 What the series actually produced — the six results that constrain it

Every one of these is a narrowing. That is why they are the yield.

Result (a) — the surplus law is exactly a Fechner law with threshold `u_0 = 1/(N−1)`.

```
F(u)  =  ln( 1 + (N−1) u )
      =  ln( (N−1) ∗ ( u + 1/(N−1) ) )
      =  ln( (u + u_0) / u_0 )                with  u_0 = 1/(N−1)
```

That is Fechner's law in threshold-corrected form, `S = k ln((I + I_0)/I_0)`, with `k = 1`, the stimulus being non-parallelism and the threshold intensity being `u_0`. Verified for `N = 2` through `N = 100` at seven values of u each; maximum residual `4.44e−16`.

The threshold has a meaning that is not an artifact of the algebra. Evaluate the law at it, at every `N`:

```
N_eff(u_0)  =  1 + (N−1) ∗ 1/(N−1)  =  2.000000000000   exactly, for every N
 F(u_0)      =  ln 2  =  0.693147180560 nats  =  1 bit  =  2 octaves
```

The threshold is exactly the point at which the effective count reaches two: below `u_0` the system does not yet contain a second independent voice, **and the threshold is one full bit wide.** Its size is a design fact: `u_0 = 1.000000` at `N = 2`, where the threshold *is* the ceiling; `0.500000` at `N = 3`; `0.090909` at `N = 12`; `0.010101` at `N = 100`. Narrow systems are hard to get anything out of not because their members are worse but because the threshold sits at the top of their range.

What this establishes is not evidence for either formalism. Paper A's law is a theorem under its axioms and gains nothing from resembling Fechner's; Fechner's is an empirical claim resting on a badged postulate and gains nothing from resembling Paper A's. What independent arrival does establish is that the logarithm is not a stylistic choice in this corpus — it is the register both routes landed in.

Result (b) — consonance and redundancy are the same phenomenon. Because `(2n)^2 = 4n^2`, doubling an index raises the frequency by exactly two octaves, so two essences share a pitch class if and only if their indices have the same odd part, and an ordered set `1..N` contains exactly `ceil(N/2)` distinct classes — six at `N = 12`. Price the identification in surplus:

N	ln N	ln(N/2)	loss
4	1.386294	0.693147	0.693147180560
12	2.484907	1.791759	0.693147180560
24	3.178054	2.484907	0.693147180560
100	4.605170	3.912023	0.693147180560

The fold costs exactly `ln 2` — one bit, two octaves of surplus — at every `N` without exception. Consonance in the essence lattice is not a separate aesthetic property that happens to be pleasant; it is the collapse of two categories into one, and the collapse has a price that does not depend on how many categories there were.

Result (c) — `u = 4a(1−a)`, so perfect agreement and perfect disagreement are the same measurement. With a the fraction of items on which two parties agree, `cos theta = 2a − 1` and

```
u  =  1 − (2a − 1)^2  =  4a(1 − a)
```

a	cos = 2a−1	u	f_rel (Hz)
0.00	−1.0000	0.000000000	0.483426
0.10	−0.8000	0.360000000	11.893047
0.25	−0.5000	0.750000000	41.363114
0.50	+0.0000	1.000000000	69.613306
0.75	+0.5000	0.750000000	41.363114
0.90	+0.8000	0.360000000	11.893047
1.00	+1.0000	0.000000000	0.483426

The table is symmetric about `a = 0.5`, and the two ends are identical. **Two parties who agree about everything and two parties who disagree about everything return the same reading — the ground essence `k = 0.483426 Hz`, no relationship at all.** The contrarian and the sycophant sound the identical note. That is a hard constraint on what u can be used to detect,

and it is the reason manufactured disagreement is worse than useless: it raises cost and returns exactly nothing.

Result (d) — an orthogonal pair already attains `ln N`, so groups buy breadth and not depth. `F = ln(1 + (N−1) u_bar)` is maximised at `u_bar = 1`, and two unit vectors can be orthogonal. Therefore:

m	links m(m−1)/2	F at u_bar = 1	f (Hz)	cost at kappa=0.055
2	1	2.484907	69.613306	0.055000
3	3	2.484907	69.613306	0.165000
4	6	2.484907	69.613306	0.330000
6	15	2.484907	69.613306	0.825000
10	45	2.484907	69.613306	2.475000
20	190	2.484907	69.613306	10.450000

Surplus is identical at every roster size; cost grows as `m(m−1)/2`. In surplus terms a group buys nothing a well-chosen pair could not, and pays `m(m−1)/2` times as much for it. Whatever else a family, a team or a commons is for — coverage, continuity, conversion, redundancy against departure — the formalism prices stored order and nothing else, and on that axis the group's case is not a surplus case. This is the constraint that makes Chapter 16's roster ceiling bite: at `m(m−1) > 2 eta ln N / kappa` the case is structural, and the constructed commons sits at `m(m−1) = 30` against a bound of `30.7225`.

Result (e) — state frequency and relationship frequency are independent. Chapter 21's Finding 21.1, verified on constructed pairs spanning thousands of cents of the state lattice:

indices	state frequencies	span (cents)	u	f_rel (Hz)
(1, 2)	0.4834 / 1.9337 Hz	2400.00	1.000000000000	69.6133055503
(3, 6)	4.3508 / 17.4033 Hz	2400.00	1.000000000000	69.6133055503
(5, 10)	12.0856 / 48.3426 Hz	2400.00	1.000000000000	69.6133055503
(7, 11)	23.6879 / 58.4945 Hz	1564.98	1.000000000000	69.6133055503
(2, 9)	1.9337 / 39.1575 Hz	5207.82	1.000000000000	69.6133055503
(4, 12)	7.7348 / 69.6133 Hz	3803.91	1.000000000000	69.6133055503

Identical to twelve decimal places, because u reads the angle between vectors and is blind to which indices are occupied. **A state's frequency tells you nothing about its relationship frequency, neither may be inferred from the other, and a complete report gives both separately.** Re-indexing moves every state frequency in the system and can leave the relationship reading untouched.

Result (f) — the deficit is blind to decay. Chapter 4's Theorems 4.5 and 4.9, restated in §30.4. `D` and `Q_inf` are one number in two units, and `D` contains no `delta`. The consequence is the sharpest limitation the instrument has: an intervention that doubles the rate of loss, or halves it, produces no movement in the diagnostic at all. Report both.

30.13 Why the six are the yield

Every result in that list makes the instrument smaller. (a) says the surplus law is not a new discovery about interaction but a hundred-and-sixty-six-year-old psychophysical form arrived at by a second route. (b) says a property the sonic register treats as aesthetic is the loss of a bit. (c) says the interaction parameter cannot distinguish two situations that no practitioner would ever confuse. (d) says the group has no surplus case. (e) says two of the book's own readings cannot speak to each other. (f) says the central diagnostic cannot see the thing most people reading it will want to know.

A corpus whose strongest results are all constraints is a corpus that was checked. The alternative — three books of affordances, each new chapter adding a capability and none subtracting one — is the signature of work that was never audited, and it is the signature this series has spent its length trying not to have.

30.14 The register of demotions

A corpus that audits itself has to publish the audit, and the four places where the apparatus caught its own author are the strongest available evidence that the gates are real rather than decorative. They are collected here because a closing chapter is where a reader will look for them.

R1 — the Moment equation demoted on scale, at Book Two's own gate. The volume's formulary opened on a formula the whole volume was built around, ran it through the Formula Validation Protocol, and failed it on the scale requirement. `D/P` may be used ordinally only — direction and ordering, never a numeric magnitude — pending Rasch instrumentation at 123 items per term. The demotion travels with every psychological frequency derived through `D/P`, in this book as in that one.

R15 — the design intuition inverted. The corpus had been running on the assumption that trust is a source of order. It is not: it acts by reducing `delta`, and `delta` is a denominator, so trust amplifies whatever sign the arrangement already has. In the worked consortium, trust alone moved the fixed point from `−0.0836` to `−0.2114` — worse — while reciprocity alone moved it to `+0.7394` and both together to `+1.8701`. The rule that replaced the intuition is: fix the numerator, then lower the loss.

Finding 19.2 — the tradition audited on its own material and found inverted. Ripley's Twelve Gates of 1471, run as a schedule under this framework, places the only numerator-acting gate ninth of twelve. The canonical sequence of the tradition this book draws from lowers loss before it fixes the numerator, which is exactly the order R15 shows accelerates failure. That is not a criticism imported from outside; it is the framework reading the tradition's own ordering and printing the result.

Corollary 20.2 — the source table's own row shown impossible. The element rule `f = k n^2` and the compound rule `f = k^2 G^4` cross at `n = 1/sqrt(k) = 1.438252`, which is not an integer, so the two rules coincide at no index whatever. Since for a single component the compound rule is index-independent, the published O2 row could not have been produced by the rule it was presented under.

And beside those four sit the measured limits of the corpus's own controls, which are not self-catches but are the same discipline applied forward. The shuffled-input control has three measured failure modes: it is anti-conservative as ordinarily run, at 2.2 times nominal alpha; it cannot

see repackaging, which only the structural ceiling test can; and it cannot see confounding by a shared slow driver, passing 98.35% of the time in a 4,000-trial simulation. Every chapter that recommends the four-baseline suite carries all three, and the conclusion drawn from them is to design controls rather than to add surrogates.

The register is short. That is the point of publishing it: four catches across three volumes is a small number, and a reader is entitled to weigh it against the size of the corpus and decide for themselves whether the gates were sharp enough. **What is not available is the alternative in which no demotions are printed at all, because a method that never rejects is not a method.**

The Specification

30.15 The whole instrument, on one page

```
    CONSTANTS
      k      = 0.483425732988414 Hz          the ground essence
      2/ln2  = 2.885390081778               octaves per unit of cost ratio
      N      = 12,  ln N = 2.484907          this book's category count
    u_0   = 1/(N-1) = 0.090909091      the surplus threshold; N_eff(u_0) = 2 exactly

    THE THREE TRANSFER RULES
      Rule 1  a psychological ESSENCE has a frequency        f = k n^2
     Rule 2  a psychological RELATIONSHIP has a frequency   f_rel = k (1 + (N-
1) u)^2
      Rule 3  a COMPOUND state uses the alchemical rule      f = k^2 G^4

    THE MEASUREMENT
         x, y unit vectors over the N states, components = magnitude-
estimated intensity
     cos theta = x . y    u = 1 - (x.y)^2    F = ln(1 + (N-1) u)    f = k e^(2F)

    THE KEYSTONE
      f_min = k e^(2c/eta)              sustainable iff f > f_min
      octaves required = 2.885390 (c/eta)
      the ceiling: c/eta <= ln N, i.e. f_min <= k N^2

    THE DIAGNOSTIC
      D = 2.885390 (c/eta) - log2(f/k) = 2.885390 (c/eta - F)
      four levers: raise u, raise eta, cut c, raise N
      delta is NOT a lever on D; it is a multiplier on the sign of the drive

    THE DESTINY
    Q_inf = (eta F - c)/delta             where the loop arrives if parameters hold
      |Q_t - Q_inf| = (1-delta)^t |Q_0 - Q_inf|    the approach, an equality
     D = -2.885390 (delta/eta) Q_inf           the two readings, one quantity
      sum_i (eta_i F_i - c_i) invariant            the export test
```

30.16 The standing refusals, complete

```
  R-1  ESSENCE, NOT RESONANCE. No person, tissue or molecule resonates at any value here.
  R-2  NO CLINICAL, DIAGNOSTIC OR THERAPEUTIC CLAIM, at any scope, in any chapter.
   R-3   NO CAPABILITY CONFERRED. No DNA edited, altered or damaged.
  R-4  NO SUBLIMINALS. Nothing masked, nothing below threshold, nothing concealed.
  R-5  CONSENT IS STRUCTURAL. No procedure may reduce anyone's ability to decline.
  R-6  R1 STANDS. The Moment equation is demoted on scale; D/P is ordinal only.
  R-7  MAGNITUDE ESTIMATION IS REQUIRED before any relationship number is printed.
   R-8   NO SINGLE-PARTY TERM. No reading is a verdict on a person.
  R-9  PRECEDENT IS NOT EVIDENCE. The (-1) column carries independent arrival, not proof.
  R-10 TESTIMONIALS ARE EVIDENTIALLY INERT. Likelihood ratio exactly 1.000000.
    R-11  THE OPEN PROBLEM STANDS. No established mechanism for sequence-
specific
          acoustic demodulation.
   R-12  EVERY CASE IN THREE BOOKS IS CONSTRUCTED AND LABELLED AS SUCH.
```

30.17 What may be claimed

The list is short and it is the honest one.

That the join is exact. Given the essence bridge and the storage fixed point, `f_min = k e^(2c/eta)` follows by substitution. Nothing was added to obtain it and it can be checked in three lines.

That the diagnostic is computable and actionable. Given `u`, `eta`, `c`, `delta` and `N`, the deficit, the destiny, the approach time and the four lever reserves are all determined, and the procedure says which lever binds and which is absent.

That the constraints are real. The six results of §30.12 are algebraic identities or verified computations, not interpretations, and each of them can be reproduced from the formulas in §30.15.

That the apparatus catches its own author. R1 demoted the volume's own keystone at the gate its author wrote. R15 corrected the design intuition the series had been running on. Finding 19.2 audited a tradition the corpus draws from and found its canonical sequence inverted. Corollary 20.2 proved a row of the source table impossible.

And what may not be claimed is everything else. In particular: that any of this describes any actual person, pair or group; that a larger fixed point is a better life; **that any frequency here is a physical vibration; that belief moves anything through a channel not named and estimated independently of the outcome; or that the absence of a known mechanism is evidence of anything whatever, in either direction.**

The Failure Modes

FM 30.1 — THE KEYSTONE QUOTED WITHOUT ITS INPUTS. `f_min = k e^(2c/eta)` reported as a result about relationships in general. It is a result about an arrangement whose `c` and `eta` were measured, and it inherits their defensibility exactly.

FM 30.2 — THE OCTAVE FIGURE TAKEN AS AUDIO. `2.885390` octaves is an exchange rate between nats and a logarithmic scale. No sound is implied, none is prescribed, and R-1 binds.

FM 30.3 — THE CEILING IGNORED. A lever plan produced for an arrangement at `c/eta > ln N`. Worked in Chapter 29 at `c/eta = 3.000000`: even perfect orthogonality leaves `+1.486245` octaves, and twenty-one categories are required.

FM 30.4 — `D` WITHOUT `delta`. The single most common incomplete reading in the book. The same `D` is consistent with destinies differing by a factor of thirty.

FM 30.5 — THE TWO FREQUENCY READINGS CONFLATED. Inferring a relationship reading from state frequencies, or the reverse. Six constructed pairs spanning 1565 to 5208 cents return identical `u` and `f_rel` to twelve decimals.

FM 30.6 — THE GROUP JUSTIFIED ON SURPLUS. An orthogonal pair already attains `ln N`. Make the roster's case on conversion, coverage or continuity, which are real and outside this reading.

FM 30.7 — MANUFACTURED DIFFERENCE. `u` depends on `|rho|`; opposition reads `u = 0` exactly as agreement does, while the cost rises.

FM 30.8 — PRECEDENT PROMOTED TO EVIDENCE. The (-1) column establishes independent arrival, which is worth having and is not proof. Two sources agreeing does not double the evidence when only one of them is evidence.

FM 30.9 — TESTIMONIALS. Likelihood ratio exactly `1.000000` at any corpus size. A thousand accounts carry the weight of zero.

FM 30.10 — THE OPEN PROBLEM SOFTENED. Any phrasing that converts "no established mechanism" into "an emerging area", "not yet fully understood", or "preliminary evidence suggests". The sentence is about what is known, and it is not a prediction.

FM 30.11 — THE ABSENCE OF MECHANISM READ AS PERMISSION. The symmetric error. Not proven impossible is not partial support. A claim with no mechanism and no demonstration has zero evidential standing, and the correct number of qualifiers before demonstration is not "few" but "it is not asserted".

Worked Example

30.18 The smallest complete instance

CONSTRUCTED. Not an observation of any person or pair. Two parties, twelve categories, one measured geometry and three declared parameters. Every line below is derivable from §30.15 and nothing else.

```
INPUTS      u = 0.35    eta = 0.50    c = 0.70    delta = 0.12    N = 12

SURPLUS      F      = ln(1 + 11 * 0.35)  =  1.578978705 nats
             N_eff  = 1 + 11 * 0.35      =  4.850000

PITCH        f_rel  = k * N_eff^2        =  11.371381804 Hz
```

```
    THRESHOLD    f_min  = k * e^(2*0.70/0.50) =  7.949765419 Hz

   CEILING     c/eta = 1.400000  vs  ln N = 2.484907       ->  UNDER, tunable

     DEFICIT        D        = 2.885390 * (1.400000 - 1.578978705)   =   -
0.516423380 octaves
                                                                  SUSTAINABLE

   DESTINY     Q_inf  = (0.50 * 1.578978705 - 0.70) / 0.12   =  +0.745744604
               check   D = -2.885390 * (0.12/0.50) * Q_inf   =  -0.516423380
                 residual                                          0.00e+00

    APPROACH     half-life = ln(0.5)/ln(0.88)  =   5.422271 passes
                 t(1%)     = ln(0.01)/ln(0.88) =  36.024800 passes

    RESERVES     R_u   = 2.885390 * (ln 12 - F)          =  2.613956 octaves
                 R_eta = 2.885390 * c * (1/eta - 1)      =  2.019773 octaves
                 R_c   = 2.885390 * (c/eta)              =  4.039546 octaves
           R_N  = 2.885390 * u / N_eff        =  0.208224 octaves per category

    EXPORT TEST  single arrangement, one drive: b = 0.089486952
              with any second participant, compare sum_i b_i before and after

    LABEL        CONSTRUCTED
   REFUSALS    essence not resonance; no clinical claim; consent structural;
                 two readings reported separately; D reported with delta
```

Read it as a whole and notice how little it says. It says this configuration of four numbers accumulates stored order, that it is currently 0.52 octaves above its own threshold, that it would take about thirty-six passes to settle, and that if the deficit ever went positive the cost side has four octaves of room and the geometry side two and a half. That is the entire content, it is reproducible from five formulas, and everything a reader might additionally want to know about the two people in question is outside it.

The gap between what the reading contains and what a reader wants is not a defect to be closed in a later edition. It is the boundary of the instrument, and the whole discipline of three books has consisted of drawing it in the right place and then not stepping over it.

30.19 The second instance — an arrangement the instrument refuses

A worked example that only shows a reading coming out well is a worked example that has not demonstrated the instrument. Here is the same procedure run on the constructed commons of SPINE §3.3, where it produces a refusal rather than a plan.

CONSTRUCTED. Not an observation of any group. Six members drawn from one doctrine, so their state vectors are near-parallel by construction — the Parallel Wall, priced. Fifteen pairs at `kappa = 0.055` per link.

```
    INPUTS       mean pairwise u = 0.018614     eta = 0.34
                 C = 0.055 * 15 = 0.825         delta = 0.09      N = 12
```

```
 SURPLUS      F      =  0.186272005 nats        N_eff = 1.204754
 PITCH        f_rel  =  0.701659675 Hz
 THRESHOLD    f_min  = 61.934885399 Hz

CEILING     c/eta = 2.426471  vs  ln N = 2.484907   ->  passes, by 0.058436

 DEFICIT      D     = +6.463836987 octaves          SHORT BY 6.46 OCTAVES
 DESTINY      Q_inf  = -8.462959612                   DECAYS
              drive b = -0.761666365 per pass
              check   D = -2.885390 * (0.09/0.34) * Q_inf = +6.463836987

 APPROACH     half-life 7.349615 passes; 99% of the way down by pass 48.83

 RESERVES     R_u   = 6.632448 octaves    exceeds the deficit by 0.168611
            R_eta = 4.620867 octaves    does not close it, even at eta = 1
             R_c   = 7.001314 octaves    exceeds the deficit, at zero cost
              R_N   = 0.044581 octaves per added category

SINGLE-LEVER raise u to 0.95     -> deficit -0.0332   at near-orthogonality
              raise eta to 0.90    -> deficit +2.1075   still failing
              cut kappa to 0.020   -> deficit +2.0085   still failing

 ROSTER       m(m-1) = 30  against  2 eta ln N / kappa = 30.722482
        a seventh member takes it to 42 and makes the case structural outright

 VERDICT      THE PROCEDURE HALTS.
 LABEL        CONSTRUCTED
```

Read the reserve block and the single-lever block together, because they say different things and the difference is the whole lesson. The reserves say two levers close the deficit in principle. The single-lever table says what those in-principle closures actually require: six people who share almost no state at all, or coordination at zero cost. Neither of those describes a commons, so the arithmetic that says the case is closeable and the arithmetic that says it is not are both correct, **and the second one is the one a planner has to act on.**

Three things this refusal is not. It is not a recommendation that the group disband — the formalism prices stored order and nothing else, and a family, a team or a commons is for a great many things the formalism does not represent. It is not a verdict on any member, because there is no single-party term anywhere in the apparatus. And it is not a claim that nothing can be done; it is a claim that nothing can be *tuned*, which is a narrower and more useful statement. The remedy available to this arrangement is restructuring — a different roster, a different cost structure, or a description with genuinely more categories in it — and every one of those is a change of what the arrangement is rather than an adjustment of how it is run.

A book that only showed fixable examples would be lying by selection, and the commons is carried through five chapters of this volume for exactly that reason. The instrument's most valuable output is the one that costs nothing to produce and saves a year: *this cannot be tuned, and here is the division that shows it.*

The Open Problem

30.20 Where the work stops

Part V of this book specified a complete procedure. Chapter 25 built the ethical architecture and stated the no-subliminals constraint absolutely. Chapter 26 gave the encoder, the addressing convention, the sonifier, the transposition, the rendering within the Gabor bound and the `N <= 25` single-transposition capacity, with preregistration and falsification criteria attached. All of it is deterministic and all of it was verified.

And Chapter 27 says that nobody knows how a cell could read a message out of a sound.

> **There is no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal.**

The gap decomposes into three requirements. Symbol separation is solved — the alphabet separates, and that is demonstrated. Order memory across the symbol interval has no identified mechanism. Mapping from a decoded symbol string to sequence-specific binding at a genomic locus has no identified mechanism. Eleven upstream components are established and one link is missing, and the missing link is specifically the carriage of sequence over force.

The operational form is the one that matters, because it is what an experiment would have to do. Take a stimulus and its symbol-permuted twin. The two have identical total energy, identical duration, identical long-term power spectrum and identical symbol counts. If a cell cannot be shown to distinguish them, no sequence-specific channel has been demonstrated, and nothing else in the experiment matters.

Two symmetric errors sit on either side of that sentence and both are refused. **It is not a claim of impossibility** — absence of a known mechanism is a statement about the literature, and the burden of demonstration sits entirely on the claimant, which is the ordinary position for any unestablished claim. **Nor is it a licence** — "not proven impossible" is not partial support, and a claim with no mechanism and no demonstration has zero evidential standing.

30.21 Why the book ends here

Part V could have ended at Chapter 26 with a complete, verified, disclosed procedure and a reader's impression that something had been achieved. Something has been achieved — an encoder that round-trips, an alphabet that separates, a rendering a listener can follow and refuse — and none of it is the thing the Part is named for.

The same choice is available to this chapter and it is declined for the same reason. A closing chapter is where a corpus is most tempted to convert its constraints into a narrative of arrival, because the reader is at the end and the material is fresh and a summary is easy to write in the key of achievement. **This one ends on the gap instead, because the gap is the load-bearing item and putting it last is the only placement in which it cannot be forgotten.**

There is a second open question and it belongs beside the first, because they fail in opposite directions. Book Two's closing chapter asked whether the storage law is closed under any natural aggregation operator — whether a population of systems each obeying the law, combined by that

operator, yields a system obeying the law with derived parameters. If such an operator exists, the scale correspondence stops being a modelling convention and becomes a theorem. If none exists, the correspondence is exactly what Book Two Chapter 88 says it is and nothing more, and the volumes are three separate applications of one equation that happen to look like a ladder. Which of those is true is not known here, and it requires no laboratory — only somebody to define the operator and try to prove closure, and to publish the result **including, especially, if the result is that the law is not closed.**

So: two problems. One needs an experiment and one needs an afternoon with a proof. Neither has been done, and the series stops at both of them rather than past them.

30.22 What is on the bench

Three books produced one substitution, one diagnostic, one definition of destiny, one conservation identity, six constraints that narrow the instrument, a catalogue of failure modes long enough to be embarrassing, and a list of refusals that is longer than the list of claims.

The claims are these, and there are no others. The join is exact. The diagnostic is computable from measurements a practitioner can take. The deficit and the fixed point are one number in two units. The deficit cannot see decay, so both must be reported. Cost export is detectable on the first pass by an arithmetic test on a conserved sum. A destiny has four arguments and changing one is the only way to change it. Belief acts through channels with estimable gains and does nothing where no channel exists. Testimonial evidence has a likelihood ratio of exactly `1.000000`.

Everything else in three books is either a badged postulate, a chosen constant, a constructed example, a rejection, or a statement of what is not known.

And the sentence at the head of Chapter 27 is exactly as true at the end of this book as it was at the beginning of that one. Nothing here changes it, nothing here claims to, and the parts of the corpus that are correct are correct independently of how it comes out.

The instrument is on the bench. It reads one gauge, it refuses more than it asserts, it has caught its own author four times in three volumes, and it stops at a gap that will be settled by an experiment rather than by an argument.

Anyone may take it further. The only condition this house asks is the one it has tried to keep for three books: say which claims are proved, badge the ones that are not, name the ones that are open, and publish the demotions when the gates catch you.

They will.

Chapter 30 closes Book Three and the Creature Comforts series. The keystone is CANON3 §1 and Chapter 4; the transfer rules are CANON3 §2; the diagnostic is CANON3 §3 and Chapter 4; destiny is CANON3 §4 and Chapter 28; the levers are Chapters 4, 15, 18 and 29; the six constraints are Chapters 10, 14, 16, 21 and 4, with Book Two Chapter 90's results (a) and (b); the belief postulate and its coupling requirement are Book Two Chapters 76 and 77, and R23 is Book Two Chapter 80; the open problem is Chapter 27 of this book and Chapter 68 of Book Two. The appendices carry the reference card, the tables, the protocols, the concordance register including every rejection, the honesty appendix, and the correspondence to Books One and Two.

Appendices

These six appendices are the book's reference layer: every law the thirty chapters use, every table needed to reproduce a reading, every protocol by which a reading's inputs are obtained, the concordance register with its rejections, the honesty accounting on which the volume's warrant rests, and the correspondence back to Books One and Two.

Nothing here is new and everything here is computed. Where a number differs in its last digit from a figure quoted in a chapter, SPINE §3.4's precision rule applies: recompute from the full-precision input, never from a rounded one. A reference card invites the reader to skip the argument, and the argument is where the limits live, so every card below carries its limits inline. One warning belongs at the front rather than the back: **this book contains no clinical claim, no diagnostic instrument, and no demonstrated physical mechanism, and its central instrument is blind to one of the four parameters that decide a relation's fate.**

Appendix A — The Frequency Reference Card

Printable: one page of constants, one of laws, one of conversions, one of clauses. A reader with the four pages and nothing else can compute every reading in this book and state correctly what the reading does not mean.

A.1 The constants

```
THE GROUND ESSENCE

 k  =  (9/8)^2 / phi^2              phi = (1 + sqrt 5)/2 = 1.6180339887498949

   k  =  0.483425732988414 Hz              (computed: 0.48342573298841435)

   k^2 = 0.2337004393                      (computed: 0.23370043931538567)

THE CATEGORY COUNT USED THROUGHOUT THIS BOOK

    N       = 12
   ln N    = 2.484907                       (computed: 2.4849066497880004)
   k N^2   = 69.6133 Hz                      (computed: 69.61330555033166)
```

```
DERIVED CONSTANTS THAT RECUR

  1/sqrt(k)    = 1.438252434444351     Corollary 20.2 crossing; not an integer
     2 / ln 2       = 2.8853900817779268      octaves per nat
     1 / (4 pi)     = 0.07957747154594767     the Gabor product
  32 / (4 pi)   = 2.5464790894703255    cycles per resolved step in the 50/49 regime
```

`phi` is stored to full double precision throughout. Book Two's Chapter 29 errata traced a residual bias in the published element table to `phi` held at fifteen significant figures: carry the constant, not its printed form.

k sets the reference pitch and is invisible to every verdict. Chapter 4 §4.5 (iii) proves it: the octave deficit is a difference of logarithms in which k appears in both terms and cancels. Only absolute hertz values depend on the constant. Sustainability verdicts do not.

A.2 The three transfer rules

```
RULE 1 -- A PSYCHOLOGICAL ESSENCE HAS A FREQUENCY          owner: Ch 5, Ch 7

  f  =  k * n^2         n = the number of distinguishable components required
                                to specify the state

     n is a COUNT: true zero, equal units, ratio-scaled by construction.
     The frequency inherits exactly the defensibility of the count.

RULE 2 -- A PSYCHOLOGICAL RELATIONSHIP HAS A FREQUENCY      owner: Ch 12

     cos theta = x . y         x, y unit vectors over the N states
     u         = 1 - (x . y)^2
     F         = ln(1 + (N-1) u)          = ln(1 + 11 u) at N = 12
     f_rel     = k * e^(2F)  =  k * (1 + 11 u)^2  =  k * N_eff^2

     u = 0  ->  f = k, the ground essence: no relationship at all
     u = 1  ->  f = k N^2 = 69.6133 Hz, the ceiling

RULE 3 -- A COMPOUND STATE USES THE ALCHEMICAL RULE         owner: Ch 20

  f_compound  =  k^2 * G^4      G = count-weighted geometric mean of indices
                                   G = (prod n_i^{m_i})^(1/T),  T = sum m_i

     Values are convention-Hz and are dimensionally Hz^2. They may not be rendered
     as hertz and may not enter an acoustic calculation.
```

Rule 3 is not a generalisation of Rule 1 and the arithmetic says so twice. For a single component `G = n` and `f_compound = k^2 n^4 = f(n)^2`, which is Theorem 20.1; and **Corollary 20.2 shows the two rules agree at no index whatever, since `k n^2 = k^2 n^4` requires `n = 1/sqrt(k) = 1.438252`, which is not an integer.** At `n = 12` the two rules differ by 6.121291 octaves. The single-component index-independence of Rule 3's proportional structure is exactly what condemned the source table's O2 row in Book Two's Chapter 29 errata.

Worked compounds, recomputed here to ten places for citation:

```
    multiset                T        P           G                 f = k^2 G^4
     ------------------------------------------------------------------------
--------
    {1x1, 2x1}              2        2     1.4142135624             0.9348017573
    {1x3, 12x1}             4       12     1.8612097182             2.8044052718
    {1x1, 12x1}             2       12     3.4641016151            33.6528632614
    {2x2, 7x1}              3       28     3.0365889719            19.8702609489
    {3x1, 5x1}              2       15     3.8729833462            52.5825988460
    {4x1, 6x1, 9x1}         3      216     6.0000000000           302.8757693527
    {5x1, 10x1}             2       50     7.0710678119           584.2510982885
    {11x1, 12x1}            2      132    11.4891252931          4071.9964546313
     ------------------------------------------------------------------------
--------
```

Rule 3 is blind to scale and acutely sensitive to ratio: trebling every multiplicity leaves G unchanged, while two multisets differing only in how many index-1 components are present sit 3.584963 octaves apart.

A.3 The keystone

```
    THE MINIMUM SUSTAINABLE FREQUENCY                          owner: Ch 4

        f_min  =  k * e^( 2c / eta )

        f  >  f_min      the relation ACCUMULATES order       Q_inf > 0
        f  =  f_min      it HOLDS, and nothing more            Q_inf = 0
        f  <  f_min      it DECAYS, whatever anyone intends    Q_inf < 0

    THE DERIVATION, IN THREE STEPS

        sustainability   Q_inf > 0   iff   eta F > c   iff   F > c/eta
        the bridge       F = (1/2) ln( f / k )
        substitute       f > k e^(2c/eta)
```

`f_min` contains no `delta` and no N, and both absences are substantive: the first is the source of the delta-blindness treated in A.4, and the second means the threshold is set by cost and conversion alone, N entering only through the ceiling on what F can reach.

In octaves above the ground essence:

```
    octaves required  =  2 (c/eta) / ln 2  =  2.885390 * (c/eta)
```

Every unit of cost-to-conversion ratio costs 2.885390 octaves of required pitch, and there is no other exchange rate.

A.4 The octave deficit, and the two theorems that govern it

```
   THE OCTAVE DEFICIT                                  owner: Ch 4, Def 4.4
```

```
        D  :=  2.885390 * (c/eta)  -  log2( f / k )

        D <= 0    sustainable as it stands
        D  > 0    the relation is short by D octaves

    THEOREM 4.5 -- THE DEFICIT IDENTITY

        D  =  2.885390 * ( c/eta  -  F )  =  -2.885390 * (delta/eta) * Q_inf

        Q_inf  =  ( eta F  -  c ) / delta            Paper E Theorem 3.5
        F      =  ln( 1 + 11 u )
```

Verified on the carried dyad as found: `D = 4.338420197297714` by the definition and `4.338420197297713` by both identities, a worst residual of `8.88e-16`. SPINE §3.5 records independent verification across twelve parameter sets to `2.665e-15`.

```
    THEOREM 4.9 -- THE DEFICIT CONTAINS NO delta

        D  =  2.885390 * (c/eta) - log2(f/k)     contains k, c, eta, N, u
                                                 does NOT contain delta
```

This is the card's most important qualification and the one most easily lost when the card is used without the chapter. Worked on the carried dyad at `eta = 0.30`, `c = 0.62`:

```
    delta = 0.09   ->   Q_inf = -5.0119395464      D = +4.3384201973
    delta = 0.05   ->   Q_inf = -9.0214911835      D = +4.3384201973
```

Trust nearly doubles the negative fixed point and the deficit does not move at all. The deficit reads the sign and size of the sustainability gap; `delta` converts that gap into a level. A reading that reports `D` without `delta` cannot see decay, and it is incomplete. Report both, always.

A.5 The ceiling, the minimum category count, and the hold line

```
   THE CEILING        Paper E Cor 3.4 in the frequency register     owner: Ch 4 Thm 4.6

       Since F <= ln N,       c/eta  <=  ln N       and      f_min  <=  k N^2

     At N = 12:  c/eta may not exceed 2.484907, i.e. f_min may not exceed 69.6133 Hz.
      Above that, NO arrangement of those twelve categories is sustainable --
        not a better pairing, not a better schedule, nothing.
     The only remedies: raise eta, cut c, or add genuinely distinct categories.

   THE MINIMUM CATEGORY COUNT                                  owner: Ch 4 Cor 4.7

       N  >=  e^(c/eta)

   THE HOLD LINE                                               owner: Ch 4 Cor 4.8

       u*  =  ( e^(c/eta) - 1 ) / (N - 1)
```

The ceiling test comes first. A deficit computed above the ceiling is a number with no verdict attached: no arrangement of the declared category set can close it.

A.6 The four levers

```
    LEVER              acts on            partial derivative of D
     ------------------------------------------------------------------
------
   L1  raise u       the geometry      dD/du   = -2.885390 (N-1) / (1 + (N-1)u)
    L2  raise eta       the conversion     dD/deta  = -2.885390 c / eta^2
    L3  cut c           the cost           dD/dc    = +2.885390 / eta
    L4  raise N       the category set   dD/dN    = -2.885390 u / (1 + (N-1)u)
```

These are the only four, and `delta` is not one of them (Theorem 4.9). `delta` moves the level, not the gap.

```
    THE ORDER-OF-OPERATIONS RULE (R15)                          owner: Ch 15

        FIX THE NUMERATOR FIRST.

        Trust acts by LOWERING delta. Q_inf = (eta F - c)/delta.
      When the numerator is negative, dividing by a smaller positive denominator
       makes the result MORE negative. Trust amplifies whatever sign the system
        already has; it is not a source of order.
```

A.7 The agreement identity

```
    THEOREM 13.1                                                owner: Ch 13

        Two parties judge m shared items, each judgement coded +1 or -1.
        Let a = fraction of items on which the judgements match. Then

            cos theta  =  2a - 1              u  =  4 a (1 - a)

        Exact; no centring required, since +/-1 vectors have norm sqrt(m).
        Resolution limit:  du = 4 |1 - 2a| / m,  bounded by 4/m.
```

The identity is symmetric about `a = 0.5` and returns `u = 0` at both `a = 1` and `a = 0`, which is R8 appearing directly in the measurement channel: the sycophant and the contrarian sound the identical note. Two parties answering at random have expected `a = 0.5`, hence `u = 1`, hence the ceiling of the whole system from guessing. The formalism cannot distinguish independence from noise, because both are orthogonality, and the guard is procedural: publish each party's accuracy with any u.

A.8 The Gabor limit

```
   dt * df  >=  1 / (4 pi)  =  0.07957747154594767          owner: Book Two Ch 71

   Classical Fourier duality. No quantum content. Measurable on an oscilloscope.
```

```
THE FOUR DESIGN LAWS
 (1) single-transposition audible capacity is N <= 25, and only at j = 6
 (2) the folded-lattice minimum-gap cliff is at N = 17 (50/49 -> 289/288),
      the second at N = 41, and the RATE factor is exactly 8
 (3) resolving a surplus difference dF costs at least 1/(8 pi dF) cycles
 (4) the rectangular gate is disqualified: no finite spectral second moment
```

N = 12 sits below the N = 17 cliff, so **this entire book renders in the 50/49 regime, at a minimum separation of 34.9756 cents and 2.546479 cycles per resolved step.** A 40-cent deficit is just above the minimum resolvable step of the lattice it is expressed in; a 6-cent one would not be.

A.9 Unit conversions

```
1 nat    =  1.442695 bits    =  2.885390 octaves  =  3462.4681 cents
1 bit    =  2 octaves EXACTLY                     =  2400 cents
1 octave =  1200 cents

computed:  1/ln2   = 1.4426950408889634
           2/ln2   = 2.8853900817779268
           2400/ln2 * (ln2/2) ... 1200 * 2/ln2 = 3462.4680981335123

D in octaves  =  2.885390 * (c/eta - F)
D in cents    =  3462.4681 * (c/eta - F)
D in bits     =  1.442695 * (c/eta - F)
```

The bit-to-octave conversion is exact and not a coincidence: frequency enters as `f = k e^(2F)`, so one nat is two e-folds of frequency and one bit — being `ln 2` nats — is two doublings, which is two octaves. **The four registers of Book Two's conversion keys converge here because they are all logarithms of the same effective count, taken to different bases.** The carried dyad as found reads `+4.3384 octaves = +5206.1 cents`; after its intervention, `−3.5910 octaves = −4309.2 cents`.

A.10 The clauses, which are part of the card and not an afterword

```
C1  Essence frequencies are ESSENCE frequencies, not physical resonances. No
    person, tissue or molecule is claimed to resonate at any value in this book.

C2  No clinical, diagnostic or therapeutic claim. The twelve states are not
    diagnoses; the readings are not assessments.

C3  No intervention may remove a person's ability to decline. Consent is
    structural, not advisory.

C4  No subliminals. Nothing masked, nothing below threshold, nothing concealed.

C5  The open problem stands: no established mechanism for sequence-specific
    acoustic demodulation.
```

C6 All worked cases are CONSTRUCTED and labelled as such.

C7 R1 binds D/P specifically. It does not bind the count index n, and it does not bind magnitude-estimated intensities. State which quantity you mean.

C8 Report D AND delta. The deficit alone cannot see decay (Theorem 4.9).

C9 A state frequency and a relationship frequency are independent readings and neither may be inferred from the other (SPINE §3.6, Ch 21 Finding 21.1).

C10 (+1) carries evidence; (−1) carries precedent. Precedent is never promoted.

A.11 The card, on one page

```
================================================================================
 THE SUSTAINING FREQUENCY -- REFERENCE CARD
================================================================================
 k = 0.483425732988414 Hz = (9/8)^2/phi^2        k^2 = 0.2337004393
 N = 12     ln N = 2.484907     k N^2 = 69.6133 Hz
 --------------------------------------------------------------------------
----
 STATE          f = k n^2                        n is a COUNT
 RELATION       f_rel = k (1 + 11 u)^2           u = 1 - (x.y)^2
                F = ln(1 + 11 u)
 COMPOUND      f_compound = k^2 G^4          convention-Hz, never rendered
 --------------------------------------------------------------------------
----
 KEYSTONE       f_min = k e^(2c/eta)
 OCTAVES REQ    2.885390 * (c/eta)
 DEFICIT        D = 2.885390*(c/eta) - log2(f/k)
 IDENTITY       D = 2.885390*(c/eta - F) = -2.885390*(delta/eta)*Q_inf
 DELTA-BLIND    D contains no delta.  REPORT D AND delta.
 DESTINY        Q_inf = (eta F - c)/delta
 CEILING       c/eta <= ln N          HOLD LINE  u* = (e^(c/eta)-1)/(N-1)
 LEVERS        raise u | raise eta | cut c | raise N.  delta is not a lever.
 ORDER          fix the numerator first (R15)
 AGREEMENT      u = 4a(1-a),  cos theta = 2a - 1,  resolution 4/m
 GABOR          dt df >= 1/(4 pi) = 0.0795774715
 UNITS         1 nat = 1.442695 bits = 2.885390 octaves = 3462.4681 cents
                1 bit = 2 octaves exactly
 --------------------------------------------------------------------------
----
 NOT a resonance. NOT a diagnosis. NOT a capability. Consent is structural.
 Nothing masked. The open problem stands. All cases constructed.
================================================================================
```

Appendix B — Frequency Tables

Every table here was computed, not asserted. Where exact arithmetic matters — §B.6's minimum-gap analysis — the computation used exact rationals via `fractions.Fraction`, because the quantities compared differ in the fourth decimal place of a cent.

B.1 The twelve-state ladder

This is the ladder exactly as SPINE §1 gives it, and it is the only ladder this book uses. The folded column transposes each raw frequency by whole octaves into `[220, 440)`; the pitch class is the odd part of the index, by the odd-part theorem of Chapter 7.

n	state class	the component it adds	f = k n^2 (Hz)	folded 220-440	pitch class
1	undifferentiated activation	arousal alone	0.483426	247.5140	1
2	valenced activation	+ valence	1.933703	247.5140	1
3	directed feeling	+ intentional object	4.350832	278.4532	3
4	situated feeling	+ situation	7.734812	247.5140	1
5	attributed feeling	+ cause	12.085643	386.7406	5
6	temporal feeling	+ projected duration	17.403326	278.4532	3
7	reflexive feeling	+ self as object	23.687861	379.0058	7
8	social feeling	+ another mind modelled	30.939247	247.5140	1
9	normative feeling	+ standard applied	39.157484	313.2599	9
10	narrative feeling	+ placed in a story	48.342573	386.7406	5
11	reciprocal feeling	+ the other modelling you	58.494514	233.9781	11
12	committed feeling	+ future action bound	69.613306	278.4532	3

Twelve states, six distinct pitch classes `{1, 3, 5, 7, 9, 11}` — `ceil(12/2)`, as the odd-part theorem requires. Indices 1, 2, 4 and 8 share class 1 and fold to the identical audible frequency. **A folded reading that reports six notes from a twelve-state ladder has lost no information it was entitled to: the collision is a property of the arithmetic, not of the fold.**

The six classes, reduced to one octave and sorted by pitch:

class	folded ratio (exact)	cents above PC1	step to next
PC1	1	0.00	203.91
PC3	9/8	203.91	203.91
PC9	81/64	407.82	329.83
PC7	49/32	737.65	34.98
PC5	25/16	772.63	330.01
PC11	121/64	1102.64	97.36

steps sum to 1200.00 cents; the scale closes at the octave, as it must

PC7 and PC5 sit 34.98 cents apart — the 50/49 minimum gap of the N = 12 regime, appearing inside the ladder itself. Sounded together they beat rather than form an interval.

B.2 f = k n^2 for n = 1 to 25, raw and folded

Twenty-five is the single-transposition audible capacity of §B.7, and a table running past it would tabulate frequencies no one integer transposition can render.

n	f raw (Hz)	j	folded [220,440)	odd part
1	0.483426	+9	247.5140	1
2	1.933703	+7	247.5140	1
3	4.350832	+6	278.4532	3
4	7.734812	+5	247.5140	1
5	12.085643	+5	386.7406	5
6	17.403326	+4	278.4532	3
7	23.687861	+4	379.0058	7
8	30.939247	+3	247.5140	1
9	39.157484	+3	313.2599	9
10	48.342573	+3	386.7406	5
11	58.494514	+2	233.9781	11
12	69.613306	+2	278.4532	3
13	81.698949	+2	326.7958	13
14	94.751444	+2	379.0058	7
15	108.770790	+2	435.0832	15
16	123.756988	+1	247.5140	1
17	139.710037	+1	279.4201	17
18	156.629937	+1	313.2599	9
19	174.516690	+1	349.0334	19
20	193.370293	+1	386.7406	5
21	213.190748	+1	426.3815	21
22	233.978055	0	233.9781	11
23	255.732213	0	255.7322	23
24	278.453222	0	278.4532	3
25	302.141083	0	302.1411	25

Index 17 folds to 279.4201, six cents from index 3's 278.4532. That is the 289/288 cliff arriving,

and §B.6 prices it exactly.

B.3 `f_min` against `c/eta`

The keystone table. Column three is `log2(f_min/k)`, which is the octave form; column four is the closed-form `2.885390 (c/eta)`. They agree to the printed precision at every row, which is the verification.

c/eta	f_min (Hz)	octaves	2.885390∗(c/eta)	
0.00	0.483426	0.000000	0.000000	
0.10	0.590458	0.288539	0.288539	
0.20	0.721186	0.577078	0.577078	
0.30	0.880859	0.865617	0.865617	
0.40	1.075884	1.154156	1.154156	
0.50	1.314087	1.442695	1.442695	
0.60	1.605030	1.731234	1.731234	
0.70	1.960388	2.019773	2.019773	
0.80	2.394423	2.308312	2.308312	
0.90	2.924555	2.596851	2.596851	
1.00	3.572060	2.885390	2.885390	
1.10	4.362924	3.173929	3.173929	
1.20	5.328887	3.462468	3.462468	
1.30	6.508717	3.751007	3.751007	
1.40	7.949765	4.039546	4.039546	
1.50	9.709865	4.328085	4.328085	
1.60	11.859656	4.616624	4.616624	
1.70	14.485417	4.905163	4.905163	
1.80	17.692528	5.193702	5.193702	
1.90	21.609703	5.482241	5.482241	
2.00	26.394151	5.770780	5.770780	
2.10	32.237888	6.059319	6.059319	
2.20	39.375446	6.347858	6.347858	
2.30	48.093278	6.636397	6.636397	
2.40	58.741263	6.924936	6.924936	
[2.484907	69.613306	7.169925	7.169925]	<-- THE CEILING at N = 12
2.50	71.746740	7.213475	7.213475	
2.60	87.631666	7.502014	7.502014	
2.70	107.033559	7.790553	7.790553	
2.80	130.731084	8.079092	8.079092	
2.90	159.675307	8.367631	8.367631	
3.00	195.027860	8.656170	8.656170	

The bracketed row is not a boundary a better arrangement can cross: at `c/eta = 2.484907` the required non-parallelism is exactly 1 — perfect orthogonality — and there is nothing above it.

The companion table is the hold line, stating the same threshold in the geometry rather than the

cost:

c/eta	u∗ = (e^(c/eta)−1)/11	f_min (Hz)
0.000000	0.000000	0.483426
0.250000	0.025820	0.797034
0.500000	0.058975	1.314087
0.750000	0.101545	2.166564
1.000000	0.156207	3.572060
1.250000	0.226395	5.889331
1.500000	0.316517	9.709865
1.750000	0.432237	16.008862
2.000000	0.580823	26.394151
2.250000	0.771612	43.516598
2.484907	1.000000	69.613306

And the conversion between the two gauges, at the carried case's `delta/eta = 0.30`:

D (octaves)	f / f_min	Q_inf
−6.0	64.0000	+6.9315
−4.0	16.0000	+4.6210
−2.0	4.0000	+2.3105
−1.0	2.0000	+1.1552
0.0	1.0000	0.0000
+1.0	0.5000	−1.1552
+2.0	0.2500	−2.3105

This table is readable only because `delta/eta` was declared. Change `delta` and every `Q_inf` in the right-hand column moves while every `D` in the left-hand column does not — Theorem 4.9 in tabular form, and the reason the ratio sits in the caption.

B.4 The interval lattice

Intervals between states are ratios of squares, `f_m/f_n = m^2/n^2`, and k cancels. In cents:

	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=11	n=12
m= 1	0.00	−2400.00	−3803.91	−4800.00	−5572.63	−6203.91	−6737.65	−7200.00	−7607.82	−7972.63	−8302.64	−8603.91
m= 2	2400.00	0.00	−1403.91	−2400.00	−3172.63	−3803.91	−4337.65	−4800.00	−5207.82	−5572.63	−5902.64	−6203.91
m= 3	3803.91	1403.91	0.00	−996.09	−1768.72	−2400.00	−2933.74	−3396.09	−3803.91	−4168.72	−4498.73	−4800.00
m= 4	4800.00	2400.00	996.09	0.00	−772.63	−1403.91	−1937.65	−2400.00	−2807.82	−3172.63	−3502.64	−3803.91
m= 5	5572.63	3172.63	1768.72	772.63	0.00	−631.28	−1165.02	−1627.37	−2035.19	−2400.00	−2730.01	−3031.28

```
 m= 6   6203.91   3803.91   2400.00   1403.91    631.28      0.00   -533.74   -
996.09 -1403.91 -1768.72 -2098.73 -2400.00
 m= 7   6737.65   4337.65   2933.74   1937.65   1165.02    533.74      0.00   -
462.35  -870.17 -1234.98 -1564.98 -1866.26
 m= 8  7200.00  4800.00  3396.09  2400.00  1627.37   996.09   462.35    0.00  -
407.82  -772.63 -1102.64 -1403.91
 m= 9  7607.82  5207.82  3803.91  2807.82  2035.19  1403.91  870.17  407.82    0.00 -
364.81  -694.82  -996.09
 m=10  7972.63  5572.63  4168.72  3172.63  2400.00  1768.72  1234.98  772.63  364.81    0.00
330.01  -631.28
 m=11  8302.64  5902.64  4498.73  3502.64  2730.01  2098.73  1564.98  1102.64  694.82  330.01
301.27
 m=12  8603.91  6203.91  4800.00  3803.91  3031.28  2400.00  1866.26  1403.91  996.09  631.28
```

Three entries earn comment. `m/n = 2` gives exactly 2400 cents, because doubling an index quadruples a squared frequency — the odd-part theorem in interval form, and the reason indices 1, 2, 4 and 8 share a class. The 7:5 entry is 1165.02 cents, 35 cents shy of an octave, and is the pair that sets the `50/49` minimum gap. The 12:11 entry, 301.27 cents, is the smallest non-collapsing interval at the top of the ladder.

B.5 Pitch-class counts

```
N          1  2  3  4  5  6  7  8  9 10 11 12 13 ... 25
ceil(N/2)  1  1  2  2  3  3  4  4  5  5  6  6  7 ... 13
measured   1  1  2  2  3  3  4  4  5  5  6  6  7 ... 13
                                  ^ this book, N = 12
```

Checked programmatically at every row: the two columns are equal for all `N` from 1 to 25. **The class count is exactly `ceil(N/2)`, because the odd parts of `1..N` are precisely the odd numbers in `1..N`, of which there are `ceil(N/2)`.**

B.6 The folded-lattice minimum gap, by exact rational arithmetic

Fold every n^2 into a single octave as an exact `Fraction`, sort, and take the smallest adjacent ratio including the wrap-around from the largest back to twice the smallest. The result is a step function in `N` with four distinct values:

```
N range        minimum gap        cents         status
------------------------------------------------------------------------
N = 2              2/1         1200.0000       degenerate (one class)
N = 3 .. 6         9/8          203.9100       the Pythagorean tone
N = 7 .. 16       50/49          34.9756       THIS BOOK'S REGIME
N >= 17          289/288          6.0008       THE CLIFF
N >= 41         1682/1681         1.0296       the second threshold
------------------------------------------------------------------------
```

The cliff fires the instant index 17 enters the set, because `17^2 = 289` folds beside `288 = 2 * 12^2`; the second fires at index 41, because `41^2 = 1681` folds beside `1682 = 2 * 29^2`. Both locations are exact and neither depends on `k`.

Three distinct ratios describe the size of the cliff, all correct in their own units. Confusing them is the specific error Book Two's R21 records and reverses:

```
   cents   ln(50/49)/ln(289/288)  = 5.828   how much smaller the interval is
    cycles   288/49                  = 5.878    3.899 cycles becomes 22.918
     RATE      (1/32)/(1/256)            = 8 EXACT  symbols per second —-
the design limit
```

The rate factor is exactly 8 because the two binding pairs sit at different octave heights: the `50/49` pair straddles `49/32` and the `289/288` pair straddles `9/8 = 288/256`. Gabor resolution depends on the absolute gap in hertz — `f/32` before the cliff, `f/256` after, at every base pitch. This book renders entirely below the cliff, so the `50/49` figures are operative throughout.

B.7 Single-transposition capacity

Take the whole set `{k n^2 : n = 1..N}`, apply one integer octave transposition `2^j`, and require the entire transposed set to lie inside 20 Hz to 20 kHz. Computed over `j = 0..15` for every `N`:

```
N        admissible j
------------------------------
20            {6}
21            {6}
22            {6}
23            {6}
24            {6}
25            {6}          <-- the maximum
26            {}           <-- no j works
27            {}
------------------------------

at N = 25, j = 6:   lowest =    30.9392 Hz      highest = 19337.0293 Hz
at N = 26, j = 6:   highest = 20914.9309 Hz     -- above the band
at N = 26, j = 5:   lowest  =    15.4696 Hz     -- below the band
```

A single integer octave transposition admits at most 25 essences inside the audible band, and only at `j = 6`. A twelve-house set fits comfortably; a thirty-two-category set does not and requires split transposition. This is a hard bound on essence-set size and it constrains any extension of the ladder a reader might be tempted to make.

Appendix C — Measurement Protocols

Eight numbered protocols and four worksheets. Each names its owning chapter, states its inputs and required outputs, and ends with the gates that must pass before publication. Reports cite them by number and do not restate them.

A prohibition governs all eight and is repeated inside each: **none of these procedures is an assessment.** Nothing here evaluates, ranks or screens a person, or produces anything describable

to them or anyone else as a finding about their mind. They produce a description of a description, and a practitioner who cannot hold that distinction should not run them.

PROTOCOL C1 — Indexing a state

Owner: Chapter 5 (the index and its count argument), Chapter 6 (the construction procedure).

Inputs. One named situation, in one sentence, written before anything else is asked. One or more framings of it. A dated declaration of the decomposition rules in force, `N = 12`, the ladder in use, the intensity instrument and its range, the audible band and the folding convention.

Steps.

```
 C1.1  DECLARE THE RULE SET, DATED. Published with the reading.
C1.2  OBTAIN CONSENT, AND MAKE DECLINING STRUCTURAL. The subject is told what is
      elicited, what is computed, what it does and does not mean, and that any
      question may be skipped without a reason and without the procedure being
      abandoned. Nothing masked; nothing concealed; nothing below awareness.
 C1.3  FIX THE SITUATION in one written sentence.
C1.4  FIX THE FRAMINGS to be elicited, and name them, before eliciting any.
      Two is the minimum for a relational reading; three is the house default.
C1.5  SILENT WRITE-FIRST. Each framing written out in full, independently,
        before any two are compared and before the practitioner comments.
C1.6  READ EACH FRAMING AGAINST ALL TWELVE COMPONENTS. PRESENT or ABSENT for
        every one. No blanks.
C1.7  SUBSTITUTION TEST on every component marked PRESENT: remove it, read the
      residual back, ask whether that is still the state. If yes, unmark it.
 C1.8  NESTING CHECK. No component marked present above one marked absent.
C1.9  COUNT. n = components present after C1.7 and C1.8. Record the count AND
        the list. A count without its list is a rating and is rejected.
```

Required outputs. `n` for each framing; the component list beside each count; `f = k n^2` raw and folded with `j`; the pitch class as the odd part of `n`.

Publication requirements, from Chapter 5 §5.16, and these are requirements.

```
I1  THE DECOMPOSITION -- what counts as one component, as a rule a third party
      can apply to a state the author never considered
 I2  THE COUNT RULE -- including redundant, nested and conjoined components
 I3  N, with the ceiling ln N it implies
I4  THE TIE-BREAK -- for a count index, ties are the same state class and share
      a frequency, which is the intended behaviour
 I5  THE BAND AND j for every published frequency
I6  THE PERMUTATION TEST -- swap two adjacent indices, recompute, report the
      change in cents and in pitch class. Ten minutes.
```

The honest limit, stated wherever the index is used. The frequency inherits exactly the defensibility of the count. A contested decomposition yields a contested frequency, and a frequency published without its decomposition is a number nobody can check.

PROTOCOL C2 — Eliciting the intensities, and why magnitude estimation is required

Owner: Chapter 6. Binding on every chapter that reports a u.

This requirement is the one most often assumed negotiable, so the argument comes before the steps.

SPINE §1.1 fixes the scale status of two quantities separately and they differ. **n, the index, is a count: it has a true zero and equal units, it is ratio-scaled by construction, and it does not inherit R1's demotion — it sidesteps it.** `w`, the intensities, are elicited magnitudes and are ordinal by default, and everything computed from intensity-weighted vectors — `u`, `F`, `f_rel`, the deficit — inherits `w`'s status. So the state frequency is ratio-scaled and the relationship frequency is not, until the intensities are.

The usual response to an ordinal input is to report the output ordinally and carry on. **That response is not available here, and Chapter 6 proves it with a number.** Apply a monotone relabelling of the intensities — a transform that preserves every ordinal relation and therefore destroys no ordinal information whatever — and u moves from 0.362115 to 0.525275. Converted through Rule 2:

```
u = 0.362115   ->   F = 1.606085   ->   f_rel = 12.004877 Hz
u = 0.525275   ->   F = 1.913686   ->   f_rel = 22.209363 Hz

shift  =  0.887547 octaves  =  1065.0568 cents  =  0.307600 nats
```

Nearly an octave, from a change an ordinal scale calls no change at all. **With ordinal intensities the relationship frequency is not determined, and reporting one would be reporting an artifact of the labelling.** The ordinal fallback is therefore not a weaker option; it is not an option.

Steps.

```
C2.1  Choose the state class the subject reports most strongly. Assign it a
       reference value -- 10 on a 0-10 instrument.
C2.2  For every other class present, ask the ratio question IN WORDS:
      "compared with that one, how strong is this one -- half as strong,
      a quarter, three times?" Ratios are what the construction consumes.
C2.3  Do not average across framings. Each framing is elicited separately.
C2.4  Do not show either party another party's numbers.
C2.5  RECORD THE RAW L2 NORM in the instrument's units, then normalise.
C2.6  PUBLISH THE RAW L2 NORMS. The free multiplier of magnitude estimation is
    harmless because normalisation removes it -- but it must stay recoverable,
       and it is recoverable only from the norms.
```

Worked, on the carried dyad, so the reporting rule has a concrete instance:

```
A = {4:0.7, 6:0.5, 9:0.4}            raw ||A||_2 = 0.948683
B = {4:0.6, 6:0.55, 9:0.3, 10:0.2}   raw ||B||_2 = 0.890225
                                     ratio ||A||/||B|| = 1.065667
```

The ratio is the recoverable content: normalisation discards it and the publication rule puts it back. **Magnitude estimation (Stevens' method) produces genuine ratio-scale subjective magnitudes up to a free multiplier and replicates, which is why it is required and not merely preferred.**

PROTOCOL C3 — The three routes to u

Owner: Chapter 13. Cited by Chapters 15, 17 and 24.

Three routes, measuring different things, and the route is declared before the numbers are seen. Choosing it afterwards is a documented failure mode (FM-13.5).

```
ROUTE A -- STATE VECTORS                                  the primary route

  Represent each party as a unit vector over the twelve states, components =
    magnitude-estimated intensity, normalised to length 1.

        cos theta = x . y            u = 1 - (x . y)^2

    Carry u to twelve or more decimal places (SPINE §3.4).
    Requires: C1 and C2 completed for both parties.
    Cannot see: whether either party is right about anything.

ROUTE B -- THE AGREEMENT CHANNEL                                Theorem 13.1

  Both parties judge m shared items independently, each judgement +1 or -1.
    Let a = the fraction of items on which they match.

        cos theta = 2a - 1           u = 4 a (1 - a)

  Exact, and no centring is required: +/-1 vectors have norm sqrt(m), so the
    norms cancel identically.
    Resolution: du = 4|1-2a|/m, bounded above by 4/m. REPORT m.
    Requires: an item set agreed BEFORE it is judged.
    Cannot see: the sign. a = 0.9 and a = 0.1 both give u = 0.36.

ROUTE C -- ERROR DECORRELATION                                  Definition 13.2

  Both parties estimate m quantities whose true values later arrive. Form each
    party's error vector e = (estimate - truth), let rho be the Pearson
    correlation of the two error vectors.

        u = 1 - rho^2

    REPORT THE SIGN OF rho EXPLICITLY. It is not recoverable from u.
    Requires: ground truth, which is the cost and it is not small.
   Can see, and the others cannot: being right for different reasons, and
    being wrong in different ways.
```

The agreement channel, so a practitioner can read u off an agreement rate directly:

a	cos theta	u	f_rel (Hz)	cents above k
1.000000	+1.000000	0.000000	0.4834	0.00
0.990000	+0.980000	0.039600	0.9963	1251.97

```
0.950000    +0.900000    0.190000        4.6158        3906.26
0.900000    +0.800000    0.360000       11.8930        5544.82
0.800000    +0.600000    0.640000       31.2494        7217.27
0.700000    +0.400000    0.840000       50.6909        8054.75
0.600000    +0.200000    0.960000       64.6019        8474.57
0.500000     0.000000    1.000000       69.6133        8603.91
0.400000    -0.200000    0.960000       64.6019        8474.57
0.300000    -0.400000    0.840000       50.6909        8054.75
0.200000    -0.600000    0.640000       31.2494        7217.27
0.100000    -0.800000    0.360000       11.8930        5544.82
0.000000    -1.000000    0.000000        0.4834           0.00
------------------------------------------------------------------
```

The table is exactly symmetric about `a = 0.5`, and its first and last rows are identical. Two parties who agree on every item and two parties who disagree on every item produce the same u, the same frequency and the same reading. That is R8 arriving through the measurement channel rather than through the algebra, and Chapter 14 owns its consequences.

THE ACCURACY GUARD, binding wherever Route B or Route C is used. A u reading is published with each party's accuracy on the item set. Two parties answering at random have expected `a = 0.5`, hence `u = 1`, hence `f_rel = 69.6133 Hz` — the ceiling of the whole system, from guessing. Surplus measures distinctness, never correctness, and this book has no formalism converting one into the other.

THE OVERLAP GATE, not optional, and it runs before anything is computed. If two framings have disjoint support, u = 1 exactly regardless of the intensities — verified across five constructed disjoint pairs with random intensities, `u = 1.000000` in every case. A degenerate reading is not a high reading; it is no reading. If the gate fires, report DEGENERATE, publish no `f_rel`, and return to the situation statement.

PROTOCOL C4 — Estimating `eta`

Owner: Chapter 15 §15.9b at dyadic scope, Chapter 11 §11.13 at personal scope.

`eta` is the fraction of a period's joint work still doing something at the start of the next period. It is countable in principle and the count is the protocol.

```
C4.1  Enumerate what the parties produced in a period: decisions, agreements,
        artefacts, practices. Write the list before counting it.
C4.2  Count how many survive intact into the following period without being
        re-made.
C4.3  eta_hat = survivors / total.
C4.4  Declare what got called a thing. The estimate is sensitive to it and the
        sensitivity is the reader's business.
```

A pair that settles six things a month and re-settles four next month is converting about a third. **The count is crude and far better than a guess, because the objection to it is visible and arguable in a way that an objection to a guess is not.**

PROTOCOL C5 — Estimating c

Owner: Chapter 15 §15.9b.

c is what the arrangement costs to run before it produces anything: coordinating rather than deciding, re-explaining, the friction of getting to where work can happen. It is measured in the same units as the surplus — nats per period — and therefore requires a declared conversion, and the conversion is where most of the error lives.

```
C5.1  Identify the period budget and state it.
C5.2  Express eta*F and c as fractions of that same budget.
C5.3  Report c/eta directly. It is recoverable even when neither term is well
      determined on its own, and it is the only combination the deficit uses.
C5.4  Attach an error bar to c/eta, not a decimal point. Its elasticity is the
      largest in the reading.
```

The ratio c/eta carries the entire cost side of the reading, and a verdict resting on it deserves an interval. Where a structure prices its coordination per link, the aggregate is `C = kappa * m(m−1)/2`, and the commons of Chapter 17 is the worked instance: `kappa = 0.055` across 15 links gives `C = 0.8250`.

PROTOCOL C6 — Estimating delta

Owner: Chapter 8, Procedure TRAIT-2.

delta is how much of what was accumulated is still there after a period of doing nothing. The estimation handle is the half-life relation.

```
t_half  =  ln(0.5) / ln(1 - delta)

delta = 0.05   ->   t_half = 13.5134 periods
delta = 0.09   ->   t_half =  7.3496 periods
```

```
C6.1  Identify a practice that lapsed, and the number of periods it lapsed for.
C6.2  Judge what fraction of it survived.
C6.3  Invert the half-life relation.
C6.4  DECLARE THE PERIOD. delta is a per-cycle rate and it is meaningless
      without its cycle. A delta quoted without a period is not a number.
```

A pair finding a lapsed practice half-forgotten after seven periods is running near `delta = 0.09`, since `ln(0.5)/ln(0.91) = 7.3496`.

PROTOCOL C7 — Producing the reading: the deficit AND delta, together

Owner: Chapter 4. This is the protocol the book exists to provide.

```
C7.1  CEILING TEST FIRST.  Is c/eta <= ln N?
   If NO: the case is STRUCTURAL. No arrangement of the declared categories
     is sustainable. Report the ceiling breach and stop. Do not compute a
   deficit and do not offer levers; the levers do not include one that helps.

C7.2  COMPUTE F.        F = ln(1 + (N-1) u)
```

```
C7.3  COMPUTE f_rel.    f_rel = k (1 + (N-1) u)^2, raw and folded with j
C7.4  COMPUTE f_min.    f_min = k e^(2 c/eta)
C7.5  COMPUTE D.        D = 2.885390 (c/eta) - log2(f/k)
   CROSS-CHECK against 2.885390 (c/eta - F). The two must agree to at least
      twelve decimal places. If they do not, an input was rounded.

C7.6  COMPUTE Q_inf.    Q_inf = (eta F - c)/delta
      CROSS-CHECK against -D * eta / (2.885390 * delta).

C7.7  REPORT D AND delta, IN THE SAME SENTENCE.
C7.8  REPORT THE HOLD LINE u* = (e^(c/eta)-1)/(N-1) and the ratio u*/u_now.
C7.9  CLASSIFY.
          D <= 0                              SUSTAINABLE
          D > 0, c/eta <= ln N, u* reachable  TUNABLE AND REACHABLE
          D > 0, c/eta <= ln N, u* not        TUNABLE, UNREACHABLE
          c/eta > ln N                        STRUCTURAL
   Reachability is a declared judgment about the parties, not a property of
      the formalism, and it is declared rather than assumed.
```

Step C7.7 is the step this protocol exists for. Theorem 4.9 proves `D` contains no `delta`, and the consequence is a reading that can be entirely correct and entirely uninformative about rate:

```
eta = 0.30, c = 0.62, u = 0.068734665264634

delta = 0.09   ->   D = +4.338420197   Q_inf = -5.011939546
delta = 0.05   ->   D = +4.338420197   Q_inf = -9.021491184
```

Halving the loss rate nearly doubles the negative destination and the deficit does not move by one digit in ten. **The deficit reads the sign and size of the gap; `delta` converts it to a level; a report with one and not the other is half a reading.** Chapter 19's Ripley schedule contains a row where exactly this happens — step 5 moves `Q_inf` from −5.582082 to −2.511937 while `D` sits at +4.831945 in both rows — and it would have been invisible in a `D`-only report.

PROTOCOL C8 — Controls, and what the surrogates cannot do

Owner: Book Two Chapters 55, 79-81; carried here as R4, R13 and R24.

Any empirical test of any operationalisation in this book beats four baselines: persistence, direct-input, matched-autoregressive, and shuffled-input. **The shuffled-input control has three measured failure modes and every protocol that recommends the suite carries all three.**

```
(1) IT IS ANTI-CONSERVATIVE AS ORDINARILY RUN.  Against a nominal alpha of 0.05
     across 500 null trials, measured false-rejection rates were:
          iid permutation      0.108   (2.2x nominal)
          circular shift       0.120   (2.4x nominal)
          block permutation    0.060   (near nominal)
   The matched-autoregressive comparison ALONE false-passed 35.8% of the time.
    REMEDY: calibrate the surrogate by simulation on the study's own design
     before use. Not after.
```

(2) IT CANNOT SEE REPACKAGING. A filtered repackaging of the input passed all four baselines including the shuffle (p = 0.0005) and was caught only by the structural ceiling test, its single-step jump being 38.5x the theoretical maximum.
REMEDY: the structural tests. Only they detect repackaging.

(3) IT CANNOT SEE A SHARED SLOW DRIVER. In a 4,000-trial simulation the shuffle worked in the clean case (48.17% detection) and failed against a shared slow driver, passing 98.35% of the time.
REMEDY: design controls.

AND SEPARATELY: no surrogate detects FABRICATED TIMING. With a rater whose next rating is nudged by the previous period's input -- no real relationship present at all -- the full four-baseline suite passed 52.7% of the time. More surrogates do not fix this.

The design-control remedy is the whole answer and it is not substitutable: blinding, channel separation, prospective sampling, and a yoked control. Adding surrogates to fix a design problem is a documented failure mode; the suite is a weak, mis-calibrated empirical audit and was never the load-bearing control.

The worksheets

WORKSHEET 1 — ONE FRAMING

```
================================================================================
 RULE SET ID _______________  DATE _____________  N = 12   BAND _____________
 SITUATION (one sentence, written first):
 ____________________________________________________________________________
 FRAMING NAME: ___________________   ELICITED BY: _________   CONSENT: [ ]

 COMPONENT                          PRESENT   SUBSTITUTION TEST   INTENSITY
  1 arousal                           [ ]           [ ]            _________
  2 valence                           [ ]           [ ]            _________
  3 intentional object                [ ]           [ ]            _________
  4 situation                         [ ]           [ ]            _________
  5 cause                             [ ]           [ ]            _________
  6 projected duration                [ ]           [ ]            _________
  7 self as object                    [ ]           [ ]            _________
  8 another mind modelled             [ ]           [ ]            _________
  9 standard applied                  [ ]           [ ]            _________
 10 placed in a story                 [ ]           [ ]            _________
 11 the other modelling you           [ ]           [ ]            _________
 12 future action bound               [ ]           [ ]            _________

 NESTING CHECK PASSED [ ]      n = _______      COMPONENT LIST ATTACHED [ ]
 REFERENCE CLASS FOR MAGNITUDE ESTIMATION: _______   VALUE: _______
 RAW L2 NORM (before normalisation): ___________________  <-- PUBLISH THIS
 f = k n^2 = _____________ Hz     j = _______   folded = _____________ Hz
```

```
  PITCH CLASS (odd part of n) = _______
================================================================================
```

WORKSHEET 2 — THE PAIR

```
================================================================================
 PARTY 1 ___________  PARTY 2 ___________   BOTH CONSENTED SEPARATELY [ ]
 SITUATION AGREED IN WRITING BY BOTH [ ]  SILENT WRITE-FIRST OBSERVED [ ]
 ROUTES DECLARED BEFORE COMPUTATION: A [ ]  B [ ]  C [ ]

 OVERLAP GATE:  shared state classes = _______   PASSES [ ]  DEGENERATE [ ]
 RAW NORMS:  ||x|| = ___________   ||y|| = ___________  ratio = ___________

 ROUTE A   cos theta = _________________   u = ____________________ (12 dp)
 ROUTE B   m = _______  a = ___________   u = 4a(1-a) = _________________
           resolution 4/m = ___________
 ROUTE C   rho = ___________ SIGN: _____  u = 1 - rho^2 = _________________
           accuracy party 1 _________   accuracy party 2 _________

 F = ln(1 + 11u) = _____________ nats      N_eff = _____________
 f_rel = k(1+11u)^2 = _____________ Hz     folded _____________ (j = ____)
================================================================================
```

WORKSHEET 3 — THE READING

```
================================================================================
 eta = _________ (Protocol C4, list attached [ ])
 c   = _________ (Protocol C5, period budget declared [ ])
 delta = _______ (Protocol C6, PERIOD DECLARED: _________________)

 STEP 1  CEILING TEST      c/eta = ___________   ln N = 2.484907
       c/eta <= ln N ?   YES [ ] -> continue    NO [ ] -> STRUCTURAL, STOP

 STEP 2  f_min = k e^(2c/eta)                = _____________ Hz
 STEP 3  D = 2.885390(c/eta) - log2(f/k)     = _____________ octaves
STEP 4  cross-check 2.885390(c/eta - F)    = _____________  AGREE TO 12 dp [ ]
 STEP 5  Q_inf = (eta F - c)/delta           = _____________
 STEP 6  cross-check -D*eta/(2.885390*delta) = _____________  AGREE [ ]
 STEP 7  hold line u* = (e^(c/eta)-1)/11     = _____________
         u* / u_now                          = _____________
 STEP 8  CLASSIFICATION: SUSTAINABLE [ ]  TUNABLE+REACHABLE [ ]
         TUNABLE/UNREACHABLE [ ]  STRUCTURAL [ ]
         reachability judgment DECLARED, not assumed [ ]

 THE REPORT LINE (both numbers, one sentence):
 "D = _________ octaves at delta = _________ per _____________."
================================================================================
```

WORKSHEET 4 — THE BOUNDS STATEMENT, WHICH IS PART OF EVERY REPORT

```
================================================================================
```

```
 [ ] These are ESSENCE frequencies, not physical resonances. Nothing is
     claimed to resonate.
[ ] The twelve states are not diagnoses. This is not a clinical, medical or
     therapeutic assessment of anyone.
 [ ] The frequency inherits exactly the defensibility of the count. The
     decomposition is published on this page.
[ ] The intensities were magnitude-estimated. The raw L2 norms are published.
 [ ] The state frequency and the relationship frequency are INDEPENDENT
     readings. Neither was inferred from the other.
 [ ] D is reported with delta. The deficit alone cannot see decay.
 [ ] Consent was obtained separately from every party and declining was
     available at every step.
[ ] Every case in this report is CONSTRUCTED and labelled as such, OR is an
     observation with its provenance stated.
================================================================================
```

Appendix D — The Concordance Register

This volume's audit trail. Every claim on which the book runs a dual concordance appears here with four fields: the claim; the (+1) source and what it measured; the (−1) source and what it recorded; the status.

The two columns do different work and the difference is the method's whole content. (+1) carries evidence — a measurement, a derivation, a published result with a number attached. (−1) carries precedent — a tradition, a corpus, a practice, a record that somebody once thought this. Precedent is never promoted to evidence. A (−1) entry can suggest where to look, can supply a structure worth testing, and can constitute a striking convergence when an independent (+1) lands on the same object. It can never, by accumulation or by antiquity or by consensus, become the reason a claim is believed.

The sharpest demonstration of why is R23. Under a selection process that publishes successes and discards failures, the likelihood ratio of testimonial evidence is exactly `1.000000`. **A likelihood ratio of 1 means the evidence does not move the posterior at all — not weakly, not by a little, not at all — because the selection guarantees you observe successes whether or not the method does anything.** The old corpus is a survivorship-filtered record, which is exactly that condition, and that is why the (−1) column is a separate column rather than a weaker part of the first.

Rejections are printed at the same length as acceptances and in the same tables. A register listing only what survived would be a sales document.

D.1 The laws and the join

#	Claim	(+1) — evidence, and what it measured	(-1) — precedent, and what it recorded	Status
D1.1	`f = k n^2` maps an essence index to a frequency	Book Two Ch 29: the law reproduces all 118 published element values, worst relative deviation `9.0e−15`	the sonic-chemistry source table, which asserted the correspondence without a derivation	**ESTABLISHED as an exact reproduction of the source law; the source's prose formula was missing its parentheses (R5)**
D1.2	`f_compound = k^2 G^4`	Book Two Ch 29: reverse-engineered and matched to 37/44 published composites to `<1e−12`; the 7 outliers are data-entry errors, and the O2 row is provably impossible under the stated rule	the composite table itself	**ESTABLISHED with errata; NOT a generalisation of D1.1 (Cor 20.2: the rules agree at no index)**
D1.3	`F = ln(1 + (N−1)u)` is the unique surplus form	Paper A: uniqueness from S1, S2, S4 given S3; every step after S3 forced	—	**PROVED CONDITIONALLY. S3 is a chosen design axiom**
D1.4	`Q_inf = (eta F − c)/delta`	Paper E Thm 3.5, with the exact solution and the contraction	Book One Node 10 FIXEDPOINT, which named the object without computing it	**PROVED. The precedent is a naming, not a derivation**
D1.5	`f_min = k e^(2c/eta)`	Ch 3: substitution verified at twelve parameter sets	—	**PROVED. An exact restatement of a sustainability condition in frequency units, and nothing more**

#	Claim	(+1) — evidence, and what it measured	(-1) — precedent, and what it recorded	Status
D1.6	`D = 2.885390(c/eta − F) = −2.885390(delta/eta − 1/4)` across twelve	Ch 4 Thm 4.5; verified to `2.665e−15` atoms, twelve parameter sets, worst residual `8.88e−16` on the carried case	—	**PROVED**
D1.7	`D` contains no `delta`	Ch 4 Thm 4.9; worked: `delta` 0.09 to 0.05 moves `Q_inf` from `−5.0119` to `−9.0215` and leaves `D` at `+4.338420`	—	**PROVED, and it is a limitation of the book's own central instrument**
D1.8	`dt df >= 1/(4 pi)` binds every rendering	Gabor; classical Fourier duality, measurable on an oscilloscope	—	**ESTABLISHED. No quantum content and none claimed**

D.2 The index and the psyche

#	Claim	(+1) — evidence, and what it measured	(-1) — precedent, and what it recorded	Status
D2.1	The component count is ratio-scaled and escapes R1's demotion	Ch 5: a count has a true zero and equal units by construction; R1 binds `D/P` specifically	—	**ESTABLISHED for `n`. NOT extended to the intensities**
D2.2	Arousal as an essence index	—	the arousal tradition in affect research	**REJECTED.** V2 fails — all twelve tie, 11.17 bits free; V3 fails; V7 fails, no tie-break
D2.3	Hierarchical need as an essence index	—	the need-hierarchy literature	**REJECTED.** V1 fails — needs are not states; V2 fails — five levels against twelve states

#	Claim	(+1) — evidence, and what it measured	(-1) — precedent, and what it recorded	Status
D2.4	Developmental stage as an essence index	—	stage theories of development	**ADMISSIBLE AS AN ORDINAL, NOT ADOPTED.** V1 fails — it indexes persons, not states; V4 fails — no true zero, so R1 applies
D2.5	Valence magnitude as an essence index	—	dimensional affect models	**REJECTED.** V2, V3, V7 fail; V4 fails by collapsing opposites, which is R8 in a different costume
D2.6	Magnitude estimation is required for the intensities	Ch 6 / SPINE §1.1: a monotone relabelling moves u from 0.362115 to 0.525275 — 1065.0568 cents	Stevens' method, which produces ratio-scale subjective magnitudes up to a free multiplier and replicates (Book Two Ch 38)	**REQUIRED, not preferred. The ordinal fallback is not available**
D2.7	Consonance and redundancy are the same phenomenon	Ch 10: Tenney harmonic distance doubled in the essence lattice; the most consonant pairs are exactly the pitch-class collisions	classical consonance theory, which recorded the ratios without the identity	**PROVED at internal scope; nothing about distress follows, and Ch 10 §10.6 states the firewall formally**
D2.8	One-hot framings are degenerate	Ch 6 / Ch 9: five constructed disjoint-support pairs return u = `1.000000000000` regardless of intensities	—	**PROVED. The gate runs before anything is computed**

D.3 Relationship

#	Claim	(+1) — evidence, and what it measured	(-1) — precedent, and what it recorded	Status
D3.1	`u = 4a(1−a)` from an agreement rate	Ch 13 Thm 13.1, proved from the norms of +/−1 vectors; exact, no centring	inter-rater agreement practice, which recorded a without converting it	**PROVED. Resolution `4/m`, and `m` is reported**
D3.2	Perfect opposition reads as perfect agreement (R8)	Book Two Ch 46: `u = 1 − (x.y)^2` depends only on `|rho|`; Ch 13's agreement table is exactly symmetric about `a = 0.5`	"creative tension" as a design principle	**PROVED, and the precedent is refuted by it: `rho ~ −1` is a parallel pair with a cost overhead, paying `C` and returning zero surplus**
D3.3	Trust amplifies an existing sign (R15)	Ch 15: `delta` 0.09 to 0.05 moves `Q_inf` from `−5.0119` to `−9.0215`; after the numerator is fixed, the same move gives `+6.0844` to `+10.9520`	the near-universal counsel to build trust first	**PROVED, and the precedent is corrected rather than rejected: trust is right, trust-first is wrong**
D3.4	An orthogonal pair already attains `ln N`	Ch 16 §16.7: orthogonal pair and orthogonal six both give `F = 2.484907`, `f = 69.6133 Hz` — identical, and the six pays 15 links against 1	the standing assumption that larger groups generate more	**PROVED in surplus terms. Groups buy breadth, redundancy and succession, which this formalism does not price**

#	Claim	(+1) — evidence, and what it measured	(-1) — precedent, and what it recorded	Status
D3.5	Ostrom's design principles map to the parameters	Book Two Ch 49/51: P4 lowers `c_v` and does NOT raise a; P5 priced at `−7.7%` of `eta S`; P8 turns `O(N^2)` coordination into `O(N)`	Ostrom's commons scholarship	**ESTABLISHED where it applies and PRICED where it does not. P4's effect is not the one usually claimed for it**
D3.6	A commons can be formally tunable and practically unreachable	Ch 17: `c/eta = 2.426471 < ln 12`, so TUNABLE; but u∗ required is 0.938078 against `u_bar = 0.018614`, a factor of 50.4	—	**ESTABLISHED. Reachability is a declared judgment about the parties, not a property of the formalism**
D3.7	Drop-one improves on every member of a healthy team	Ch 17 §17.11b: all five rows improve, `Q_inf` rising by between 1.97 and 2.80	performance-ranking practice	**PROVED, and it refutes the practice: a favourable drop-one row carries no information about the member it names**

D.4 The tradition audited — Part IV

#	Claim	(+1) — evidence, and what it measured	(-1) — precedent, and what it recorded	Status
C23.1	Staged process with per-stage checks	Shewhart 1931; Deming 1986; §23.8 prices staged control at 47.83% to 93.21% yield at `d = 0.90`	the Zosimos corpus; Newman & Principe 1998; Principe 2013	**ESTABLISHED**

#	Claim	(+1) — evidence, and what it measured	(-1) — precedent, and what it recorded	Status
C23.2	Cadence beats intensity	this work's concavity and ceiling results; Cepeda et al. 2006	fixed-hour practice; Durkheim 1912; Rappaport 1999	**ESTABLISHED, and folklore's REASON REJECTED. See D.5**
C23.3	Correspondence tables are relational graphs	graph theory; the gcd invariant tabulated for `N = 3..12`	the tables themselves; Yates 1966	**ESTABLISHED AS FORM ONLY. Two classical tables graded against V1-V7 and rejected as indices**
C23.4	The operator is part of the apparatus	Rosenthal & Fode 1963; Polanyi 1958; Collins 1985; §23.16 computes mirroring at 536.4074 cents	the pure-artisan requirement	**ESTABLISHED, MECHANISM REJECTED. Moral purity is not the mechanism**
C23.5	The colour sequence is a necessary order	—	precedent only	**REJECTED.** C2 fails: identification is circular
C23.6	"As above, so below" as a transfer licence	R16's three computed disanalogies: mean-field decay error 92% by `t = 12`; two nine-member groups with identical `u = 0.5999` losing 77% against 21% to a three-person departure; a 90.1x period-normalisation trap	the maxim	**REJECTED AS STATED. The correspondence is of dynamics and geometry, not of agency**

#	Claim	(+1) — evidence, and what it measured	(-1) — precedent, and what it recorded	Status
C23.1b	This tradition's canonical stage ORDER	R15; Ch 19 Finding 19.2	Ripley's *Compound of Alchymy* (1471), the Twelve Gates	**REJECTED (order only; staging stands)**
C24.1	Material transmutation by chemical means	the energy-register argument: `2e6` on bonds, `8.24e7` on the furnace-to-barrier ratio; transmutation is real by nuclear means at 15.52 GW-station-years per gram under the most favourable constructed assumptions	the transmutation corpus	**REJECTED.** C1-C3 fail; AUDIT-1 steps 2, 4, 5, 6, 7, 8 fail
C24.2	The corpus establishes a success rate	R23: `LR = 1.000000`, invariant to corpus size; with a denominator the ratio goes to about `1e304`	the published successes	**REJECTED.** AUDIT-1 step 2
C24.3	Secrecy protects a valuable result	error persistence under secrecy: `r = 0` gives infinite expected detection time; Deckname search cost `3^12 = 531,441` readings, 19.02 bits, 1,455 years at a trial a day	the secrecy tradition	**REJECTED AS AN EPISTEMIC PRACTICE.** AUDIT-1 step 8

#	Claim	(+1) — evidence, and what it measured	(-1) — precedent, and what it recorded	Status
C24.4	Shared values or correspon-dences are evidence	R12 recomputed: 234,454 words to 305 distinct outputs, 99.995% collision, 10.47 bits destroyed against a 9.5863-bit uniform bound — worse than its own worst case; in an 8,000-word corpus `P(a value-twin) = 0.999666`	the gematria corpus	**REJECTED; MNEMONIC READING RETAINED.** A retrieval index need not be injective, only reproducible
C24.5	The canonical gate order must not be varied	Ch 19 Finding 19.2 against R15	Ripley 1471	**REJECTED AS AN ORDERING**

D.4b Chapter 19's audit of the Twelve Gates, verdict by verdict

Seven gates name operations on this book's declared parameters, two are split, and three fail — with a fourth entry, the planetary-metal ordering, failing as an index. Each rejection states what would reverse it, because a rejection that cannot say what would reverse it is an opinion.

The `(-1)` column for the whole set is Ripley's *Compound of Alchymy* (1471). The seven that map do so on named parameters — calcination on `n` downward, solution as the inverse of Rule 3, separation on `N`, putrefaction on `delta`, congelation on `G`, cibation on `c`, fermentation on `G` and `u`. Conjunction maps but SPLITS, its two readings returning incommensurable units, and the schedule halts there. The failures:

entry	(+1) — evidence, and what it measured	Verdict, and what would reverse it
8 sublimation	folding by `2^j` preserves every interval and every pitch class exactly; nothing in the parameter set moves, and `k` cancels from `D`	**RENDERING — rejected as an operation on a state.** Reverses if `j` entered a verdict. It does not
10 exaltation	"nobler" is a value ordering, not a coordinate (Ch 5 §5.18); the rescues collapse to gates 1 or 9, or hit the ceiling `F <= ln 12`, `f <= 69.6133055503`	**REJECTED.** Reverses if a count-valued definition of nobility is supplied. None is, in the source or since

entry	(+1) — evidence, and what it measured	Verdict, and what would reverse it
11 multiplication	contraction: `|Q_t − Q_inf| = (1−delta)^t |Q_0 − Q_inf|`, 99% closure at 48.83 passes; the destination is fixed by four parameters repetition does not touch	**SPLIT: the iterative reading MAPS, the unbounded reading is REJECTED.** Reverses if a pass altered `eta`, `F`, `c` or `delta` — then it is a different gate under a borrowed name
12 projection	no base/noble coordinate; Rule 3 is blind to quantity, so the mass is not in the value; no channel (belief postulate); CONSTRAINT L67 forbids reaching the non-latent; R23 fixes the testimonial LR at 1.000000	**REJECTED** on all four conditions. Reverses if a channel with measured gain is exhibited. None is
13 the planetary-metal ordering as an essence index	completeness, non-arbitrariness, count semantics and stability all fail; one disputed swap moves a published class by 568.7174 cents	**REJECTED.** Reverses if the ordering were fixed by a count. It is fixed by worth

D.5 Findings the book records against its own sources

Four items belong in their own section because in each of them the framework audits a source on the source's own material and returns a verdict the source would not have given.

FINDING 19.2 — Ripley's ordering contradicts R15. Run as a schedule on the carried dyad, the Twelve Gates of Ripley's *Compound of Alchymy* (1471) place the only numerator-acting gate ninth of twelve. Fermentation, at step 9, moves `D` by `−5.305895` octaves, which is 88.93% of the entire move; the eight gates before it move it by `−0.660631` between them, which is 11.07%. Step 5, putrefaction, acts on `delta` — and it runs four steps before the numerator is repaired. **In the tradition's order, raising `delta` improves the destination; in R15's order, the identical move damages it; and the only thing that differs is the sign of the numerator when the move is made.** This is the strongest single item in Part IV precisely because it is the tradition being audited on its own published material, and it is recorded here as `(+1)` evidence against a `(−1)` precedent — never the reverse.

The same schedule shows the delta-blindness of D1.7 as a visible row: step 5 moves `Q_inf` from `−5.582082` to `−2.511937` while `D` sits at `+4.831945` before and after.

R11 — cadence wins, and not for the reason the tradition gives. Computed rather than assumed: the integrated effect of one large impulse and the same total input delivered periodically is identical to every digit — 433.3333 in all three regimes tested. That is exact LTI invariance, and a naive "cadence delivers more" claim is simply false for a linear time-invariant system. Cadence buys three other things instead, and Appendix E.5 states them. The honest counter-case is kept

in print: above a threshold no realistic cadence reaches, the impulse wins, and a once-a-decade rite requires a 5.63-year trace half-life to work.

Ch 21's refused move. A proposed re-indexing of the carried Subject's `mirror` framing from state 7 to state 11 was run through TR-1 and halted at step 3: three of the four added components were observed in the repertoire, and the fourth — the other's model of the subject — was not. **The move was refused under CONSTRAINT T21, and steps 4 through 10 were not run, so the arithmetic that would have been produced is not published, because publishing it would let a refused move circulate as a priced one.** What is published is the refusal, the named missing component, and what would reverse the verdict: a single observation of the component, in any state, on any occasion. A refusal that names its missing component is a scientific result; one that says the move is impossible is an opinion.

The closed lineage, read in the book's own register. Constructed, and not an observation of any lineage historical or living. A closed lineage in which the student's whole training consists of reproducing the master's framings exactly is `rho = 1`, hence `u = 0`, hence — by R8, which makes it worse rather than better — the same reading a perfectly opposed pair would receive.

```
   arrangement                    u      F      f (Hz)    c/eta     D      Q_inf
    ------------------------------------------------------------------------
------------------
   closed lineage, exact reproduction  0.000  0.000000  0.483426  2.250000 +6.492128 -
7.500000
   closed lineage, slight drift      0.050  0.438255  1.161430  2.250000 +5.227591 -
6.039150
   open school of six, u = 0.30      0.300  1.458615  8.938542  2.090909 +1.824415 -
3.091215
   open school of six, u = 0.50      0.500  1.871802  20.424737  2.090909 +0.632209 -
1.071189
    open school of six, u = 0.70       0.700  2.163323  36.590494   2.090909  -
0.208942  +0.354024
    ------------------------------------------------------------------------
------------------
   every row's c/eta is inside the ceiling ln 12 = 2.484907, so these are real
    failures and not ceiling breaches
```

The swing from the closed lineage to the open school at `u = 0.70` is 7490.44 cents and 6.701070 octaves of deficit. Openness is three levers at once: it raises u because independent practitioners frame differently, raises `eta` because a published result is stored order and an unpublished one is a memory, and lowers per-link `c` because a record replaces re-explanation. **And SPINE §3.6 binds the reading: every f in that table is a relationship frequency and says nothing whatever about the state frequencies of the people in it.**

D.6 Rejections inherited from Book Two, restated because they still bind

#	Claim	(-1) — precedent	Failing criterion	Status
B2-a	Astrology's orb as a conjugate-spread structure	the orb tradition	C2 FAILED in print, with Carlson, *Nature* 318 (1985) cited in the same passage as the definitive failed validity test	**REJECTED as a validity claim; the structural analogy contributes no number**
B2-b	64 codons to 64 hexagrams	the correspondence tradition	refused at Ch 67 §67.11	**REFUSED**
B2-c	The Heisenberg analogy as a source of numbers	quantum-mechanical framing of psychological uncertainty	every quantitative result in Ch 72 comes from an information-budget bound, the Gabor trade, or a Lyapunov horizon law — none from Heisenberg	**RETAINED AS STRUCTURE, REJECTED AS ARITHMETIC**
B2-d	Momentum descent as a formulary method	optimisation practice	failed to escape the worked landscape at every `beta` tested; the barrier is a wide plateau, not a spike	**EXPLICITLY EXCLUDED from the formulary with its failure stated. The exclusion is a feature**
B2-e	`r = 1` as an unqualified target	calibration practice	R9: headroom at the `D = P` fixed point is exactly zero by cancellation	**REJECTED AS A SIMPLE GOAL**
B2-f	Aggregating `r` across sites	reporting practice	R10: aggregate `D/P` overstated bankable input by 4.26x	**PROHIBITED. Distortion does not average**

Appendix E — The Honesty Appendix

This is the book's warrant. Everything else in the volume is an argument; this appendix is the accounting that says which arguments were paid for and in what currency. A reader who trusts nothing else here should be able to trust this.

The accounting has four drawers. **PROVED** means derived from stated premises, with the derivation in print and the arithmetic verified. **POSTULATED** means asserted as a design choice or a working assumption, badged in the sentence that states it, with the cost of the alternative computed where that was possible. **BORROWED** means taken from an established result elsewhere, cited, and used without re-derivation. **UNTESTED** means specified but not run — a claim with a falsification criterion attached and no data against it.

An item can sit in more than one drawer at different levels of description, and where it does, both placements are recorded. A theorem proved from a postulate is proved as a theorem and postulated as a system, and a reader is entitled to both facts.

E.1 The first drawer — PROVED

```
    the essence bridge       f = k e^(2F)                          Book Two Ch 30
    the surplus form           F = ln(1 + (N-1)u), unique given S3  Paper A
       the storage solution          Q_t exact, and Q_inf = (eta F -
c)/delta   Paper E Thm 3.1/3.5
   the keystone          f_min = k e^(2c/eta)            Ch 3, twelve parameter sets
    the octave form          octaves = 2.885390 (c/eta)          Ch 4 Thm 4.3
    the deficit identity       D = 2.885390(c/eta - F)
                      = -2.885390(delta/eta) Q_inf     Ch 4 Thm 4.5, 2.665e-15
    delta-blindness           D contains no delta                Ch 4 Thm 4.9
   the ceiling           c/eta <= ln N, f_min <= k N^2      Ch 4 Thm 4.6 / Paper E Cor 3.4
    the minimum category      N >= e^(c/eta)                     Ch 4 Cor 4.7
    the hold line            u* = (e^(c/eta)-1)/(N-1)            Ch 4 Cor 4.8
    the odd-part theorem       ceil(N/2) classes, verified N = 1..25   Ch 7
    the agreement identity   u = 4a(1-a), cos theta = 2a - 1     Ch 13 Thm 13.1
    R8                      opposition reads as agreement       Book Two Ch 46
    R15                   trust amplifies an existing sign    Book Two Ch 49/51
   the compound rule       f = k^2 G^4, and Thm 20.1        Book Two Ch 29, Ch 20
    Cor 20.2                  the two rules agree at no index      n = 1.438252
    Thm 22.2                    refinement cannot move Q_inf         Ch 22
    the error law             |Q_t - Q_inf| = (1-delta)^t |Q_0 - Q_inf|   Ch 22
    the Gabor bound          dt df >= 1/(4 pi)                classical Fourier
    the four design laws       N <= 25 at j = 6; cliffs at 17 and 41;
                         rate factor exactly 8; rectangular gate disqualified
   Finding 21.1            state and relationship frequency independent   Ch 21
   R23                LR = 1.000000 for testimonial evidence       Book Two Ch 80
```

Two of these are limitations rather than achievements.

The deficit is blind to decay, and it is the book's central instrument. Theorem 4.9 says `D` contains no `delta`. Worked on the carried dyad, moving `delta` from 0.09 to 0.05 takes `Q_inf` from `-5.011940` to `-9.021491` — nearly doubling the negative destination — while `D` stays at

+4.338420 to ten decimal places. The instrument this book exists to provide cannot see one of the four parameters that decide a relation's fate. The remedy is procedural and is stated everywhere the instrument appears: report D and delta together. A reading that gives only D is incomplete, and the incompleteness is not a rounding issue but a structural one.

A state's frequency and a relationship's frequency are independent, and neither may be inferred from the other. SPINE §3.6 records the verification: four constructed pairs at lattice positions spanning thousands of cents — indices [1,2] at 2400.00 cents apart, [3,6] at 2400.00, [5,10] at 2400.00, [7,11] at 1564.98 — all return u = 1.000000000000 and f_rel = 69.6133055503 Hz, to twelve decimal places. The reason is structural: u depends on the angle between state vectors, not on which indices are occupied. Re-indexing moves every state frequency and can leave the relationship reading untouched. A complete report therefore gives both numbers, and any chapter tempted to derive one from the other is making an error the arithmetic forbids.

E.2 The second drawer — POSTULATED

S3, the affine effective count, is a chosen design axiom and this entire book leans on it. The surplus form is uniquely characterised by four axioms: S1 geometric dependence, S2 zero at zero, S3 that g(u) = e^{F(u)} is affine in u, and S4 normalisation F(1) = ln N. Every step after S3 is forced. S3 itself is forced by nothing. It asserts that the *effective count*, not the surplus, is the quantity that behaves linearly in the geometry, and that is a modelling choice about what the natural unit of the system is.

The cost of a different choice is computed rather than waved at. F_alt(u) = u ln N satisfies S1, S2 and S4, violates only S3, and agrees at both endpoints — yet at u = 0.5 the two disagree by 33.62%, 1.871802 against 1.242453. S3 selects one curve from a family that agree at the ends and diverge by a third in the middle, and the middle is where every real case lives. Nothing here is safe from it: N_eff is defined by S3, the bridge identifies the essence index with N_eff, f = k N_eff^2 inherits it, and f_min is a statement about F = ln N_eff. Every chapter using the keystone is using S3.

The storage law's linear form is a modelling choice, not a discovery. Q_{t+1} = (1−delta)Q_t + eta S_t − C_t is linear in Q, with constant eta and constant delta. That is what makes the fixed point exist, makes the convergence geometric, and makes Q_inf computable in closed form. Real systems can have superlinear cost, Q-dependent conversion, or exogenous accretion, and where they do, the model does not apply and Chapter 3 §3.14 computes where it stops. The linear form is retained because it is the simplest form that supports the join, and its simplicity is the reason the join is checkable — not evidence that it is right.

```
    OTHER BADGED POSTULATES, WITH THEIR OWNERS

   Postulate R       eta = eta_max * rho, reciprocity as a multiplier  Book Two Ch 49/51
      Postulate  SO1            Rasch  instrumentation  makes  Delta  ratio-
scaled     Book Two Ch 54
   the belief postulate belief acts only through a coupling with real gain Book Two Ch 76
   the trait cut      a declared convention, not a natural kind       Ch 8 §8.6
    the twelve-state ladder itself: a constructed decomposition, defended
                      against four rivals and adopted, not discovered    Ch 5
   the ladder's ordinal-to-frequency mapping: defensible, not unique     Ch 5 §5.4
```

Belief acts only through a coupling with real gain, and where no channel exists nothing happens. **The book must never imply that wishing moves the fixed point.** Changing a destiny means changing `(eta, F, c, delta)` and nothing else — a definition, not a promise.

E.3 The third drawer — BORROWED

```
  Gabor's limit and Fourier duality
  Stevens' magnitude estimation, and its replication record
 Rasch measurement, as the route out of R1's demotion (123 items per term)
  Shewhart 1931 and Deming 1986, on staged process control
  Cepeda et al. 2006, on distributed practice
  Rosenthal & Fode 1963; Polanyi 1958; Collins 1985, on the operator
  Ostrom's design principles, mapped to parameters and priced
  Brooks' quadratic coordination cost
  Tenney's harmonic distance
Granovetter's threshold model, with mu* = 0.408522 and a 99.0% / 14.5% knife edge
Carlson, Nature 318 (1985), as the definitive failed astrological validity test
Church, Gao & Kosuri, Science 337:1628 (2012), on data in nucleotide sequence
Goldman et al., Nature 494:77 (2013); Erlich & Zielinski, Science 355:950 (2017)
 Zetsche et al., Cell 163:759 (2015); Fonfara et al., Nature 532:517 (2016)
  Qi et al., Cell 152:1173 (2013); Gilbert et al., Cell 159:647 (2014)
Patapoutian, Nobel 2021, on mechanically gated ion channels (PIEZO1/PIEZO2)
Tajik et al., Nature Materials 15:1287 (2016), on force reaching the nucleus
  Ibsen et al., Nature Communications 6:8264 (2015); Duque et al. (2022)
Takahashi & Yamanaka, Cell 126:663 (2006), Nobel 2012, on latent reprogramming
  Ripley, Compound of Alchymy (1471) -- borrowed as (-1) PRECEDENT only
 the Zosimos corpus; Newman & Principe 1998; Principe 2013 -- likewise (-1)
```

Every borrowing is used for what its source established and nothing further. The biological citations establish sequence operations and mechanical operations; they do not establish the link between them.

E.4 The fourth drawer — UNTESTED

```
the operationalisation of the twelve-state ladder: FVP-6 SPECIFIED, NOT RUN
    design: preregister rule set, ladder, elicitation script and analysis;
      fit on training episodes; FREEZE; report held-out performance
     kill condition: if counts under the frozen protocol fail to beat the
    persistence baseline on held-out prediction, the operationalisation is
      dead and the ladder is withdrawn

the acoustic -> c-factor causal claim: NOT CONFIRMED (R7). Any chapter invoking
     acoustic c-factor engineering carries that status.

the genomic audio protocol of Ch 26: a RESEARCH PROCEDURE, labelled as one.
     Nothing in it has been run on anything.

the decisive experiment of Ch 27: designed, preregistered in outline, not run.
```

```
every parameter estimate in this book: eta, c and delta have no instrument
   comparable to u's, and Protocols C4–C6 are estimation practice badged as
     practice.

the research programme's own odds: 21 preregistered kill conditions across five
   stages, four terminal; 35.8% completion probability; 55.3% chance of dying
     at or before cross-subject recovery; 8,554.8 researcher-hours, 5.35
     FTE-years, 71 months.
```

The last item is a result, not a caveat. A programme that computes and publishes its own probability of failure is more credible than one claiming certainty, and the coherence with R1 is no coincidence: the gate deciding the hard-science question is also the stage most likely to kill the programme.

E.5 The specific disclaimers, each stated in full

Essence frequencies are essence frequencies, not physical resonances. No person, tissue, cell or molecule is claimed to resonate at any value in this book. The frequencies represent a count and an angle; the hertz are a unit chosen so the representation can be compared with a lattice and, where a rendering is called for, heard. Nothing in the formalism asserts a physical oscillation and nothing in it would be evidence of one.

The scale asymmetry, stated as SPINE §1.1 fixes it. Two quantities, different scale status, and no blanket verdict for "the ladder" or "the method" is permitted. `n` is a count: true zero — zero components is no state — and equal units, hence ratio-scaled by construction, and `f = k n^2` is ratio-scaled with it. Neither inherits R1's demotion; they sidestep it. `w`, the intensities, are ordinal by default, and `u`, `F`, `f_rel` and the deficit inherit `w`'s status. So the state frequency is ratio-scaled and the relationship frequency is not, until the intensities are. R1's demotion binds `D/P` specifically — pending Rasch instrumentation at 123 items per term — and binds neither this index nor magnitude-estimated intensities. Two phrasings are forbidden because both are false: "the ladder is ordinal per R1", and "the method is ratio-scaled" without qualification.

And the ordinal reading is not merely weaker; it is unusable. An ordinal-preserving relabelling of the intensities moves u from 0.362115 to 0.525275, which is 0.887547 octaves — 1065.0568 cents, nearly an octave — from a change an ordinal scale says is no change at all. Magnitude estimation is therefore required rather than preferred, and the raw L2 norms are published so the free multiplier stays recoverable.

R23 stands, and its number is exact. The survivorship likelihood ratio for testimonial evidence is `1.000000` — not approximately, exactly. A thousand testimonials carry precisely the evidential weight of zero, because the selection guarantees you observe successes whether or not the method does anything. The companion result: a prediction specifying only a direction has `LR = 2.0000` against `11.3577` for one specifying a magnitude, so a belief predicting only a direction is not yet testable.

R11, stated exactly, because the easy version of it is false. The integrated effect of one large impulse and the same total input delivered periodically is identical to every digit — 433.3333 in all three regimes tested. That is LTI invariance and it is exact. Cadence wins by strict concavity (Theorem 59.1, which needs no ceiling at all) and by the surplus ceiling clipping the impulse —

90.44% of the nominal budget lost at N = 12 — and not by delivering more total input. It does not deliver more.

R15, stated exactly. Trust acts by reducing delta, and Q_inf = (eta F − c)/delta. When the numerator is negative, dividing by a smaller positive denominator makes the result more negative. Trust does not add order; it amplifies whatever sign the system already has. Fix the numerator first.

R8, and the identity that carries it into measurement. Because u = 1 − (x.y)^2 depends only on |rho|, perfect opposition yields u = 0 exactly as perfect agreement does. The agreement identity u = 4a(1−a) exhibits it directly: the channel is symmetric about a = 0.5 and returns u = 0 at both ends. The contrarian who inverts you carries the same information as the sycophant who echoes you — none — and manufactured opposition pays C and returns zero surplus.

Chapter 16's result about groups, stated before its qualifications. A single orthogonal pair attains u_bar = 1, F = ln 12 = 2.484907, f = 69.6133 Hz; six mutually orthogonal members attain exactly the same and pay 15 links instead of 1. In surplus terms a group buys nothing a well-chosen pair could not. The qualifications are real and are printed with it: orthogonality is an idealisation and achievable u_bar falls with roster size; the formalism has no term for redundancy against loss, for succession, or for the fact that a pair ends when one of its members does; eta and kappa are not roster-independent in practice; and the ceiling is on F, not on output. Groups buy breadth, not depth, and the formalism prices only the depth.

Chapter 17's commons, and Chapter 18's window. The commons is formally tunable — c/eta = 2.426471 sits inside ln 12 = 2.484907 by 0.058436 nats, or 0.1686 octaves — and practically unreachable, because the hold line demands u* = 0.938078 against a present u_bar of 0.018614, a factor of 50.4. No single lever closes a 6.4638-octave gap: u to 0.95 leaves −0.0332, and 0.95 is near-orthogonality; eta to 0.90 leaves +2.1075; kappa cut to 0.020 leaves +2.0085. Some systems cannot be tuned and must be restructured, and a book showing only fixable examples would be lying by selection. Just as awkward, the drop-one analysis improves the reading on *every* member of the healthy team — Q_inf rising by between 1.97 and 2.80 in all five rows — so a favourable drop-one row carries no information about the member it names. And Chapter 18's re-read window, bounded below by convergence and above by drift, can be computably empty: the tuned family's window is [13.51, 16.23] at 1% drift and empty at 2%, 3% and 5%. A margin thin enough to be unverifiable is a margin too thin.

The open problem, carried without softening. There is no established mechanism by which a cell demodulates sequence-specific information from an acoustic signal. The channel needs three things: D1 symbol separation, satisfiable in principle and its engineering exists; D2 order memory, for which there is no identified mechanism — this is the heart of the gap, since a permutation-invariant response is not a weak version of a sequence-specific one but a different quantity; and D3 mapping to a locus, for which there is likewise none. Eleven upstream rows are established and one is open. Reporting only the eleven is advocacy; reporting only the one is a different dishonesty, implying the whole edifice is speculation when most of it is textbook. Chapter 27 is why the rest of the book can be trusted.

The remaining absolute constraints. No clinical, medical, diagnostic or therapeutic claim is made anywhere in this book; the twelve states are not diagnoses and the readings are not assessments. No claim is made that any DNA is edited. No capability is conferred on anyone by anything here. No subliminals, ever — nothing masked, nothing below threshold, nothing concealed; open-label placebo works, so disclosure costs nothing that matters. No intervention may

remove a person's ability to decline, and consent is structural rather than advisory. All worked cases in all thirty chapters are constructed and are labelled as such, and no invented case is presented anywhere as an observation.

E.6 What would change these verdicts

An honesty appendix that could not be moved by evidence would be a creed. Each drawer has an exit. PROVED is exited by an error in a derivation or a failed verification, and every verification here states its residual so a reader knows what tolerance was claimed. POSTULATED is exited by exhibiting a system in which the postulate fails — for S3, one whose effective count is not affine in the geometry, in which case the whole functional form changes and every number in this volume moves with it. BORROWED is exited by retraction or supersession. UNTESTED is exited by running the test, and every item in it carries a kill condition stated in advance rather than a hope.

The open problem is exited by one experiment, described in Chapter 27, with both outcomes written before it is run. Until then the position is the one stated throughout: the architecture upstream is real, the architecture downstream is real, and the link between them does not exist yet.

Appendix F — Correspondence to Books One and Two

Three books, one instrument, built in three stages. Book One taught the operator to read the comfort gradient. Book Two taught the engineer to build the circumstance and laid down both halves of one equation without multiplying them. This volume performs the multiplication, and this appendix says item by item which part of the earlier work each part of this one extends, what the extension adds, and what it costs.

Two conventions govern the tables. Where Book One's nodes are named the name is used; where they are not, the node is referred to by index only, because inventing names would violate Book One's own CANON §5.3. And where an extension costs something the cost is printed in the same row as the gain.

F.1 Book One's twelve nodes

The nodes sit on the same twelve-index ladder this book uses, so each has a frequency and a pitch class under Rule 1. Those values are Book Two Chapter 29's and agree digit for digit with B.1.

node	f = k n^2 (Hz)	class	what Book One taught	extended by	what the extension adds	what it costs
01	0.483426	1	the ground: undifferentiated activation	Ch 5, Ch 12	`n = 1` is the count's first rung, and `u = 0` gives the ground essence `f = k` — the same pitch from two directions	the reading at `f = k` is *no relationship*, not a low one, and readers are told so every time
02 GRADIENT	1.933703	1	read the desire line; move along the gradient of comfort	Ch 11, Ch 18	`grad h = −grad C` (R19) makes a habit survey a headroom measurement; Ch 18 turns the gradient into a numbered intervention procedure	the consent axiom: no intervention may remove the ability to decline, which rules out the cheapest interventions
03 HORIZON	4.350832	3	the many windows of the one tower — several framings of one situation	Ch 9, Ch 10	the framings become a chord with computable intervals, and its width is the internal surplus `F_int`	the one-hot trap: framings without overlapping support score `u = 1` for free and mean nothing by it

node	f = k n^2 (Hz)	class	what Book One taught	extended by	what the extension adds	what it costs
04 WAVE	7.734812	1	the future as a spread rather than a point	Ch 26, Book Two Ch 70-72	the spread is priced by Gabor: `dt df >= 1/(4 pi)`, an exact classical bound	the quantum analogy contributes no number; a badged analogy may organise, it may not compute
05	12.085643	5	set the idea of greater comfort where the self will pass; lower the cost of the destination	Ch 4 lever L3, Ch 18	cutting `c` is now one of exactly four levers, with its partial `dD/dc = +2.885390/week`	R20: lowering cost only at the origin shortens the wait from 32.0 to 9.9 weeks but leaves a 37% smaller permanent fund
06	17.403326	3	the minimal reversible probe; the veil that measures the measurer	Ch 18, Ch 23	reversibility is classified formally, and the operator's own state is computed into `u` — mirroring costs 536.4074 cents	an observation taken by a reactive instrument is not free, and the price is now a number rather than a warning

node	f = k n^2 (Hz)	class	what Book One taught	extended by	what the extension adds	what it costs
07 COL-LAPSE	23.687861	7	the moment the many become one	Ch 10, Ch 14	consonance and redundancy are proved identical, so collapse into agreement is collapse into redundancy	an entirely consonant self is an entirely redundant one, and nothing about distress follows — Ch 10 §10.6 states the firewall formally
08 ECHO	30.939247	1	the world's answer, read over what was sent	Ch 13 Route C	the return difference becomes error decorrelation, `u = 1 − rho^2` — the only route that sees being wrong in different ways	it needs ground truth, unavailable for most of what two people actually disagree about
09 RECURSE	39.157484	9	the loop that returns changed	Ch 22	the transient `(1−delta)^t Q_0` is exactly the error law, and the three constants of arrival depend on `delta` alone	refinement cannot move the destination (Thm 22.2): the seventh distillation reaches nowhere the first was not already headed

node	f = k n^2 (Hz)	class	what Book One taught	extended by	what the extension adds	what it costs
10 FIXED-POINT	48.342573	5	where the loop arrives	**Ch 28**, and Ch 4	**Node 10 FIXED-POINT = Paper E Theorem 3.5 = this book's Chapter 28.** `Q_inf = (eta F − c)/delta` is destiny defined precisely; and by Thm 4.5 it is the octave deficit in different units	the definition is not a promise: changing a destiny means changing `(eta, F, c, delta)` and nothing else, and belief acts only through a coupling with real gain
11 SEAL	58.494514	11	when not to intervene	Ch 18 §18.13	a computable stop rule: ceiling test, empty re-read window, unreachable hold line each halt the procedure	the halt is a finding, not a failure; Ch 19's schedule halts at gate 4 for the same class of reason

node	f = k n^2 (Hz)	class	what Book One taught	extended by	what the extension adds	what it costs
12	69.613306	3	the committed state, the top of the ladder	Ch 4, Ch 16	`k N^2 = 69.6133` Hz is now the system ceiling, reachable by an orthogonal pair and by nothing larger	above `c/eta = ln 12 = 2.484907` no arrangement of the twelve is sustainable, and the only remedies leave the ladder

Node 10's row is the load-bearing one and is stated once more in isolation, because three books converge on it: **Book One's tenth node, Paper E's Theorem 3.5, and this volume's Chapter 28 are the same object under three descriptions.** Book One named it and could not compute it; Paper E computed it and did not connect it to pitch; Chapter 4's deficit identity connects it, and Chapter 28 states what the connection means — a pitch reading and a destiny reading are the same reading.

One structural observation about the ordering, offered as arithmetic and not as evidence for it: indices 10 and 5 share pitch class 5 and sit exactly two octaves apart, since `10 = 2 * 5`, placing the fixed point in octave relation to an earlier node — the relationship an accumulator has with its own transient. Had the nodes been numbered differently the relation would not hold, and the sensitivity test says so: swapping 09 and 10 displaces a pitch class by 364.81 cents and is load-bearing, while swapping 03 and 12 displaces nothing, both being class 3.

F.2 Book Two's parts

Book Two part	subject	extended by	what the extension adds	what it costs
I — The Circuit	recursion as a circuit, not a roadblock	Ch 2, Ch 22	the storage law read as a circuit with a computable settling time	the linear form is a modelling choice; Ch 3 §3.14 computes where it stops

Book Two part	subject	extended by	what the extension adds	what it costs
II — The Geometry of Difference	`u`, `N_eff`, the participation ratio	Ch 12, Ch 13, Ch 16	three measured routes to `u` between persons, and the aggregation gap at group scope	the gap is larger than Book Two's: 0.394666 octaves on the constructed family
III — Engineering Space	the room as a circuit element	not extended	—	this volume is about persons and bonds; the spatial scope stays where Book Two left it
IV — The Sociology	collective intelligence as an engineerable quantity	Ch 16, Ch 17	Brooks priced in octaves, the roster ceiling, three structures read end to end	an orthogonal pair already reaches `ln N`, so the group's case rests on grounds the formalism does not price
V — Tradecraft	the survey, the baselines, the traps	Appendix C, Protocol C8	the four-baseline suite carried with all three measured holes attached	design controls are mandatory and not substitutable
VI — The Essence Law	`f = k n^2`, folding, the class rule	Ch 5, Ch 7, Ch 20	the law applied to psychological categories via a count-based index; the compound rule as coagula	the index sidesteps R1 only because it is a count; the intensities do not, and must be magnitude-estimated
VII — The Perceptual Lens	the Moment equation `M = (D/P)E`	not extended quantitatively	—	R1's demotion: `D/P` is ordinal pending Rasch instrumentation at 123 items per term, a cost this book does not spend

Book Two part	subject	extended by	what the extension adds	what it costs
VIII — The Personal Formulary	the self as storage	Ch 8, Ch 11	traits as low-`delta` states, and a complete personal worksheet	`delta` is not a lever on the deficit, so the personal reading needs both numbers too
IX — The Interpersonal Formulary	the dyad, trust, reciprocity, Ostrom	Ch 12-18	the dyad given a frequency, a threshold, and an ordering rule	R15: the cheapest-looking intervention accelerates the failure when the numerator is negative
X — Validation	the Formula Validation Protocol	Ch 6 §6.14, Appendix E	`f_min` and the index run through the gates, FVP-6 marked SPECIFIED, NOT RUN	the protocol demotes its own author's formulas, and the demotions are printed
XI — The Old Notation	correspondence tables, gematria	Ch 19-24	the twelve gates audited operation by operation; Finding 19.2	seven rejections across Ch 23 and 24, each with its failing criterion named
XII — The Concordance Method	the two columns	Appendix D	this volume's register, every rejection printed at the length of an acceptance	precedent is never promoted, and R23 explains why in one number
XIII — Telecommunications to the Cell	the cell as receiver	Ch 25, Ch 26	the protocol restated for psychological content, with the no-subliminals constraint absolute	the open problem, carried whole and unsoftened
XIV — The Uncertainty	Gabor, and Heisenberg correctly stated	Ch 26, Appendix B.6	the four design laws applied: capacity 25, cliffs at 17 and 41, rate factor 8	the rectangular gate is disqualified; implementations need a windowed envelope

Book Two part	subject	extended by	what the extension adds	what it costs
XV — Belief and Mechanism	the belief postulate	Ch 29	the postulate applied to the four levers: belief acts only through a coupling with real gain	R23 stands, and testimonial evidence has LR = `1.000000`
XVI — Rewriting the Narrative	narrative as regulatory input	Ch 25	determination by choice, proved: a corpus closed under negation has resultant exactly zero	a contradictory corpus cannot command — the content-layer protection, and it is structural
XVII — Synthesis	the three instruments, the complete concordance	Ch 30, Appendices D and F	the arc across three books, closed on the open problem rather than a triumph	R16: the correspondence is of dynamics and geometry, not of agency; nine disanalogies named

F.3 What this volume adds that neither predecessor had

Stated tightly, because the temptation at the end of a correspondence table is to claim the whole inheritance as new. The join itself, `f_min = k e^(2c/eta)`. The octave deficit as a computable reading, with the proof that it is blind to `delta`. The count-based index, which escapes R1's demotion by construction and states its own exposure in the same breath. The magnitude-estimation requirement, with the 1065-cent demonstration that makes it a requirement. The proof that state and relationship frequency are independent readings. And the audit of the old corpus on its own material, of which Finding 19.2 is the sharpest instance.

What it does not add is a mechanism. That is Chapter 27's subject and Appendix E's fourth drawer, and the series ends where it began: the archive is real, the regulatory layer is real, and the route from an acoustic symbol stream to either does not exist yet.